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Examiners' report

FURTHER MATHEMATICS A

H245

For first teaching in 2017

Y541/01 Summer 2022 series

Version 1

Contents

Introduction

Our examiners’ reports are produced to offer constructive feedback on candidates’ performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates’ performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers are also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

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Paper Y541/01 series overview

This paper, along with Y540, assesses the compulsory core content of the A Level Further Mathematics A – H245 qualification. Questions in each paper can assess any part of the core specification. Due to Covid-19 this is only the second year in which these papers have been sat by a full cohort. There was some evidence that many candidates, while confident with standard techniques, had perhaps not been sufficiently exposed to minor points from the specification and lacked confidence in communication and problem solving.

Most candidates appeared to be able to complete the paper in the time available.

Candidates who did well on this paper generally did the following:	Candidates who did less well on this paper generally did the following:
• showed a secure grasp of all standard techniques • communicated well using mathematical language correctly • applied the breadth of their mathematical knowledge to find ways to tackle problems unfamiliar to them.	• showed a limited knowledge of standard techniques across the breadth of the specification • produced unclear or incomplete mathematical arguments • struggled to bring together different aspects of the course to find solution routes into problems unfamiliar to them.

Key point call out

Where a question uses a command word such as "**determine**", "**show that**" or requests "**In this question you must show detailed reasoning**" a clear mathematical argument will be expected. When proving an identity (or showing it to be true), it is vital that the candidate's argument unambiguously equates left hand side to right hand side.

Assessment for learning

Where a question requests proof of an identity (or demonstration of a result) examiners expect candidates to present a joined-up argument where LHS is shown to equate to RHS. The most straightforward structure for this would be a complete argument within the form LHS = …=…= RHS, although sub-arguments that compress the middle section can be used. See Question 5, Question 6 and Question 9.

Question 1 (a) and (b)

1 **(a)** Find a vector which is perpendicular to both $3\mathbf{i} - 5\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$. **[1]**

The equations of two lines are $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 11\mathbf{j} - 4\mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$.

(b) Show that the lines intersect, stating the point of intersection. **[5]**

Basic concepts involved in working with vectors were well understood. Some candidates treated part (b) as though it had asked solely to find the point of intersection. Good responses showed clear evidence that the three, non-vector equations were consistent, mostly by showing that the solution to two of those equations obeyed the third one. Other acceptable methods included showing that the solution point lay on both lines or that the distance between the lines was zero.

Question 2 (a), (b) and (c)

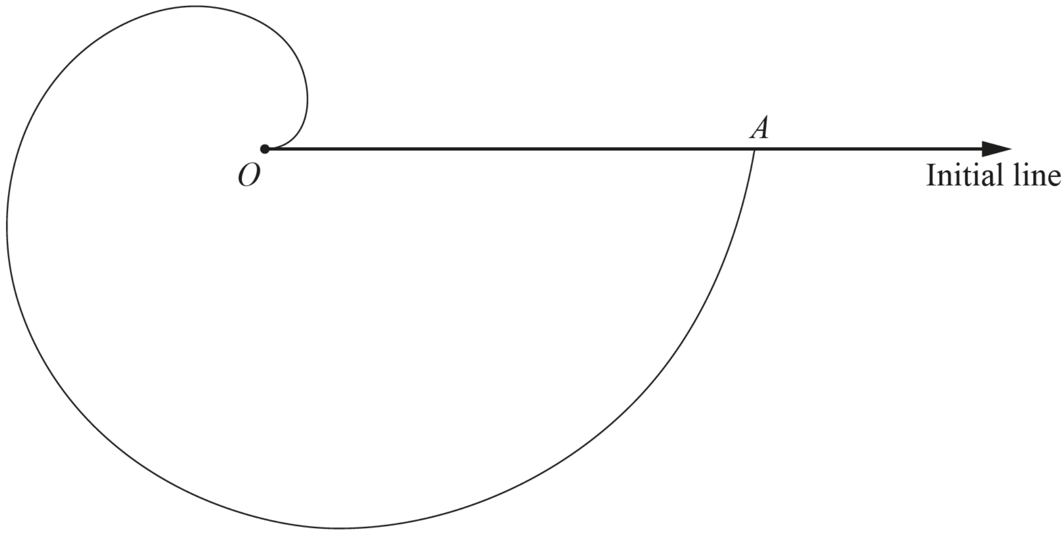
2 Two polar curves, C_1 and C_2, are defined by $C_1: r = 2\theta$ and $C_2: r = \theta + 1$ where $0 \leqslant \theta \leqslant 2\pi$.

