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Examiners' report

FURTHER MATHEMATICS B (MEI)

H645

For first teaching in 2017

Y433/01 Summer 2022 series

Version 1

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Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

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Paper Y433/01 series overview

This is a minor option paper for MEI Further Mathematics. It assesses Modelling with Algorithms under the three strands of algorithms, networks and linear programming.

The evidence suggested that candidates had found the paper long and had struggled with Question 7.

Candidates who did well on this paper generally did the following:	Candidates who did less well on this paper generally did the following:
• attempted all questions and gave responses that were appropriate for the number of marks available • worked neatly and explained their working where appropriate • answered written responses precisely and unambiguously • checked all arithmetic.	• did not give written responses to questions asking for an explanation • worked in a muddled way and misread their own letters or numerical values • did not read the questions carefully enough • made errors in simple arithmetic.

Assessment for learning

Centres should advise candidates that erased work can sometimes still show through on the scanned copy and may lead to ambiguities especially when it has been overwritten or in diagrams.

Question 1 (a)

1

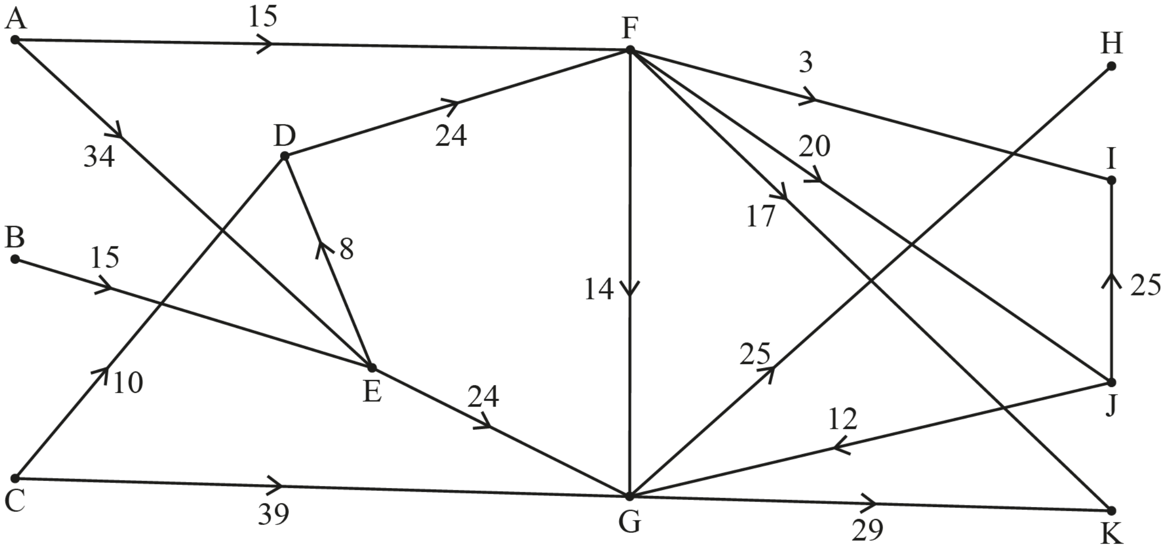


Fig. 1.1

The diagram in **Fig. 1.1** represents a system of pipes through which a fluid can flow from three sources to three sinks. The weights on the arcs show the capacities of the pipes in litres per minute.

(a) Add a supersource S and a supersink T to the network in the Printed Answer Booklet. Give appropriate weightings and directions to the connecting arcs. **[2]**

Most candidates were able to add appropriately weighted directed arcs from a supersource S to A, B and C and to a supersink T from H, I and K.

A few candidates did not use directed arcs or had unsuitable weights (weights that were too small to supply A, B or C or to take the flow from H, I or K) and some candidates also had an arc JT.

Question 1 (b)

(b) The cut α partitions the vertices into sets $\{S, A, B, C, D\}, \{E, F, G, H, I, J, K, T\}$. Calculate the capacity of cut α. **[1]**

Mostly correct, but some candidates added or subtracted 8 for the arc ED, instead of having the backflow empty.

