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Examiners' report

FURTHER MATHEMATICS B (MEI)

H645
For first teaching in 2017

Y435/01 Summer 2019 series

Version 1

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Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y435 series overview

This was the first sitting of this paper which contains the Extra Pure content of H645, A Level Further Mathematics B (MEI).

The paper consisted of 6 compulsory questions which nearly all candidates managed to attempt although there was some evidence of time pressure. Overall standards were quite high and, although presentation was variable, solutions were generally set out well.

Throughout the paper it was expected that candidates were able to demonstrate a high level of understanding of the (sometimes abstract) topics. Several questions required proof of a given result and the advice is that candidates should provide a detailed explanation of all their working; the examiner should not need to fill in any gaps of reasoning or calculation even if some of the steps appear obvious.

	AfL	Questions which involve proving a given statement should be treated with rigour and as such should always end with a conclusion following correct reasoning. Omitting this step will usually lose the final mark.

The following questions were answered well: 1(a), 1(b), 2(a), 3, 4(c)(i), 4(c)(ii), 5(a), 6(b)

The following questions proved more demanding: 2(b)(ii), 2(c), 5(d)(i), 5(d)(ii), 6(c), 6(d)

Question 1 (a)

1 The matrix **A** is $\begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}$.

(a) Given that **A** represents a reflection, write down the eigenvalues of **A**. **[1]**

Although some candidates showed working here it was expected that the eigenvalues for a reflection would be known and just written down.

Question 1 (b)

(b) Hence find the eigenvectors of **A**. **[3]**

This question was answered well. Some credit could be gained for correct working for one eigenvector. Many candidates were able to produce the matrix equation, but then some made simple algebraic errors in the linear equation and many were unable to convert this equation into the correct eigenvector.

Question 1 (c)

(c) Write down the equation of the mirror line of the reflection represented by **A**. **[1]**

Again, some candidates showed unnecessary working and a surprising number wrote down the perpendicular line $y = -2x$.

Question 2 (a)

2 A surface S is defined by $z = 4x^2 + 4y^2 - 4x + 8y + 11$.

(a) Show that the point P $(0.5, -1, 6)$ is the only stationary point on S. **[2]**

The first method mark was given for showing the correct partial derivatives and obtaining the x and y coordinates. To achieve both marks it was necessary to show the substitution of these values into the expression for z to obtain 6.

Question 2 (b) (i)

(b) **(i)** On the axes in the Printed Answer Booklet, draw a sketch of the contour of the surface corresponding to $z = 42$. **[2]**

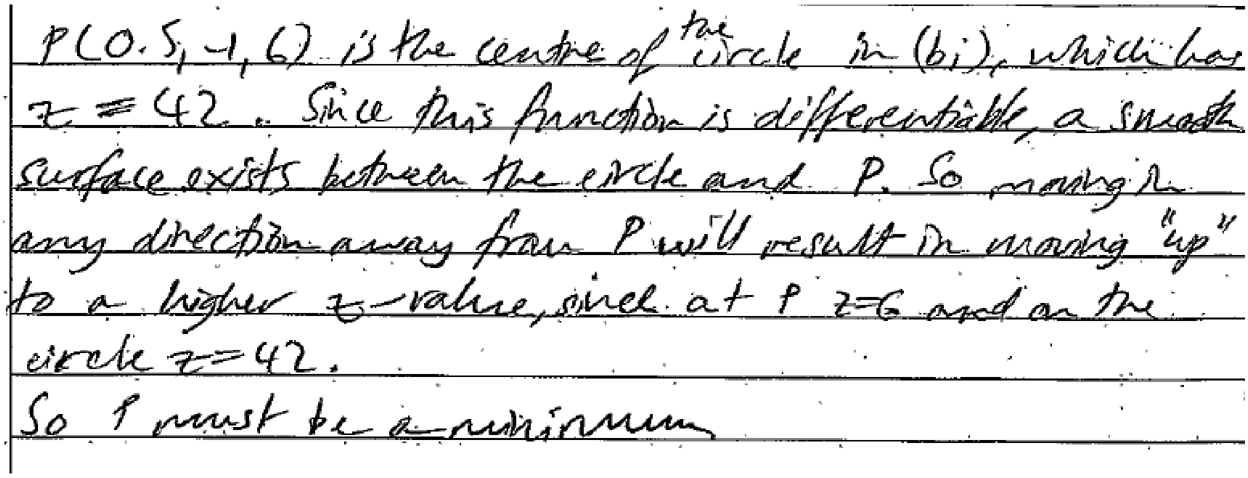
Nearly all candidates gained the first B mark for drawing a circle. The second mark was for both the centre, nearly always correct, and the radius which had to be clearly indicated either in the sketch or stated in the working. $(x-0.5)^2 + (y+1)^2 = 9$ was not sufficient to imply the candidate knew the coordinates of the centre or that the radius was 3.

