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Examiners' report

FURTHER MATHEMATICS A

H245
For first teaching in 2017

Y541/01 Summer 2019 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y541 series overview

This paper (along with Y540) assesses the compulsory core content of the new (from 2017) A Level Further Mathematics A qualification. Questions in each paper can assess any part of the core specification. This is the first year in which these papers have been sat.

Candidates who did well on this paper generally were familiar with the whole breadth of the specification, using standard techniques correctly. They used the required prior knowledge effectively, for instance in questions involving calculus. They selected efficient methods to problem solve and communicated their arguments efficiently.

Key point call out

Centres may find it useful to work at developing candidates' ability to communicate effectively when writing extended mathematical arguments. Two particular areas that allow strong candidates to do well are effective use of equals signs in an argument and control of lengthy algebraic expressions with due care over sign changes.

Candidates who did less well on this paper generally demonstrated unfamiliarity with some topics, for example appearing unaware of the standard matrix for a shear, or with the process used to find a Maclaurin expansion. Their mathematical arguments lacked structure, omitted key steps or demonstrated conceptual misconceptions.

Most candidates were able to complete the paper in the time available, although there was some evidence that a few had not left themselves enough time with the final questions.

Question 1 (a)

1 In this question you must show detailed reasoning.

(a) By using partial fractions show that $\sum_{r=1}^{n} \frac{1}{r^2+3r+2} = \frac{1}{2} - \frac{1}{n+2}$. **[5]**

Almost all candidates were successful in forming the partial fractions and most recognised the need to use the method of differences. While many clear answers were seen, there were two common errors.

- Not showing sufficient terms to demonstrate cancellation properly.
- Not equating the two sides of the identity.

A few candidates took a perfectly acceptable algebraic approach to get an interim expression, such as

$$\sum_{r=1}^{n} \frac{1}{r+1} - \sum_{r=2}^{n+1} \frac{1}{r+1}.$$

Question 1 (b)

(b) Hence determine the value of $\sum_{r=1}^{\infty} \frac{1}{r^2+3r+2}$. **[2]**

Although candidates generally scored well on this question, a lot of conceptual misunderstandings were in evidence (which would result in greater problems in Question 3). Many candidates seemed confused about the notion of limit and statements using infinity as a number were common, as were statements such as

$$\sum^{\infty} \cdots \to \frac{1}{2}$$

Question 2 (a)

2 (a) A plane Π has the equation $\mathbf{r}.\begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} = 15$. C is the point $(4, -5, 1)$.

Find the shortest distance between Π and C. **[3]**

The formula for shortest distance from a point to a plane is given in the formulae booklet and most candidates successfully used it, however a small number used the wrong vector for the denominator. Some candidates selected alternative valid methods. The most common of these was to substitute into the equation of the plane a general point on the normal from C; although this generally proved successful, it generally took more time.

Question 2 (b)

(b) Lines l_1 and l_2 have the following equations.

$$l_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

Find, in exact form, the distance between l_1 and l_2. **[5]**

To find the distance between parallel lines is an explicitly stated part of the specification, yet many candidates appeared unfamiliar with the topic. A significant number started by attempting to use the formula for distance between skew lines. If they were successful in performing the cross-product they usually realised that the lines were parallel, but quite a few candidates then did not know how to proceed. Some simply found a vector from one line to the other and reasoned that its magnitude must be the required distance, even though they had made no attempt to make sure that it was perpendicular to the lines. Others formed planes and found the distance between them, but these were often the wrong planes.

Key point call out

Candidates, who cannot see their way into a problem should be encouraged to, where relevant, draw a sketch. In this question, most candidates found the vector joining $\begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$ to $\begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}$. Those who then drew a sketch showing two parallel lines and this vector could easily see that they could use a right angled triangle to find the needed distance.