A few candidates appeared to have used the cut {S, A, B, C, D, E}, {F, G, H, I, J, K, T} with E included in the source set instead of the sink set.

Question 1 (c)

(c) Explain why a flow of 34 litres per minute along AE cannot be achieved. **[1]**

Most candidates recognised that the maximum flow out of vertex E was 8 + 24 = 32 and deduced that the flow into E could not be as big as 34. Some said that the flow out of vertex E is 32 (rather than a maximum of 32) but the argument was still essentially correct.

A few candidates only looked at the arc EG for the flow out of vertex E, this may have been because the direction of the arc ED was missed.

Question 1 (d)

(d) An LP formulation is set up to find the maximum flow through the network.
Write down a suitable objective function for the LP formulation. **[1]**

Most candidates gave a suitable objective function and said that it was to be maximised.

Some candidates included the arc weights, e.g. maximise 15AF + 19AE + … and some candidates included all the arcs instead of a set that formed a cut.

Question 1 (e)

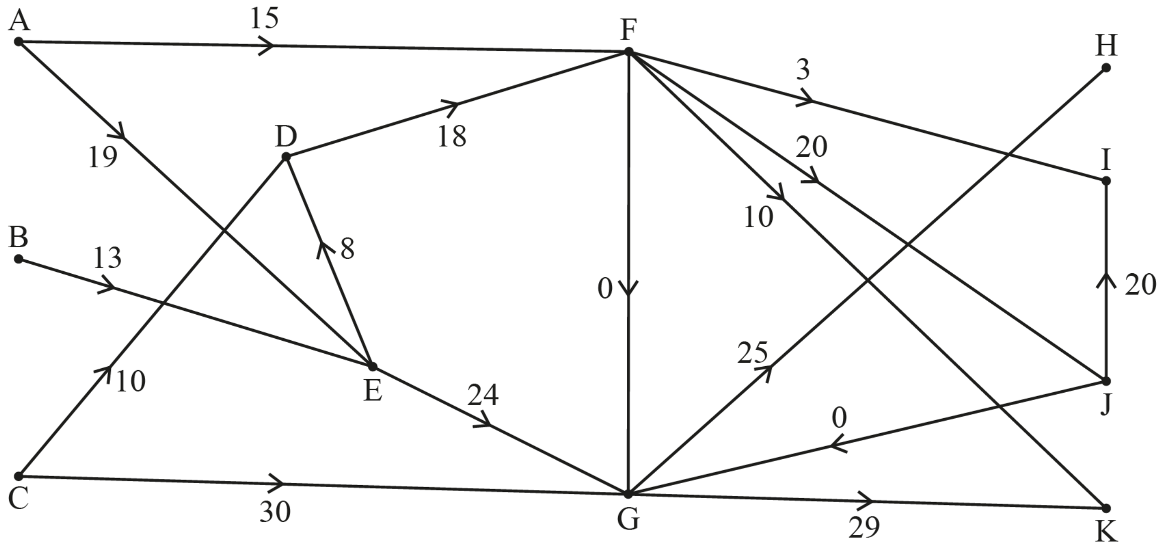


Fig. 1.2

The diagram in **Fig. 1.2** shows a feasible flow through the network.

(e) Show that this is a maximum flow. **[2]**

Most candidates realised that they needed to find a minimum cut and use the max flow = min cut theorem. Several were able to find the minimum cut but there were also several candidates who gave an incorrect cut or who made arithmetic mistakes in adding up the flow shown in Fig. 1.2.