C_1 intersects the initial line at two points, the pole and the point A.

(a) Write down the polar coordinates of A. **[2]**

(b) Determine the polar coordinates of the point of intersection of C_1 and C_2. **[2]**

The diagram below shows a sketch of C_1.



(c) On the copy of this sketch in the Printed Answer Booklet, sketch C_2. **[1]**

Most candidates appeared confident when working with polar equations. Centres may wish to reinforce the fact that, in coordinate notation, a point is written as (r,θ) and not the other way round. Some candidates incorrectly drew curves which started at the pole instead of at (1, 0); others had the two curves intersecting outside the first quadrant; yet others drew a curve which continued beyond $\theta = 2\pi$

$\theta = 2\pi$.

Question 3

3 **In this question you must show detailed reasoning.**

The roots of the equation $4x^3 + 6x^2 - 3x + 9 = 0$ are α, β and γ.

Find a cubic equation with integer coefficients whose roots are $\alpha + \beta$, $\beta + \gamma$ and $\gamma + \alpha$. **[6]**

In this question, candidates demonstrated varying levels of competence. Some candidates understood but were unable to use the relationships between the symmetric functions of the roots of polynomial equations and the coefficients (see OCR ref 4.05a). Others showed varying levels of algebraic adeptness when trying to evaluate those relationships for the new coefficients. A few candidates, for their final answer, wrote an expression instead of an equation; omission of " = 0" meant that the final mark could not be given.

Exemplar 1

3

OR

let $u = \Sigma\alpha - x$

$\Sigma\alpha = -\frac{b}{a} = -\frac{6}{4} = -\frac{3}{2}$

$\Rightarrow u = -\frac{3}{2} - x \Rightarrow x = -\frac{3}{2} - u$

$\Rightarrow 4\left(-\frac{3}{2} - u\right)^3 + 6\left(-\frac{3}{2} - u\right)^2 - 3\left(-\frac{3}{2} - u\right) + 9 = 0$

$\Rightarrow 4\left(-\frac{27}{8} + 3\left(\frac{-3}{2}\right)^2 \times -u + 3\left(\frac{-3}{2}\right)(-u)^2 + (-u)^3\right)$

$+ 6\left(\frac{9}{4} + 3u + u^2\right) + \frac{9}{2} + 3u + 9 = 0$

$\Rightarrow -\frac{27}{2} - 27u - 18u^2 - 4u^3 + \frac{27}{2} + 18u + 6u^2 + \frac{9}{2} + 3u + 9 = 0$

$\Rightarrow -4u^3 - 12u^2 - 6u + \frac{27}{2} = 0 \quad \times 2$

$\Rightarrow -8u^3 - 24u^2 - 12u + 27 = 0$

The example above is typical of a candidate who efficiently answered the question by using substitution (OCR ref 4.05b). Candidates may find that this method reduces the risk of algebraic errors.

Question 4

4 In this question you must show detailed reasoning.

Determine the smallest value of n for which $\dfrac{1^2+2^2+...+n^2}{1+2+...+n} > 341$. **[4]**

The vast majority of candidates were able to identify the two series and quote the standard series results. Subsequently, the most common approach was to cancel the common terms and solve the linear inequality. A small number of candidates, however, formed a cubic inequality; if they did so, they were expected to properly solve said inequality before reaching their conclusion. The "**In this question you must show detailed reasoning**" means that candidates need to fully develop an accurate mathematical argument; so those who solved an equation rather than inequality (often seen using a "critical value") didn't tend to gain full marks. A few candidates evaluated the first few terms of the series and inferred (rather than deduced) an arithmetic series. This approach not only implies a lack of knowledge of the specification but is also not a watertight method. Consequently, fewer marks were available for this incomplete method.

Some candidates did not realise the implication of applying a strict inequality within $\mathbb{N}$ and left their final answer as 511 rather than 512.