There were nonetheless a number of different successful ways found to reach the solution. Where candidates did identify a correct approach to the problem they generally came up with a good response, barring arithmetic errors. The main problem that these candidates had was ensuring that their answer was an exact value (as was requested). Some candidates appeared unable to convert cosine to sine and presented a rounded answer. Others presented their answer using the form $\sin(\cos^{-1}p)$, which was not acceptable for a final answer.

Question 3

3 In this question you must show detailed reasoning.

Show that $\int_5^\infty (x-1)^{-\frac{3}{2}}\,dx = 1$. **[5]**

This question requested detailed reasoning. Good responses to this problem were elegant and showed a real understanding of the process. Usually these candidates could string together their whole argument via a series of equalities, starting with $\int_5^\infty (x\text{-}1)^{-\frac{3}{2}}dx = \lim_{n\to\infty} \int_5^n (x\text{-}1)^{-\frac{3}{2}}dx$. They then justified their evaluation of the limit with an aside on the right hand side of their page at the relevant place in the argument.

Many candidates, however, simply did not approach this problem with sufficient rigour, for instance

- treating infinity as a number (a common starting point for candidates was 'Let $a = \infty$' which immediately cut down the number of marks available), or
- not presenting the full argument or not even mentioning the entity whose value was supposed to be being shown to be equal to 1, or
- using poor, confusing or ambiguous notation.

As mentioned in Q1(b) there seems to be quite a lot of confusion around the whole issue of limits. For example, candidates should realise that a limit is a value and so cannot, in itself, tend to anything.

Exemplar 1

$$\int_5^\infty (x-1)^{-\frac{3}{2}}\,dx = \left[-2(x-1)^{-\frac{1}{2}}\right]_5^\infty$$

$$\left(-2(\infty-1)^{-\frac{1}{2}}\right) - \left(-2(5-1)^{-\frac{1}{2}}\right)$$

$$-2(\infty)^{-\frac{1}{2}} + 2(5-1)^{-\frac{1}{2}}$$

$$-2\left(\frac{1}{\sqrt{\infty}}\right) + 2\left(\frac{1}{\sqrt{4}}\right)$$

$$-\frac{2}{\infty} + \frac{2}{2}$$

Because this candidate doesn't use a limiting process applied to a proper integral, they cannot access most of the marks available.

Question 4 (a)

4 A 2-D transformation T is a shear which leaves the y-axis invariant and which transforms the object point (2, 1) to the image point (2, 9). **A** is the matrix which represents the transformation T.

(a) Find **A**. **[3]**

Most candidates successfully found **A**. Many benefited from the ability to recall the general form of the matrix representing a shear, *y*-axis invariant. They used this to find the unknown element by correctly multiplying their matrix by the object vector and equating to the image vector. Candidates who had not learnt the shear matrix forms often started from a more general 2x2 matrix with two or four unknown elements (depending on their conceptual grasp of invariance). Frequently those trying to determine four unknowns were unsuccessful. Other errors sometimes observed were attempts to right-multiply by **A** or consideration of the stretch matrix with zeros in the leading diagonal.

Question 4 (b)

(b) By considering the determinant of **A**, explain why the area of a shape is invariant under T. **[2]**

For candidates that had answered part (a), finding the determinant generally presented no problems. To gain the second mark here, candidates needed to explain that the (absolute value of) the determinant gives the **area** scale factor of the transformation. Without the word 'area', the candidate could be mistakenly talking about a length scale factor (which, of course, also yields no change in area).

Question 5 (a)

5 A particle of mass 2 kg moves along the x-axis. At time t seconds the velocity of the particle is $v\,\text{ms}^{-1}$.

The particle is subject to two forces.

- One acts in the positive x-direction with magnitude $\frac{1}{2}t$ N.
- One acts in the negative x-direction with magnitude v N.

(a) Show that the motion of the particle can be modelled by the differential equation

$$\frac{dv}{dt} + \frac{1}{2}v = \frac{1}{4}t.$$ **[1]**

In order to show how to derive the given differential equation, candidates were required to start by quoting $F = ma$ before using it.