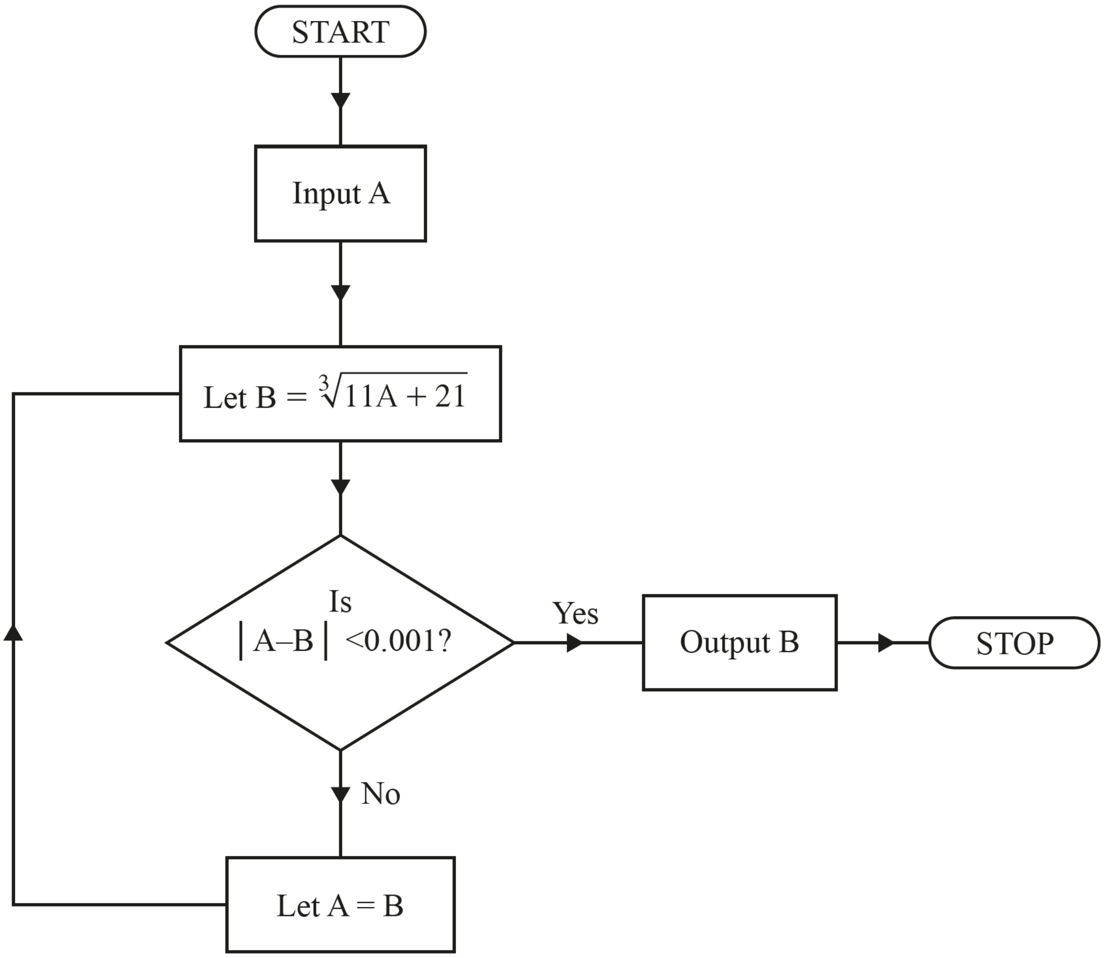
Some candidates gave written explanations outlining what would happen if certain arcs were saturated and some gave a list of all the saturated arcs but did nothing with it.

Misconception

Some candidates confused the flow across the cut (the flow shown in Fig. 1.2) with the capacity of the cut (the maximum possible flow across the cut (without consideration of any other arcs).

Question 2 (a)

2



The flow chart shows an algorithm that gives a numerical approximation for the real root of the cubic equation $x^3 - 11x - 21 = 0$.

(a) Work through the algorithm using the input $A = 5$. Record the values of A and B correct to **7** decimal places every time they change. Give the final output correct to **3** decimal places. **[3]**

Most candidates were able to use their calculators efficiently to work through the algorithm and recorded their results appropriately. Some used a line for each change, which resulted in unnecessary duplication of values unless the unchanged entries were left blank.

A few candidates continued to an extra pass, this was allowed provided the preceding values were correct and there was only one extra pass.

The algorithm is 'self correcting' so candidates who made arithmetic mistakes could still get to the final output, although usually after performing many iterations.

Some candidates did not use the accuracy specified in the question.

Assessment for learning

If a question specifies a certain accuracy this must be used and overrides the general guidance on the front of the question paper.