Question 5 (a)

5 (a) By using the exponential definitions of $\sinh x$ and $\cosh x$, prove the identity $\cosh 2x \equiv \cosh^2 x + \sinh^2 x$. **[2]**

In this question, most candidates knew the exponential definitions of the hyperbolic functions and attempted to prove RHS = LHS. Some, however, omitted starting their proof with "$\cosh^2 x + \sinh^2 x =$"

(or "RHS =") – see above advice.

Working the proof from LHS to RHS was equally acceptable, albeit not the most intuitive way to work.

Assessment for learning

Refer to the assessment for learning point in the overview section at the start of this report involving questions that require the proof of an identity or demonstration of a result.

Exemplar 2

5(a)	$\cosh x = \frac{1}{2}(e^{x}+e^{-x})$ $\sinh x = \frac{1}{2}(e^{x}-e^{-x})$
	$\cosh 2x = \frac{1}{2}(e^{2x}+e^{-2x})$
	$\cosh^2 x + \sinh^2 x = \frac{1}{4}(e^{x}+e^{-x})^2 + \frac{1}{4}(e^{x}-e^{-x})^2$
	$= \frac{1}{4}[e^{2x}+2+e^{-2x}+e^{2x}-2+e^{-2x}]$
	$= \frac{1}{4}[2e^{2x}+2e^{-2x}]$
	$= \frac{1}{2}(e^{2x}+e^{-2x})$
	$= \cosh 2x$

The exemplar above shows a comprehensive proof.

Question 5 (b)

(b) Hence find an expression for $\cosh 2x$ in terms of $\cosh x$. **[1]**

Most candidates made use of the identity from part (a) together with the relationship between $\cosh^2 x$ and $\sinh^2 x$ to derive the required identity. Those who simply quoted this identity did not gain the mark since they hadn't shown that they were deriving it from part (a) as requested by the command word "hence".

Question 5 (c)

(c) Determine the solutions of the equation $5\cosh 2x = 16\cosh x + 21$, giving your answers in exact logarithmic form. **[4]**

Almost all candidates obtained the right quadratic equation and solved it. Since this question asked candidates to "determine", examiners expected candidates to justify their rejection of the result $\cosh x = -1$, thus demonstrating their familiarity with OCR ref 4.07e. More successful responses tended to state that they were rejecting it since the range for $f(x) = \cosh x$ is $f(x) \geq 1$. Other reasons seen included "$\cosh x$ is positive" (credited) and "$\cosh x > 1$" (not credited, since incorrect). Candidates then either used the formula for $\cosh^{-1} x$ or first principles to reach a solution. A significant number of those who used the formula omitted the negative logarithm from their solution.

Key specification point

4.07f Use of expression for inverse hyperbolic functions. Practice of this specification point should highlight the need to consider the symmetry of the $\cosh x$ function.

Question 6 (a), (b), (c), (d), (e) and (f)

6 A particle, P, positioned at the origin, O, is projected with a certain velocity along the x-axis. P is then acted on by a single force which varies in such a way that P moves backwards and forwards along the x-axis.

When the time after projection is t seconds, the displacement of P from the origin is x m and its velocity is $v \text{ ms}^{-1}$.

The motion of P is modelled using the differential equation $\ddot{x} + \omega^2 x = 0$, where $\omega \text{ rad s}^{-1}$ is a positive constant.

(a) Write down the general solution of this differential equation. **[1]**

D is the point where $x = d$ for some positive constant, d. When P reaches D it comes to instantaneous rest.

(b) Using the answer to part **(a)**, determine expressions, in terms of ω, d and t only, for the following quantities

- x
- v **[3]**

(c) Hence show that, according to the model, $v^2 = \omega^2(d^2 - x^2)$. **[1]**

The quantity z is defined by $z = \frac{1}{v}$.

(d) Using part **(c)**, determine an expression for z_m, the mean value of z **with respect to the displacement**, as P moves directly from O to D. **[2]**

One measure of the validity of the model is consideration of the value of z_m. If z_m exceeds 8 then the model is considered to be valid.

The value of d is measured as 0.25 to 2 significant figures. The value of ω is measured as 0.75 ± 0.02.