A small number of candidates' working showed a conceptual misunderstanding, from seemingly believing that there could be two accelerations involved.

Question 5 (b)

The particle is at rest when $t = 0$.

(b) Find v in terms of t. **[5]**

Most candidates attempted to use an integrating factor (IF) and could usually find the correct one.

Marks were lost due to

- not multiplying both sides by the IF,
- an inability to integrate $te^{t/2}$ by parts,
- poor handling of the constant of integration, often leading to the erroneous form $v = f(x) + c$.

These problems did not present themselves to candidates who solved the equation by finding the complementary function (CF) and a particular integral (PI). Some who took this equally valid approach did not however know the most efficient trial function to use in order to discover their PI.

Question 5 (c)

(c) Find the velocity of the particle when $t = 2$. **[1]**

Virtually all the candidates who obtained the correct equation for v in part (a) were able to find the value of the velocity at $t = 2$, mostly preferring the exact value e^{-1}.

Question 5 (d)

When $t = 2$ the force acting in the **positive** x-direction is replaced by a constant force of magnitude $\frac{1}{2}$N in the same direction.

(d) Refine the differential equation given in part **(a)** to model the motion for $t \geqslant 2$. **[1]**

Most candidates derived the new equation of motion successfully. A few candidates however read the unit symbol for Newton in $\frac{1}{2}$N as a constant and inserted $\frac{N}{4}$ as the constant term in their differential equation. Had they followed the example given in part (a), they should have been able to correct their error.

Question 5 (e)

(e) Use the refined model from part **(d)** to find an exact expression for v in terms of t for $t \geqslant 2$. **[3]**

Most candidates did not take advantage of their prior work but started to solve the refined differential equation from scratch. The most efficient course of action was to recognise the effect of an unchanged term in v. A significant number of candidates opted for the separation of variables method, which led to a more complicated procedure when finding v explicitly. Some also applied the incorrect boundary condition at $t = 0$ when attempting to find the new arbitrary constant.

Question 6 (a)

6 A is a fixed point on a smooth horizontal surface. A particle P is initially held at A and released from rest.

It subsequently performs simple harmonic motion in a straight line on the surface. After its release it is next at rest after 0.2 seconds at point B whose displacement is 0.2 m from A. The point M is halfway between A and B.

The displacement of P from M at time t seconds after release is denoted by x m.

(a) On the axes provided in the Printed Answer Booklet, sketch a graph of x against t for $0 \leqslant t \leqslant 0.4$. **[4]**

In answering this question a cosine graph for $x(t)$ was often (correctly) selected, although the sine graph was seen. Most candidates did show a single cycle of their wave function with the correct period 0.4s, however very few candidates understood that the displacement at $t = 0$ was negative and draw the correct 'inverted' cosine graph. While the lack of 'inversion' was not penalised at this stage, it did lead to an incorrect value for the displacement in part (b).

Some candidates gave the amplitude as 0.2m instead of 0.1m and many neglected to label the intersections with the t axis at 0.1s and 0.3s.

Question 6 (b)

(b) Find the displacement of P from M at 0.75 seconds after release. **[2]**

Many candidates were able to find $\omega = 5\pi$, usually gaining one mark for the absolute displacement. A few started from the general expression $a\cos(\omega t) + b\sin(\omega t)$ for $x(t)$, despite having drawn a cosine graph in part (a) and these usually did not arrive at the conclusion that $b = 0$.

Question 7 (a)

7 In an Argand diagram the points representing the numbers $2 + 3\mathrm{i}$ and $1 - \mathrm{i}$ are two adjacent vertices of a square, S.