Question 2 (b)

(b) Show that the algorithm does give the real root of $x^3 - 11x - 21 = 0$ correct to **3** decimal places. **[2]**

This part covered specification reference A5: Understand that algorithms can sometimes be proved correct or incorrect.

Taking the roots from a calculator and identifying that the real root agreed with the value from the algorithm to 3 d.p. accuracy was not enough to constitute a proof.

Question 2 (c)

(c) Explain how to adapt the algorithm so that a numerical approximation for the real root of the cubic equation $x^3 - 12x - 23 = 0$ can be found. **[1]**

This part covered specification reference A3: Be able to repair, develop and adapt given algorithms.

Almost all candidates were able to identify the appropriate changes needed.

Question 3 (a)

3

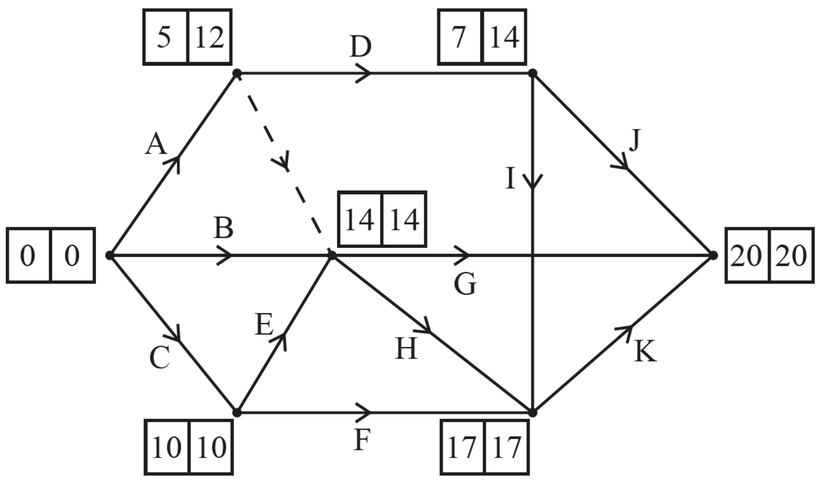


Fig. 3.1

Activity	Independent Float
A	0
B	7
C	0
D	0
E	0
F	2
G	2
H	0
I	0
J	1
K	0

Fig. 3.2

Fig. 3.1 shows an activity network for a project. Each activity is represented by an arc. The duration of each activity is measured in hours. The early event times and the late event times are shown at each vertex.

Fig. 3.2 shows the independent float for each activity.

(a) State which activities are critical. **[1]**

Many candidates identified the critical activities correctly.

Some candidates listed all the activities with 0 independent float, although sometimes they then corrected their response, possibly after attempting part (b).

Question 3 (b)

(b) Complete the table in the Printed Answer Booklet showing the duration of each activity. **[3]**

Many correct answers, some candidates with just one or two errors (often on activities G or J) and some candidates with up to 5 errors.

Hardly any candidates who attempted this part got fewer than 6 correct durations.

Question 3 (c)

Each activity requires one worker. Once a worker has started an activity they continue with that activity until it is complete.

(c) Draw a schedule to show how three workers can complete the project in the minimum completion time. Each box in the Printed Answer Booklet represents 1 hour. For each worker, write the letter of the activity they are doing in each box, or leave the box blank if the worker is not required for that 1 hour. **[2]**

Many good answers. Some candidates did not include all the activities and some split activities between more than one worker.