(e) Determine what can be inferred about the validity of the model from the given information. **[1]**

(f) Find, according to the model, the least possible value of the velocity with which P was initially projected. Give your answer to **2** significant figures. **[2]**

This modelling question addressed some of the assessment objectives from AO3 (section 3b in H245 specification). Knowledge of a variety of subject content areas was required within the different parts. So, in part (a), candidates were expected to recognise the differential equation for simple harmonic motion (SHM) and to understand the term 'general solution'. In (b), they needed to apply boundary conditions. In (d), they needed to apply the formula (given in the Formula Booklet) for the mean value of a function and integrate using a method from the further calculus section of the subject content.

In parts (a) and (b), candidates who were well-practised in use of SHM were able to quote the general formula, to identify the amplitude, and to easily reach a particular solution. Some, less successful, responses included an attempt to find a particular solution by first using " $x = d$ when $v = 0$ " within $x = \mathrm{f}(t)$ before any application of the other boundary condition – this tended to lead them nowhere.

Part (c) was well answered by most who had correctly answered (b) although some didn't properly equate the two sides. (See Assessment for Learning point given in the overview section).

Parts (d) to (f) were less well answered. In (d), many candidates appeared unfamiliar with the topic of mean value of a function and showed workings that never attempted any integration. In (e) and (f) there were a lot of candidates who ignored the implication of the value of *d*, 0.25, being a rounded value.

Part (f) should have been accessible to all candidates since part (c) had given them a formula to apply, but many didn't attempt the question. Candidates did need to give units (ms^{-1}) for their final answer since assessment addressed the application of a model.

Question 7 (a) and (b) (i)

7 You are given that a is a parameter which can take only real values.

The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & 4 & -6 \\ -3 & 10-4a & 9 \\ 7 & 4 & 4 \end{pmatrix}$.

(a) Find an expression for the determinant of **A** in terms of a. **[2]**

You are given the following system of equations in x, y and z.

$$\begin{aligned} 2x + 4y - 6z &= 6 \\ -3x + (10-4a)y + 9z &= -9 \\ 7x + 4y + 4z &= 11 \end{aligned}$$

The system can be written in the form $\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -9 \\ 11 \end{pmatrix}$.

(b) **(i)** In the case where **A** is **not** singular, solve the given system of equations by using $\mathbf{A}^{-1}$. **[5]**

Most candidates had a valid method for finding the determinant frequently using expansion of the determinant by the first row. Some candidates, attempting to use this approach, neglected to change the sign for the middle term.

Similarly, most candidates had a valid method for finding the inverse matrix. A few, erroneously, having found minor determinants, multiplied each one by the related element.

Having found their inverse of **A**, most candidates also knew which matrix multiplication to apply in order to solve the system. However, accuracy marks were often lost due either to earlier errors or to not realising that the solution could be simplified to numerical values.

Question 7 (b) (ii)

(ii) In the case where **A is** singular describe the configuration of the planes whose equations are the three equations of the system. **[3]**

Most candidates evaluated the determinant of the singular matrix and formed the equation of the second plane. Some realised that this equation was equivalent to the first one. Fewer interpreted this fact correctly with some mistakenly concluding that these planes were parallel rather than identical. Examiners expected a complete answer to include the information that the two planes intersect in a line or form a sheaf.

Question 7 (c) (i) and (ii)

The transformation represented by **A** is denoted by T.
A 3-D object of volume $|5a-20|$ is transformed by T to a 3-D image.

(c) **(i)** Determine the range of values of a for which the orientation of the image is the reverse of the orientation of the object. **[1]**

(ii) Determine the range of values of a for which the volume of the image is less than the volume of the object. **[2]**

There were few good answers for these two questions, although identifying the final restriction that $a \neq 4$ was a stretch and challenge mark (see section 3a in H245 specification).

However, a significant number of candidates demonstrated little understanding of this section of the topic (OCR ref 4.03k). One common error was to form inequalities based on the volume of the shape; another was to not recognise that a negative determinant could still lead to a reduced volume.

Question 8

8 In this question you must show detailed reasoning.

It is given that $\sum_{r=k}^{98} \frac{5r+2}{r(r+1)(r+2)} = \frac{20539}{34650}$ for some k.

Determine the value of k. **[7]**

This question produced a good spread of marks, with most candidates able to tackle the partial fractions, but not all candidates spotted that they could use the method of differences between the limits of k and 98.

Being a "**detailed reasoning**" question, examiners expected to see sufficient terms from the series so that the cancellation pattern was evident. A bare minimum of the 6 non-cancelling terms and at least one complete set of cancelling terms was expected here. Examiners also expected to see a valid reason given, at the end, for rejecting $k = -\frac{2}{9}$. Rejecting it simply because it is negative was insufficient.