(a) Find the area of S. **[3]**

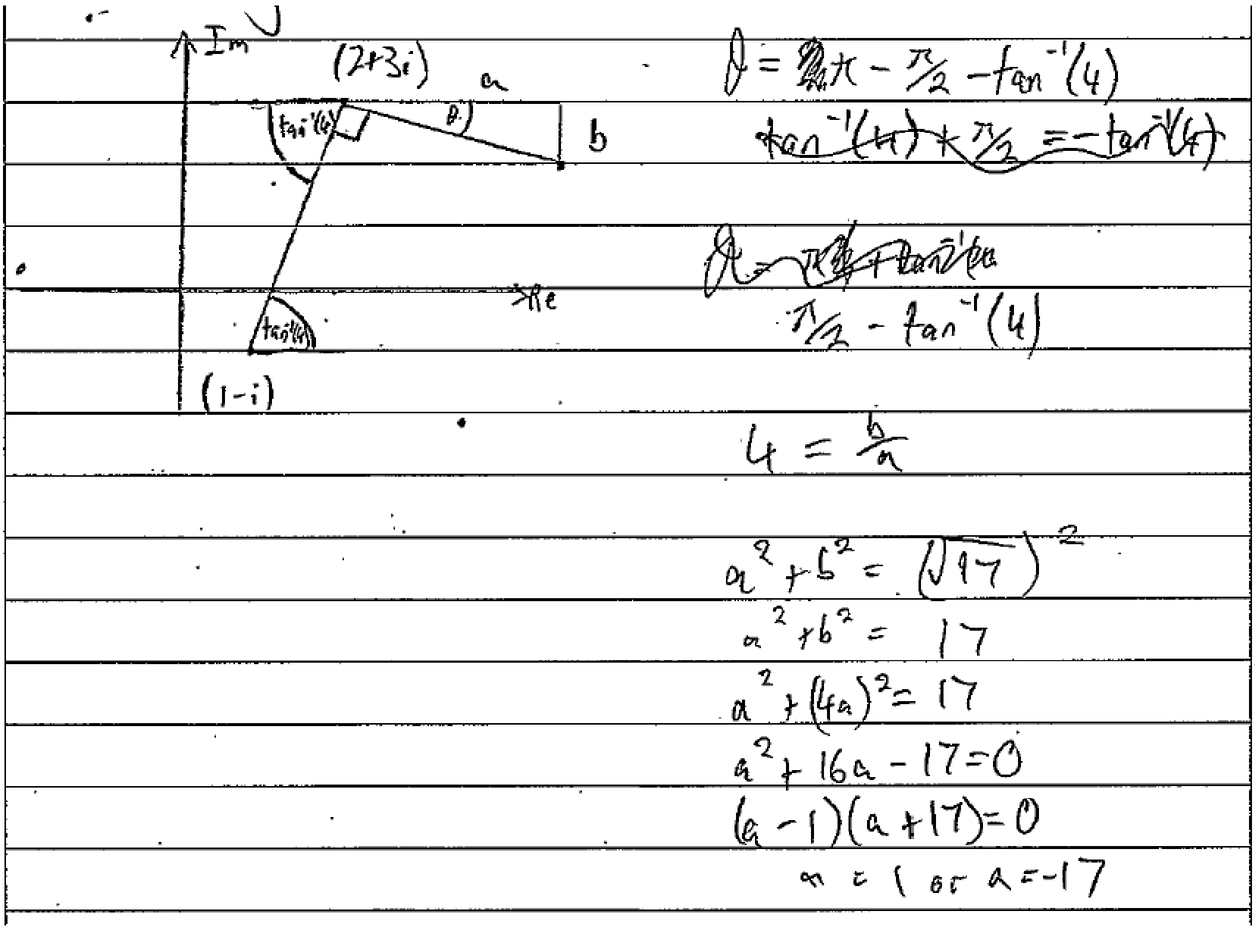
This part was usually done correctly. Most candidates subtracted the two numbers, occasionally with errors and used the correct method to find the distance. Complex number notation was often replaced by vector notation and/or a diagram. The great majority of candidates then squared their distance to find the area, although a small number thought that the given points were opposite, rather than adjacent, and hence arrived at half the correct answer.