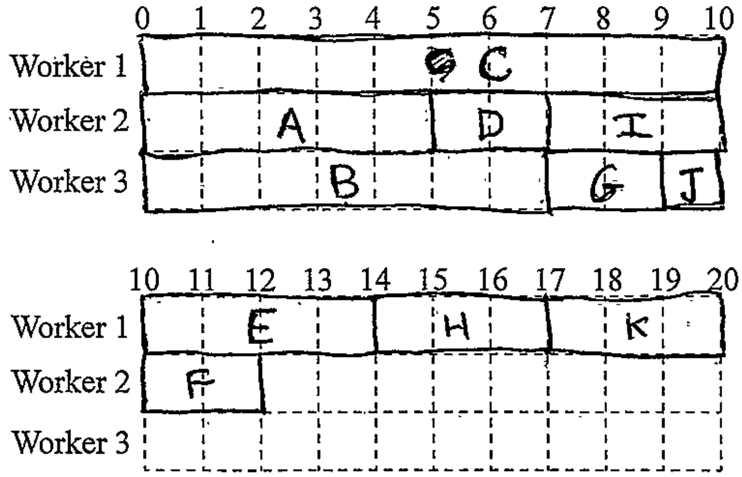
Assessment for learning

It is important for students to always keep the context of the question in mind and check that their answer is consistent with the information given.

In this question some candidates gave schedules that contravened the precedences shown in Fig. 3.1, for example starting G before E had finished.

Exemplar 1

3(c)



This candidate has all the activities, although some of the durations are wrong (from part b). However activity G is starting before activity E when it should come after A, B and E.

Question 4 (a)

4 A list of n numbers is to be sorted into descending order using the quick sort algorithm.

The number of comparisons made in each pass of the algorithm is used as a measure of the complexity of the quick sort algorithm.

(a) Show that, in the worst case, the quick sort algorithm has complexity $\mathrm{O}(n^2)$. **[2]**

Candidates needed to explain the number of comparisons used in each pass of the algorithm, in the worst case, and sum these to give a correct quadratic expression.

Some candidates counted the number of times each value was used in a comparison instead of the number of comparisons used in each pass. Some candidates appeared to be counting the number of comparisons for shuttle sort instead of quick sort.

Assessment for learning

With a 'show that' question, candidates must give an explanation that is sufficiently detailed to cover every step of their working.

Question 4 (b)

(b) A computer takes 2.3×10^{-7} seconds to sort the list of 100 ascending integers 1, 2, 3…, 99, 100 into descending order using the quick sort algorithm.

Calculate approximately how long it will take the computer to sort the list of 500 000 ascending integers 1, 2, 3,…, 499 999, 500 000 into descending order using the quick sort algorithm. You should assume that the computer uses the first value as the pivot for each sublist. **[2]**

Many correct responses.

Assessment for learning

To avoid an ambiguity when times are in seconds, candidates should be encouraged to write 'sec' or 'seconds' rather than 's'.

e.g. 5.75s can sometimes look like 5.755.

Question 4 (c)

A student attempts to use the quick sort algorithm to sort a list of seven random integers into descending order. After two passes through the list the student produces the following list.

6 8 10 5 7 11 4

(c) Explain how you know that the student has made at least one error in these two passes. **[1]**

Several good responses but also several where candidates had made assumptions about which values had been used as the pivots.

The easiest way to answer this was to state that each pass fixes the pivot (and possibly other values) in its correct final position so after two passes there should be at least two values in their correct final positions, but only 4 is in its correct final position.

Some candidates explained that in each pass 11 must move at least one place towards the left-hand end of the list so after two passes there must be at least two values to the right of 11, and not just one value.

Some candidates seemed to think that the list was being sorted into ascending order and were concerned about the position of the value 4.

Question 5 (a)

5 Consider the following LP problem.

Maximise $P = 2x + 3y - z$

Subject to

$3x + y - 4z \leqslant 70$
$5x + 4y \leqslant 60$
$x \geqslant 4,\ y \geqslant 2,\ z \leqslant -2$

(a) Explain why the simplex algorithm cannot be used to solve this LP problem. **[1]**

Many candidates said that the origin is not in the feasible region so cannot be used as the starting point for the simplex algorithm.

Some candidates identified that the constraints $x \geq 4$ and $y \geq 2$ and said that two-stage simplex would be needed, or artificial variables would be needed. Others just said that they were not in the standard form but did not explain that if written using $\leq$ they would give $-x \leq -4$ and $-y \leq -2$, but we cannot have negative values in the RHS of such constraints.

Some candidates identified the constraint $z \leq -2$ and said that a variable cannot be negative.