Question 2 (b) (ii)

(ii) By using the sketch in part **(b)(i)**, deduce that P must be a minimum point on S. **[3]**

This proved a difficult question to answer fully. Some candidates chose to ignore the sketch and just describe a minimum point which gained no marks. Others identified P as being inside and below the contour but the idea of moving upwards in every direction from P to the contour proved more difficult to explain. There were some sketches which were helpful.

Exemplar 1


P(0.5, -1, 6) is the centre of the circle in (bi), which has z = 42. Since this function is differentiable, a smooth surface exists between the circle and P. So moving in any direction away from P will result in moving "up" to a higher z-value, since at P z=6 and on the circle z=42.
So P must be a minimum

A complete, concise response is illustrated in Exemplar 1.

Question 2 (c)

(c) In the section of S corresponding to $y = c$, the minimum value of z occurs at the point where $x = a$ and $z = 22$.
Find all possible values of a and c. **[4]**

This was another question which candidates found challenging. The best responses expressed *z* as a function of *x* and *c*, solved *dz/dx* = 0 giving *x* = 0.5 = *a*. Substitution then gave the values of *c.* Candidates who substituted *x* = *a* and *y* = *c* immediately misunderstood the idea of the section and were penalised. Similarly, repetition of the work in part 2(a) using partial derivatives gained no credit.

Question 3

3 The matrix $\mathbf{A}$ is $\begin{pmatrix} -1 & 2 & 4 \\ 0 & -1 & -25 \\ -3 & 5 & -1 \end{pmatrix}$.

Use the Cayley-Hamilton theorem to find $\mathbf{A}^{-1}$. **[8]**

This question was attempted successfully by most candidates and it was possible to gain 6 marks of the available 8 even with a minor error in finding the characteristic polynomial. Candidates were expected to find the matrix A^{-1} , not just an expression for it in terms of A. It was preferable, but not essential, for candidates to show the matrix A^2. Good use was made of calculators in this question with some candidates indicating when they were used.

Question 4 (a)

4 T is the set $\{1, 2, 3, 4\}$. A binary operation $\bullet$ is defined on T such that $a \bullet a = 2$ for all $a \in T$. It is given that $(T, \bullet)$ is a group.

(a) Deduce the identity element in T, giving a reason for your answer. **[2]**

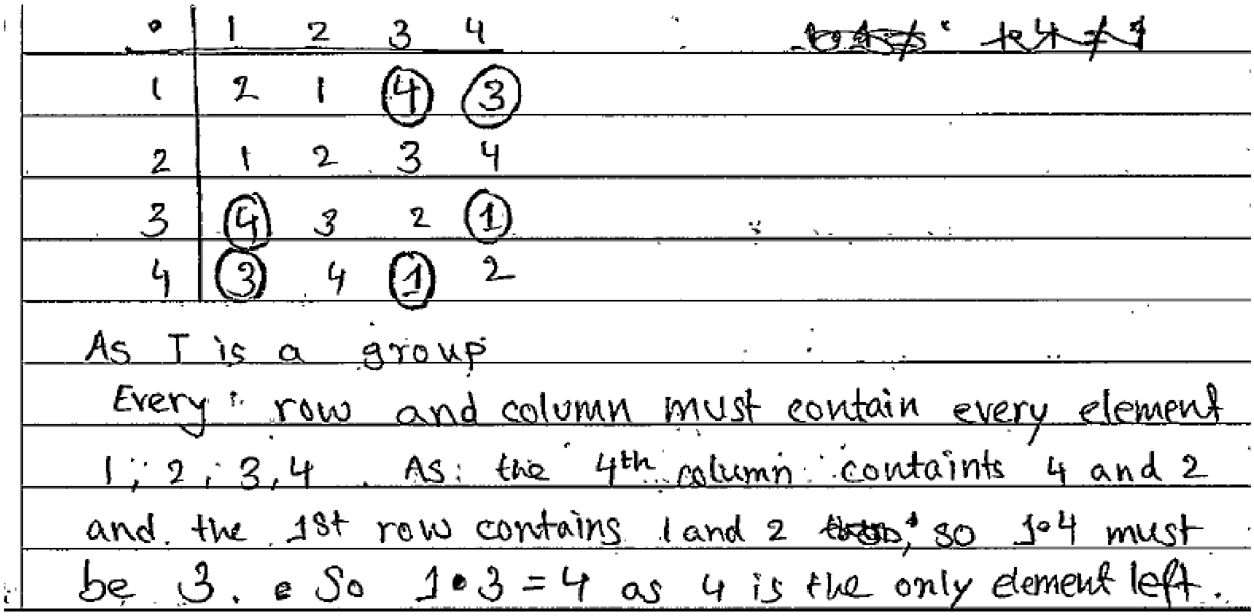
Nearly all candidates correctly recognised 2 as the identity. The statement $2 \cdot 2 = 2$ was the most economical way to achieve full marks although some candidates chose to explain this in words. Stating that all elements were self-inverse did not gain the second mark.