Question 7 (b)

(b) Find all the possible pairs of numbers represented by the other two vertices of *S*. **[4]**

There were many correct answers to this part, although most candidates proceeded geometrically (either in vector notation or diagrammatically) to find the perpendicular vector/s, rather than multiplying 1 + 4i by (±)i. Many candidates then translated and found the correct vertices, although some did leave them in vector notation, rather than giving the (complex) numbers. The majority, but by no means all, showed the correct pairings for the numbers. A few candidates attempted the circuitous route of finding the Cartesian equation of one or both of the perpendicular sides and considering the points on such at distance $\sqrt{17}$ from the given points. This rarely met with success, with candidates either giving up or becoming embroiled in miscalculations.

Exemplar 2



Here is an example of a candidate who tries to find the translation vector via simultaneous equations using gradient and distance. They get the gradient wrong and neglect to fully square (4*a*) resulting in the breakdown of the method. Selecting efficient methods is one key to success in these papers.

Question 8 (a)

8 In this question you must show detailed reasoning.

(a) By writing $\sin\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$ show that

$$\sin^6\theta = \tfrac{1}{32}(10 - 15\cos 2\theta + 6\cos 4\theta - \cos 6\theta).$$ **[5]**

A considerable number of candidates derived this correctly, although sometimes losing a mark through omitting sufficient detailed reasoning. This usually happened where candidates did not explicitly group corresponding positive and negative exponentials together before converting to the cosine forms. One surprisingly common error was to quote the complex exponential expression for $\sin\theta$ with 2 in the denominator, rather than $2i$. This is such a fundamental mistake that it cost candidates, but they could go on to pick up a method mark were they to then binomially expand $(e^{i\theta} - e^{-i\theta})^6$. The standard abbreviation 'z' was often used in place of $e^{i\theta}$, which was acceptable and efficient if defined as such, but penalised otherwise. A small number of candidates dispensed with the binomial theorem and attempted to expand $(e^{i\theta} - e^{-i\theta})^6$ by repeated multiplication, usually incorrectly. The binomial expansion, along with other formulae in the formula booklet, can be used without justification even when detailed reasoning is requested. Where candidates expanded $(\cos\theta + i\sin\theta)^6$, they were unable to make progress deemed worthy of marks.

Exemplar 3

$$2i\sin\theta = e^{i\theta} - e^{-i\theta} \qquad \cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$$

$$\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$

$$\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)^6 = -\frac{1}{64}\left(e^{i\theta}-e^{-i\theta}\right)^6$$

$$= -\frac{1}{64}\Big(e^{6i\theta} + 6e^{5i\theta}(-e^{-i\theta}) + 15e^{4i\theta}e^{-2i\theta} + 20e^{3i\theta}(-e^{-3i\theta}) + 15e^{2i\theta}(e^{-4i\theta}) + 6e^{i\theta}(-e^{-5i\theta}) + e^{-6i\theta}\Big)$$

$$= -\frac{1}{64}\left(e^{6i\theta} - 6e^{4i\theta} + 15e^{2i\theta} - 20 + 15e^{-2i\theta} - 6e^{-4i\theta} + e^{-6i\theta}\right)$$

$$= -\frac{1}{32}\left(\frac{e^{6i\theta}+e^{-6i\theta}}{2} + \frac{15e^{2i\theta}+15e^{-2i\theta}}{2} - \frac{(6e^{4i\theta}+6e^{-4i\theta})}{2} - \frac{20}{2}\right)$$

$$= -\frac{1}{32}\left(\cos 6\theta + 15\cos 2\theta - 6\cos 4\theta - 10\right)$$

$$= \frac{1}{32}\left(10 - 15\cos 2\theta + 6\cos 4\theta - \cos 6\theta\right)$$

In this example we see what is otherwise a clear argument marred by the absence of a key element – namely that they never equate $\sin^6\theta$ to their solution. Demonstrations and proofs must explicitly show that what is posited has been proved.