A few candidates were concerned that *z* had a negative coefficient in the objective, but on its own this is not a problem, the problem arises when we start to use the constraints.

Question 5 (b)

(b) Use the substitutions $x = X + 4,\ y = Y + 2,\ z = -Z - 2$ and $P = Q + 16$ to reformulate this LP problem into standard form. **[2]**

Several correct responses, some arithmetic errors and some candidates who reformulated the constraints but not the objective.

Question 5 (c)

(c) Represent the reformulated problem as an initial simplex tableau. **[2]**

Most candidates knew how to set up an initial simplex tableau.

Question 5 (d)

After a first iteration of the simplex algorithm the tableau below is produced.

Q	X	Y	Z	s_1	s_2	RHS
1	$\frac{7}{4}$	0	-1	0	$\frac{3}{4}$	24
0	$\frac{7}{4}$	0	4	1	$-\frac{1}{4}$	40
0	$\frac{5}{4}$	1	0	0	$\frac{1}{4}$	8

(d) Perform a second iteration of the simplex algorithm. **[3]**

Some candidates chose an incorrect pivot and some used inappropriate pivot operations, resulting in a tableau that did not have the correct structure.

Structure of a simplex tableau

Each tableau should be a shorthand for an identity matrix (the basis columns) and another matrix (the non-basis columns) together with a vector of non-negative values (the RHS column).

Exemplar 2

Q	X	Y	Z	s_1	s_2	RHS
4	35/4	0	0	1	11/4	136
0	7/4	0	4	1	-1/4	40
0	5/4	1	0	0	1/4	8

This candidate has not divided the pivot row by 4 and has then calculated the new row 1 as 4(row 1) + (what they think is the pivot row). This gives a tableau that is equivalent to the simultaneous equations but is not valid when using matrix form because there is no identity matrix.

When the simplex algorithm is used the pivot row should be divided by the pivot entry to get the new pivot row. In this case row 2 becomes row 2 divided by 4, this is the new pivot row (= npr).

The other rows are then transformed by adding or subtracting a multiple of the new pivot row to each to achieve a basic variable in the column that had been used to choose the pivot. In this case row 1 becomes row 1 + 1(npr) and row 3 Is unchanged (becomes row 3 + 0(npr)).

The pivot variable becomes basic. In this case in the Z column row 2 becomes a 1 and the other rows become 0s.

Assessment for learning

When performing an iteration of the simplex algorithm it is usually easier to use fractions rather than decimals, unless the decimals are exact to 2 d.p.

Rounded decimals can generate errors in subsequent iterations.

Question 5 (e)

(e) Find the maximum value of P, and the corresponding values of x, y and z. **[2]**

Basic variables take the value from RHS column corresponding to the position of the 1 in the basis column, non-basic variables take the value 0. This gave the values of Q, X, Y and Z from which the values of P, x, y and z could be found by 'undoing' the coding.

Several candidates gave the correct values (or correct from their tableau). Some gave the values of Q, X, Y and Z but did not use them to find P, x, y and z. Some candidates did not know how to deal with the non-basic variable X and some made arithmetic errors, such as subtracting 16 from Q instead of adding 16 or stating that Z could not be negative.

Question 6 (a)

6 **Fig. 6.1** shows a weighted graph. The weights represent arc lengths.