Some candidates chose to consider sums from 1 to n, but unless their method also involved use of the sum from 1 to k – 1, they couldn't access the marks for showing cancellation: most who tried this method were more prone to introduce errors.

Exemplar 3

$$\sum_{r=k}^{98} \frac{5r+2}{r(r+1)(r+2)} = \sum_{r=k}^{98} \frac{1}{r} + \frac{3}{r+1} - \frac{4}{r+2}$$

$$= \frac{1}{k} + \frac{3}{k+1} - \frac{4}{k+2}$$

$$+ \frac{1}{k+1} + \frac{3}{k+2} - \frac{4}{k+3}$$

$$+ \frac{1}{k+2} + \frac{3}{k+3} - \frac{4}{k+4}$$

$$\vdots$$

$$+ \frac{1}{96} + \frac{3}{97} - \frac{4}{98}$$

$$+ \frac{1}{97} + \frac{3}{98} - \frac{4}{99}$$

$$+ \frac{1}{98} + \frac{3}{99} - \frac{4}{100}$$

$$9k^2 - 61k - 14 = 0$$

$$k = \frac{61 \pm \sqrt{61^2 + 4 \times 14 \times 9}}{18}$$

$$= 7, -\frac{2}{9}$$

k must be an integer $\therefore$

$$k \neq -\frac{2}{9} \therefore$$

$$k = 7$$

The exemplar above shows extracts from a good answer with clear cancellation pattern and well explained conclusion. Note that there is a set of cancelling terms both at the start and at the end.

Assessment for learning

Detailed reasoning.

Candidates may find it helpful, in adjudging how much detail to give, to ask themselves: "If someone who understood mathematical language read this, would they be able to easily see what I am doing at each step?"

Question 9 (a)

9 In this question you must show detailed reasoning.

(a) Show that $\text{Re}\left(e^{4i\theta}(e^{i\theta}+e^{-i\theta})^4\right) = a\cos 4\theta \cos^4\theta$, where a is an integer to be determined. **[3]**

Candidates who gained all marks here usually did so with a simple left to right demonstration: LHS =…=…=RHS. They distinctly showed the three clear steps of using Euler's formula on the inner bracket, applying De Moivre's theorem and then selecting the real part. (See Assessment for Learning point in the overview section).

Some candidates attempted to expand the brackets in this part of the question which usually led nowhere helpful.

Question 9 (b)

(b) Hence show that $\cos\frac{1}{12}\pi = \frac{1}{2}\sqrt[4]{b+c\sqrt{3}}$, where b and c are integers to be determined. **[6]**

Since this question requests that candidates use part (a), methods that didn't attempt to expand the $\left(e^{i\theta}+e^{-i\theta}\right)^4$ term, or some term derived from it, gained no marks. Although challenging, this question produced a good spread of marks, Since this was a "**detailed reasoning**" question, candidates who found a calculator method of deducing the correct answer, or who quoted $\cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$,were not credited for this.

Question 10

10 The coordinates of the points A and B are $(3, -2, -1)$ and $(13, 10, 9)$ respectively.

- The plane Π_A contains A and the plane Π_B contains B.
- The planes Π_A and Π_B are parallel.
- The x and y components of any normal to plane Π_A are equal.
- The shortest distance between Π_A and Π_B is 2.

There are **two** possible solution planes for Π_A which satisfy the above conditions.

Determine the acute angle between these two possible solution planes. **[8]**

As would be expected, this final question was challenging, requiring strong problem solving skills to complete. Nonetheless, even less successful responses often gained partial credit for making a start on finding an expression for the shortest distance or for using normal/s of the correct algebraic form. As a route into a problem, a diagram is often helpful, as could be seen from some candidates' work.

The more successful responses generally involved an equation for the shortest distance and then reducing this to a quadratic. Some responses went no further, suggesting that there may have been some confusion on how reduce two variables to a single normal vector – the fact that a normal could be of any magnitude, and hence that the variables were relative, was missed. Those that did realise this usually were able to progress as follows:

They found 2 possible normal vectors.

Then they applied the dot product to obtain the angle between these which was also the angle between the 2 possible planes.

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