Question 8 (b)

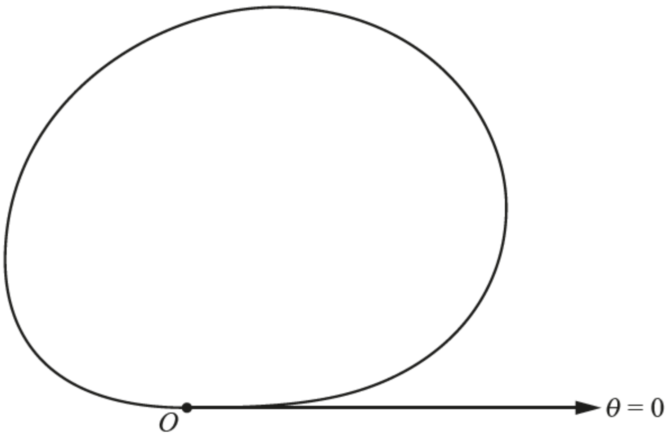
(b) Hence show that $\sin\frac{1}{8}\pi = \frac{1}{2}\sqrt[6]{20-14\sqrt{2}}$. **[3]**

This question was well answered, with most candidates substituting $\theta = \frac{\pi}{8}$ into the formula and explicitly evaluating the cosines, although sometimes with a sign error in the $\cos 6\theta$ term. This being a detailed reasoning question, candidates had to show sufficient working to convince that they had indeed evaluated the separate terms and not simply worked back from the given answer. Likewise, some intermediate step, such as writing $\frac{1}{32}$ as $\frac{2}{64}$, was required to justify the factor of $\frac{1}{2}$ in the final answer.

Question 9 (a)

9 In this question you must show detailed reasoning.

The diagram below shows the curve $r = \sqrt{\sin\theta}\,e^{\frac{1}{3}\cos\theta}$ for $0 \leqslant \theta \leqslant \pi$.



(a) Find the exact area enclosed by the curve. **[4]**

Where an integrand is the product of two functions, there are three techniques available to candidates - inspection, substitution and parts. Candidates at this level should be encouraged to develop their 'inspection' skills sufficiently to recognise this integrand to within a constant term. The purpose of the other two techniques is to produce an integrand that has become simpler to integrate. Where candidates selected 'parts' (which a lot did) they produced a more complicated integrand; at this point they should reconsider their approach. Flexibility of approach to problem-solving is an important skill to develop.

Question 9 (b)

(b) Show that the greatest value of r on the curve is $\sqrt{\frac{\sqrt{3}}{2}e^{\frac{1}{6}}}$. **[7]**

Key point call out

Centres should emphasise the key role that calculus plays in problem-solving. Whenever a (non-quadratic) problem involves optimisation it should become second nature for candidates to expect to differentiate.

Centres should also encourage candidates to take extra care when dealing with the initial stages of question with high mark allocations.

Some candidates did not realise that they needed to find $\frac{dr}{d\theta}$. Others made errors when finding it, even when writing out the product rule term-by-term. Where candidates progressed past these initial issues, they generally produced a coherent mathematical argument.

Exemplar 4

$r = \sqrt{\sin\theta}\, e^{\frac{1}{3}\cos\theta} = (\sin\theta)^{\frac{1}{2}} e^{\frac{1}{3}\cos\theta}$

$\frac{dr}{d\theta} = \frac{1}{2}\cos\theta(\sin\theta)^{-\frac{1}{2}} e^{\frac{1}{3}\cos\theta} - \frac{1}{3}\sin\theta(\sin\theta)^{\frac{1}{2}}$

Centres should stress to candidates that an early error in a major problem can lead to most marks becoming inaccessible. Here, the candidate omits one key term ($e^{\frac{1}{3}\cos\theta}$) while differentiating a product. The resultant expression becomes impossible to meaningfully factorise. Consequently, the candidate cannot solve $\frac{dr}{d\theta} = 0$, meaning that they gain only 1 of the 7 marks available.

