OCR
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A Level Further Mathematics B (MEI)

Y435 Extra Pure

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 The set $G = \{1, 4, 5, 6, 7, 9, 11, 16, 17\}$ is a group of order 9 under the binary operation of multiplication modulo 19.

(i) Show that G is a cyclic group generated by the element 4. **[3]**

(ii) Find another generator for G. Justify your answer. **[2]**

(iii) Specify two distinct isomorphisms from the group $J = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ under addition modulo 9 to G. **[5]**

2 A binary operation $*$ is defined on the set $S = \{p, q, r, s, t\}$ by the following composition table.

$*$	p	q	r	s	t
p	p	q	r	s	t
q	q	p	s	t	r
r	r	t	p	q	s
s	s	r	t	p	q
t	t	s	q	r	p

Determine whether $(S, *)$ is a group. **[4]**

3 **(i)** Find the general solution of

$$u_n = 8u_{n-1} - 16u_{n-2},\ n \geq 2. \quad (*)$$ **[4]**

A new sequence v_n is defined by $v_n = \dfrac{u_n}{u_{n-1}}$ for $n \geq 1$.

(ii) (A) Use (*) to show that $v_n = 8 - \dfrac{16}{v_{n-1}}$. for $n \geq 2$. **[2]**

(B) Deduce that if v_n tends to a limit then it must be 4. **[2]**

(iii) Use your general solution in part **(i)** to show that $\lim_{n\to\infty} v_n = 4$. **[3]**

(iv) Deduce the value of $\lim_{n\to\infty}\left(\dfrac{u_n}{u_{n-2}}\right)$. **[1]**

4 A surface S has equation $\mathrm{g}(x, y, z) = 0$, where $\mathrm{g}(x, y, z) = (y - 2x)(y + z)^2 - 18$.

(i) Show that $\dfrac{\partial \mathrm{g}}{\partial y} = (y + z)(-4x + 3y + z)$. **[2]**

(ii) Show that $\dfrac{\partial \mathrm{g}}{\partial x} + 2\dfrac{\partial \mathrm{g}}{\partial y} - 2\dfrac{\partial \mathrm{g}}{\partial z} = 0$. **[4]**

(iii) Hence identify a vector which lies in the tangent plane of every point on S, explaining your reasoning. **[3]**

(iv) Find the cartesian equation of the tangent plane to the surface S at the point $\mathrm{P}(1, 4, -7)$. **[3]**

The tangent plane to the surface S at the point $\mathrm{Q}(0, 2, 1)$ has equation $6x - 7y - 4z = -18$.

(v) Find a vector equation for the line of intersection of the tangent planes at P and Q. **[4]**

5 **In this question you must show detailed reasoning.**

You are given that the matrix $\mathbf{M} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \end{pmatrix}$ represents a rotation in 3-D space.

(i) Explain why it follows that $\mathbf{M}$ has 1 as an eigenvalue. **[2]**

(ii) Find a vector equation for the axis of the rotation. **[4]**

(iii) Show that the characteristic equation of $\mathbf{M}$ can be written as

$$\lambda^3 - \lambda^2 + \lambda - 1 = 0.$$

[5]

(iv) Find the smallest positive integer n such that $\mathbf{M}^n = \mathbf{I}$. **[6]**

(v) Find the magnitude of the angle of the rotation which $\mathbf{M}$ represents. Give your reasoning. **[1]**

END OF QUESTION PAPER