Question 4 (b)

(b) Find the value of $1 \bullet 3$, showing how the result is obtained. **[3]**

There were many varied ways of showing that $1 \cdot 3 = 4$ but elimination of 1, 2 and 3, using the identity and self-inverse property proved the most successful. Other candidates chose to draw the Cayley table here, either fully or partially with an explanation. If this table was used as the answer to 4(c) (i) it needed to be indicated in the answer space.

Exemplar 2



•	1	2	3	4
1	2	1	4	3
2	1	2	3	4
3	4	3	2	1
4	3	4	1	2

~~1•3≠3~~ ~~1•4≠4~~

As T is a group
Every row and column must contain every element
1, 2, 3, 4. As the 4th column contains 4 and 2
and the 1st row contains 1 and 2 ~~then~~, so 1•4 must
be 3. So 1•3 = 4 as 4 is the only element left.

There were some very good, well-reasoned responses to this question as shown in Exemplar 2.

Question 4 (c) (i)

(c) (i) Complete a group table for $(T, \bullet)$. **[2]**

This part was generally well done by most candidates.

Question 4 (c) (ii)

(ii) State with a reason whether the group is abelian. **[1]**

Most candidates realised that a reference to the symmetry along the **leading** diagonal of the table was required here. It was a follow through mark but the table in 4(c) (i) needed to be complete. Examples of wording that did not gain the mark were:

- The table is symmetrical
- The table is symmetrical along the diagonal(s)

Question 5 (a)

5 A financial institution models the repayment of a loan to a client in the following way.

- An amount, £C, is loaned to the client at the start of the repayment period.
- The amount owed n years after the start of the repayment period is £L_n, so that $L_0 = C$.
- At the end of each year, interest of $\alpha\%$ ($\alpha > 0$) of the amount owed at the start of that year is added to the amount owed.
- Immediately after interest has been added to the amount owed a repayment of £R is made by the client.
- Once L_n becomes negative the repayment is finished and the overpayment is refunded to the client.

(a) Show that during the repayment period, $L_{n+1} = aL_n + b$, giving a and b in terms of α and R. **[2]**

Many candidates achieved 2 marks here. The most frequent errors were:

- b = R
- a = ∝, 1 + ∝ or 1 + ∝%

Question 5 (b)

(b) Find the solution of the recurrence relation $L_{n+1} = aL_n + b$ with $L_0 = C$, giving your solution in terms of a, b, C and n. **[5]**

The first two marks were gained by most candidates either using *a* or $1 + \frac{\propto}{100}$ in the auxiliary equation. There were some examples of trying to solve the problem as a second order recurrence relation. The particular solution took the form L_n = constant. We accepted $L_n = f(n)$ if the term in *n* was correctly eliminated. Many competent candidates forfeited the final mark because they did not give their solution in terms of *a, b, C* and *n*. Poor choice of constants led to some confusion.

Alternatively, it was possible to use the formula for the sum of a geometric progression and obtain the correct solution. A few candidates wrote this down as the standard solution to $L_{n+1} = aL_n + b$ and this was given full credit if completely correct.