Question 10 (a)

10 (a) Use differentiation to find the first two non-zero terms of the Maclaurin expansion of $\ln\left(\frac{1}{2} + \cos x\right)$. **[4]**

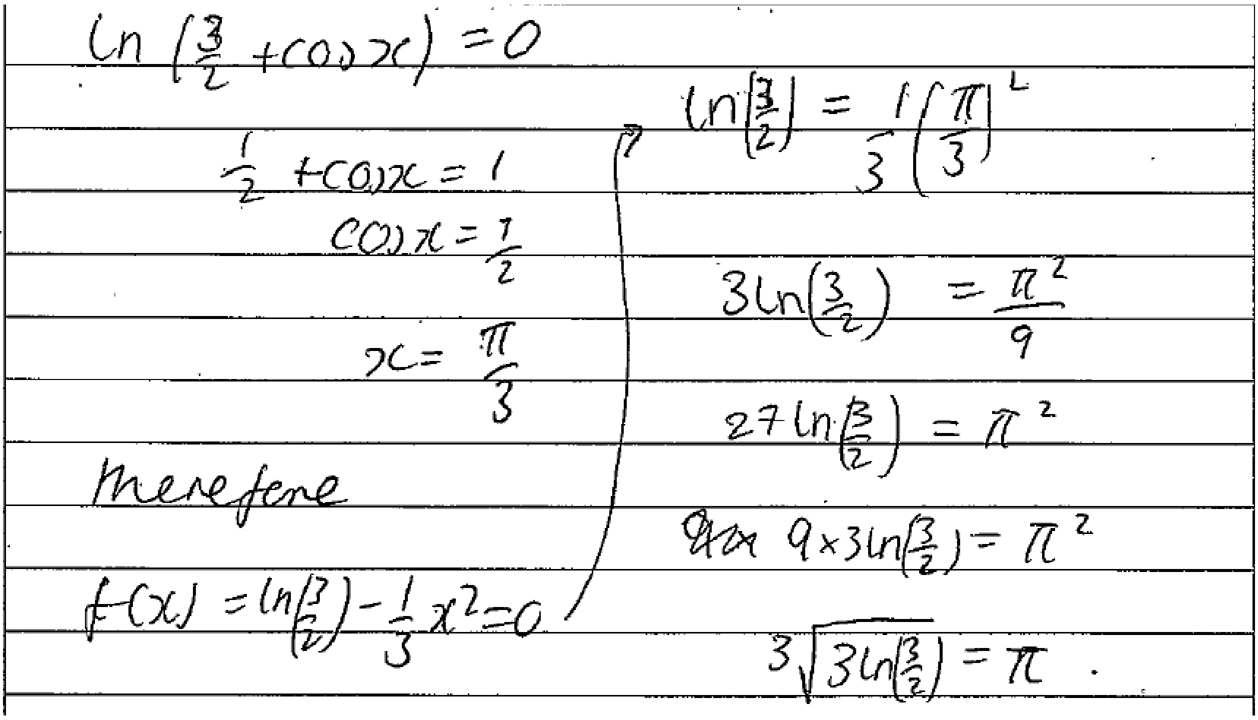
Most candidates correctly found f(0), however sign errors often crept into their expressions for f'(x) or f''(x). A few candidates ignored the requirement to 'use differentiation' and attempted (usually unsuccessfully) to find the terms via the use of standard formulae booklet expansions. Where they did find the correct expansion they were given partial credit.

Question 10 (b)

(b) By considering the root of the equation $\ln\left(\frac{1}{2}+\cos x\right)=0$ deduce that $\pi \approx 3\sqrt{3\ln\left(\frac{3}{2}\right)}$. **[3]**

Some candidates considered only the original equation and others only the modified equation with Maclaurin expansion. Those who considered both were usually able to put the two together for full method marks. Some otherwise good answers were marred by totally ignoring the 'approximately equal' sign.

Exemplar 5



This candidate appears unaware of the significance of the approximately equals sign in the question and/or of the approximation involved in using only the first few terms of a Maclaurin expansion.

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