

A Level Further Mathematics A Y540 Pure Core 1 Sample Question Paper

Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 30 minutes

You must have:

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:

- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 Show that $\frac{5}{2-4i} = \frac{1}{2} + i$. [2]

2 **In this question you must show detailed reasoning.**

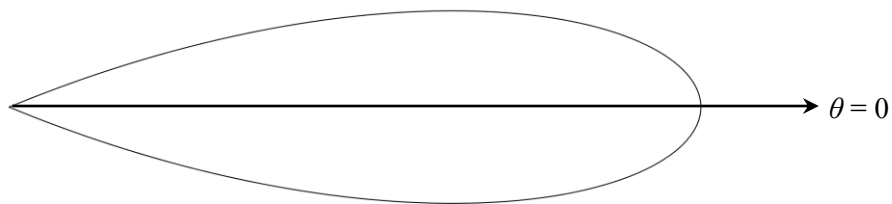
The equation $f(x) = 0$, where $f(x) = x^4 + 2x^3 + 2x^2 + 26x + 169$, has a root $x = 2 + 3i$.

(i) Express $f(x)$ as a product of two quadratic factors. [4]

(ii) Hence write down all the roots of the equation $f(x) = 0$. [1]

3 **In this question you must show detailed reasoning.**

The diagram below shows the curve $r = 2\cos 4\theta$ for $-k\pi \leq \theta \leq k\pi$ where k is a constant to be determined.



Calculate the exact area enclosed by the curve. [6]

4 Draw the region in an Argand diagram for which $|z| \leq 2$ and $|z| > |z - 3i|$. [3]

5 (i) Show that $\frac{d}{dx}(\sinh^{-1}(2x)) = \frac{2}{\sqrt{4x^2 + 1}}$. [2]

(ii) Find $\int \frac{1}{\sqrt{2 - 2x + x^2}} dx$. [3]

6 The equation $x^3 + 2x^2 + x + 3 = 0$ has roots α , β and γ .

The equation $x^3 + px^2 + qx + r = 0$ has roots $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$.

Find the values of p , q and r . [5]

7 The lines l_1 and l_2 have equations $\frac{x-3}{1} = \frac{y-5}{2} = \frac{z+2}{-3}$ and $\frac{x-4}{2} = \frac{y+2}{-1} = \frac{z-7}{4}$.

(i) Find the shortest distance between l_1 and l_2 . [5]

(ii) Find a cartesian equation of the plane which contains l_1 and is parallel to l_2 . [2]

8 (i) Find the solution to the following simultaneous equations.

$$x + y + z = 3$$

$$2x + 4y + 5z = 9$$

$$7x + 11y + 12z = 20$$

[2]

(ii) Determine the values of p and k for which there are an infinity of solutions to the following simultaneous equations.

$$x + y + z = 3$$

$$2x + 4y + 5z = 9$$

$$7x + 11y + pz = k$$

[6]

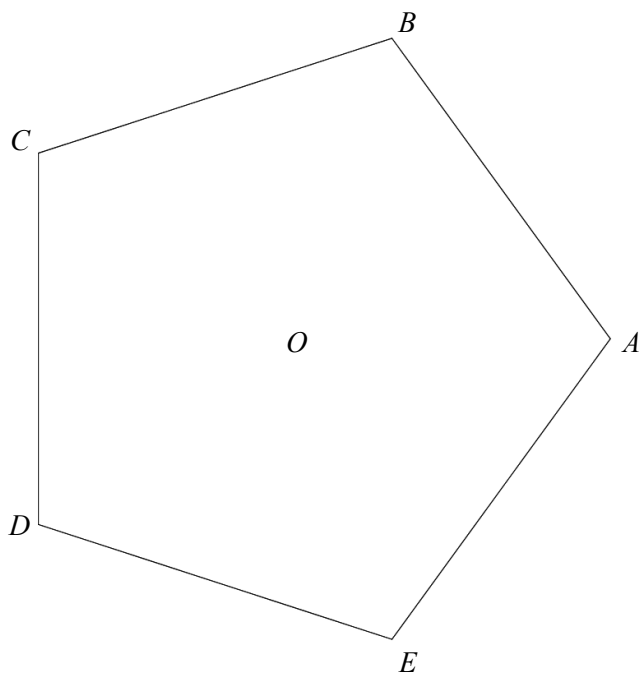
9 Prove by induction that, for all positive integers n ,

$$\sum_{r=1}^n \frac{5-4r}{5^r} = \frac{n}{5^n}.$$

[5]

- 10** The Argand diagram below shows the origin O and pentagon $ABCDE$, where A, B, C, D and E are the points that represent the complex numbers a, b, c, d and e , and where a is a positive real number.

You are given that these five complex numbers are the roots of the equation $z^5 - a^5 = 0$.



- (i) Justify each of the following statements.

(a) A, B, C, D and E lie on a circle with centre O . [1]

(b) $ABCDE$ is a regular pentagon. [2]

(c) $b \times e^{\frac{2i\pi}{5}} = c$ [1]

(d) $b^* = e$ [1]

(e) $a + b + c + d + e = 0$ [2]

- (ii) The midpoints of sides AB, BC, CD, DE and EA represent the complex numbers p, q, r, s and t . Determine a polynomial equation, with real coefficients, that has roots p, q, r, s and t . [3]

- 11** A company is required to weigh any goods before exporting them overseas. When a crate is placed on a set of weighing scales, the mass displayed takes time to settle down to its final value.

The company wishes to model the mass, m kg, which is displayed t seconds after a crate X is placed on the scales.

For the displayed mass it is assumed that the rate of change of the quantity $\left(0.5\frac{dm}{dt} + m\right)$ with respect to time is proportional to $(80 - m)$.

(i) Show that $\frac{d^2m}{dt^2} + 2\frac{dm}{dt} + 2km = 160k$, where k is a real constant. **[2]**

It is given that the complementary function for the differential equation in part **(i)** is $e^{\lambda t}(A\cos 2t + B\sin 2t)$, where A and B are arbitrary constants.

(ii) Show that $k = \frac{5}{2}$ and state the value of the constant λ . **[4]**

When X is initially placed on the scales the displayed mass is zero and the rate of increase of the displayed mass is 160 kg s^{-1} .

(iii) Find m in terms of t . **[7]**

(iv) Describe the long term behaviour of m . **[1]**

(v) With reference to your answer to part **(iv)**, comment on a limitation of the model. **[1]**

(vi) (a) Find the value of m that corresponds to the stationary point on the curve $m = f(t)$ with the smallest positive value of t . **[2]**

(b) Interpret this value of m in the context of the model. **[1]**

(vii) Adapt the differential equation $\frac{d^2m}{dt^2} + 2\frac{dm}{dt} + 5m = 400$ to model the mass displayed t seconds after a crate Y, of mass 100 kg, is placed on the scales. **[1]**

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