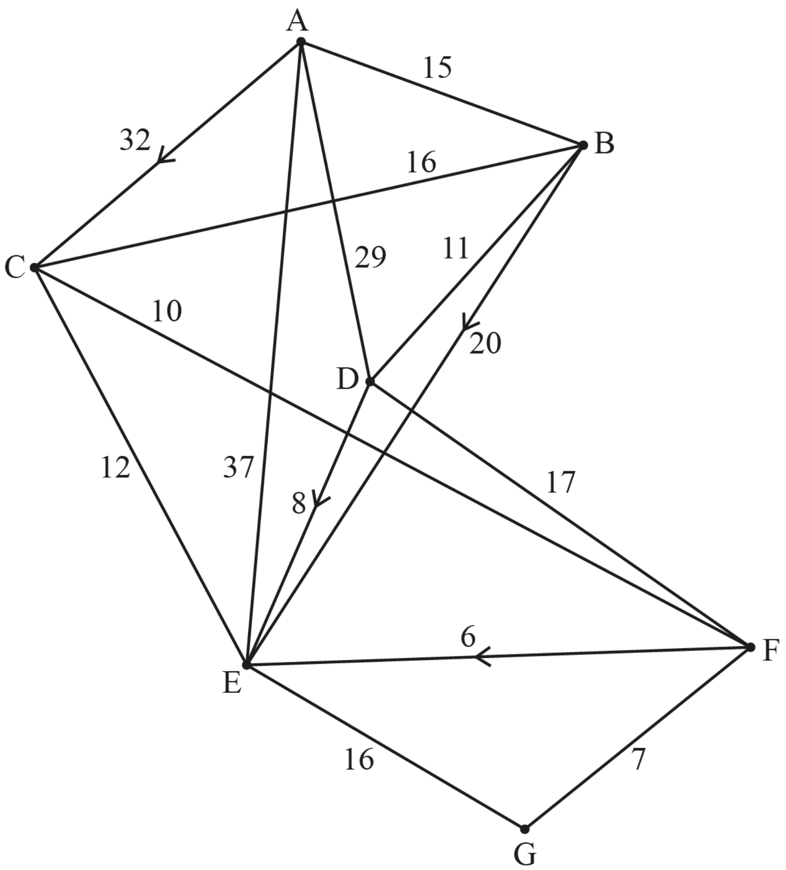


Fig. 6.1

(a) Apply Dijkstra's algorithm to the copy of the network in the Printed Answer Booklet to find the shortest path from A to G. **[5]**

Many correct responses. Some candidates who had used the algorithm correctly but did not state the route of the shortest path.

Some candidates had an arithmetic error in their working values but were apparently using Dijkstra's algorithm, while others seemed to be trying to spot the shortest path without using the algorithm.

Assessment for learning

If a question asks for the use of an algorithm, the response must show sufficient working to make it clear that the algorithm has been used correctly.

Question 6 (b) (i)

An LP formulation is set up to find the longest route between A and G in **Fig. 6.1** subject to the conditions that no arc can be repeated but nodes B, C, D, E and F can be passed through more than once.

Maximise 15AB + 32AC + 29AD + 37AE + 16BC + 11BD + 20BE
+ 16CB + 12CE + 10CF + 11DB + 8DE + 17DF + 12EC
+ 16EG + 10FC + 17FD + 6FE + 7FG

Subject to

AB + AC + AD + AE = 1
EG + FG = 1
AB + CB + DB − BC − BD − BE = 0
AC + BC + EC + FC − CB − CE − CF = 0
AD + BD + FD − DB − DE − DF = 0
AE + BE + CE + DE + FE − EC − EG = 0
CF + DF − FC − FD − FE − FG = 0
BD + DB ⩽ 1
BC + CB ⩽ 1
CE + EC ⩽ 1
CF + FC ⩽ 1
DF + FD ⩽ 1
BE ⩽ 1
DE ⩽ 1
FE ⩽ 1

(b) Explain the purpose of the following lines in the LP formulation

(i) AB + AC + AD + AE = 1 **[1]**

Candidates who had read the stem and knew that a path from A was required were usually able to interpret this line of the formulation as corresponding to A being the start vertex of a path so only one of the arcs from A is used (exactly one of these four arcs is used).

Question 6 (b) (ii)

(ii) CF + DF − FC − FD − FE − FG = 0 **[1]**

Most candidates realised that this constraint was to ensure that each time the path entered vertex F it left again. A few candidates thought that they were dealing with a flow problem and said that F is passed through exactly once, but this is not necessarily the case here.

Question 6 (b) (iii)

(iii) CE + EC $\leqslant$ 1 **[1]**

Candidates usually realised that this constraint was to prevent the undirected arc between C and E being used in both directions. Some gave very good answers about using the arc once or not at all.

Question 6 (c) (i)

The LP is run in a solver and the following output is shown in the table in **Fig. 6.2**.