Question 5 (c)

(c) Deduce from parts **(a)** and **(b)** that, for the repayment scheme to terminate, $R > \frac{\alpha C}{100}$. **[2]**

Many candidates started with $L_n < 0$ and made little progress. Those who used the coefficient of a^n generally showed the result quickly. Some chose to explain the relationship between the initial interest and the repayment amount in words which also gained the marks.

Question 5 (d) (i)

A client takes out a £30 000 loan at 8% interest and agrees to repay £3000 at the end of each year.

(d) **(i)** Use an algebraic method to find the number of years it will take for the loan to be repaid. **[3]**

Many candidates found substituting the values into L_n challenging, especially dealing with the signs. Some candidates used a numerical method instead of an algebraic method which was not acceptable here and just stating the answer gained no credit. There were a lot of candidates who gave no response to this question.

Question 5 (d) (ii)

(ii) Taking into account the refund of overpayment, find the total amount that the client repays over the lifetime of the loan. **[3]**

Candidates did not appear to understand what was required here even if they had an answer to 5(d)(i). Quite a few calculated L_{21} but got no further. Another successful approach was to calculate the amount still owing at the end of the 20th year (£2 542.82) and then add the final repayment to £60 000.

Question 6 (a)

6 **(a)** Given that $\sqrt{7}$ is an irrational number, prove that $a^2 - 7b^2 \neq 0$ for all $a, b \in \mathbb{Q}$ where a and b are not both 0. **[2]**

This question proved difficult in two ways:

- Candidates started the proof by contradiction correctly by assuming that a^2-7b^2=0, but did not complete the argument fully. It was not sufficient to state '√7=a/b which is rational'. Some discussion of rational × rational and/or rational × irrational was needed.
- Starting with √7=a/b turned into a proof that √7 is irrational which gained no marks.

Exemplar 3

assume $a^2 - 7b^2 = 0$ (where $a, b \in \mathbb{Q}$)

$a^2 = 7b^2$

$a = \sqrt{7}b$

$a - \sqrt{7}b = 0$

but since $\sqrt{7} \notin \mathbb{Q}$, $\sqrt{7}b \notin \mathbb{Q}$

$\therefore$ $a - \sqrt{7}b \neq 0$ since $a \in \mathbb{Q}$, $\sqrt{7}b \notin \mathbb{Q}$

(contradicting the initial assumption)

Exemplar 3 shows a well-reasoned, concise but complete argument.

Question 6 (b)

(b) A set G is defined by $G = \{a + b\sqrt{7} : a, b \in \mathbb{Q}, a \text{ and } b \text{ not both } 0\}$.
Prove that G is a group under multiplication. (You may assume that multiplication is associative.) **[7]**

This was a lengthy question that required a detailed proof which candidates tackled well, scoring 4 or 5 out of the 7 marks.

There were two ways in which candidates could have been more concise:

- The identity could be stated as 1+0√7. If it was given as 1 we required them to observe that 1∈G.
- The question stated that associativity could be assumed. Many candidates used valuable time giving long proofs of associativity.

Conversely, there were areas where candidates did not give enough information.

- For closure we required multiplication of distinct elements and the statement that their resulting coefficients were rational.
- For the inverse it was not sufficient to give the inverse as 1/(a+√b); candidates needed to express this in a suitable form and justify **all three** conditions for this to be in G.
- At the end of the proof we required a conclusion to include **all four** axioms.

	AfL	Careful reading of the question regarding the assumption of associativity could have saved many candidates unnecessary working.

Question 6 (c)

(c) A subset H of G is defined by $H = \{1 + c\sqrt{7} : c \in \mathbb{Q}\}$.
Determine whether or not H is a subgroup of $(G, \times)$. **[2]**

A numerical example worked best here and most candidates realised that trying to establish closure or finding the inverse was the correct method. It was necessary to conclude that *H* was not a subgroup of (*G*, ×). Some chose to use one or two general elements and the second mark was only given if candidates stated clearly that their rational part ≠1.

Question 6 (d)

(d) Using $(G, \times)$, prove by counter-example that the statement 'An infinite group cannot have a non-trivial subgroup of finite order' is false. **[2]**

Candidates either did not identify the set $\{1, -1\}$ or then did not complete the proof with a conclusion regarding the statement.

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