VARIABLE	VALUE
AB	0.000000
AC	1.000000
AD	0.000000
AE	0.000000
BC	0.000000
BD	0.000000
BE	1.000000
CB	1.000000
CE	0.000000
CF	1.000000
DB	0.000000
DE	1.000000
DF	0.000000
EC	1.000000
EG	1.000000
FC	0.000000
FD	1.000000
FE	0.000000
FG	0.000000

Fig. 6.2

(c) (i) Find the length of the longest route from A to G given by the output in **Fig. 6.2**. **[1]**

A surprising number of arithmetic errors. Some candidates gave a route instead of the length of a longest route.

Question 6 (c) (ii)

(ii) State all possible corresponding longest routes from A to G. **[2]**

Several candidates found at least one of the two possible longest paths from A to G. Some candidates gave shorter routes (routes that did not use all the arcs corresponding to variables with the value 1 in the output in Fig. 6.2).

Question 7 (a)

7 Kai makes three sizes of model car, small, medium and large, to sell each year at a fair.

Kai has enough material to make either 250 small cars or 175 medium cars or 150 large cars. Alternatively, he could make a combination of the three sizes.

Each small car requires 40 minutes to make, each medium car requires 70 minutes to make, and each large car requires 140 minutes to make.

The total time that Kai spends making all the cars must not exceed 220 hours.

From experience, small cars sell particularly well so Kai decides to make at least three small cars for every one large car.

Furthermore, Kai decides to make exactly 210 cars.

Kai plans on selling each small car for £5, each medium car for £8 and each large car for £12. Kai wants to maximise the total income from the sale of the cars at the fair (that is, money made from selling cars, ignoring the cost of materials).

Let x, y and z represent the number of small, medium and large cars respectively that Kai makes.

(a) Explain why the maximum total income is achieved when $7x+4y$ is minimised. **[2]**

Some candidates started finding the constraints and then often tried to claim that this was in some way connected to the times of 40 minutes and 70 minutes to make one small car and one of medium car, respectively. Several candidates tried to use the constraints to argue that large cars should never be made.

This part was only about the objective and using the equation from the fact that Kai decides to make exactly 210 cars (in total) to eliminate the *z* variable.

Question 7 (b)

(b) Determine the constraints of this ILP problem by listing them as simplified inequalities with integer coefficients in x and y only. **[6]**

Most candidates were able to find some of the constraints in x, y and z.

The materials constraint was often missed and presented as upper bounds for x, y and z without understanding the line 'Alternatively, he could make a combination of the three sizes.'

The units needed to be consistent for the time constraint (all in minutes or all in hours) and the demand constraint needed some care to get the factor 3 on the correct side, and the inequality the right way round. It is often useful to try out some values, e.g. there must be 3 small cars for each large car so if $z = 10$ then x must be at least 30, hence $x \geq 3z$.

Having achieved these constraints, z could then be eliminated using $z = 210 - x - y$. Some candidates made arithmetic errors and some tried to eliminate z by using $x = 3z$ (or more often $z = 3x$).

The candidates who did all this correctly needed to also remember to substitute for z in the constraint $z \geq 0$ to get $x + y \leq 210$ as the final constraint (apart from non-negativity for x and y).

Question 7 (c)

(c) By representing the feasible region for x and y graphically, determine how many of each size of car Kai should make to maximise the total income from the fair. You must show all your working. **[6]**

Many candidates were missing the constraint $x + y \leq 210$ but they were able to graph the other three constraints.

Some candidates either did not realise that the x-axis started at $x = 100$ or did not know how to deal with this.

The optimal vertex should have been (137.5, 35), which some candidates found. This was an ILP so integer-valued points near this vertex needed to be tested for feasibility and to try to find the optimum for the ILP. Most candidates who got this far opted for (138, 35), but the point (139, 33) gave a better income. The solution needed to state the number of cars that should be made of each size (including large).

Question 7 (d)

(d) Using the solution found in part **(c)**, find the maximum possible total income from the sales. **[1]**

Only a few candidates attempted this part.

Those who did usually achieved a correct income for their values of x, y and z from part (c).

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