OCR
Oxford Cambridge and RSA

…day June 20XX – Morning/Afternoon

A Level Further Mathematics B (MEI)

Y422 Statistics Major

SAMPLE MARK SCHEME

Duration: 2 hours 15 minutes

MAXIMUM MARK 120

This document consists of 20 pages

Text Instructions

1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

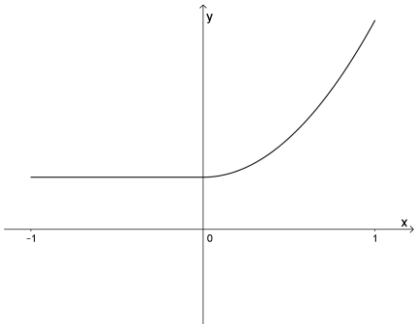
h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

k Anything in the mark scheme which is in square brackets […] is not required for the mark to be earned on this occasion, but shows what a complete solution might look like

Question			Answer	Marks	AOs	Guidance	
1	(i)		$\frac{2}{9}$	B1 [1]	1.1		
1	(ii)		r: 3, 4, 5, 6 $P(X=r)$: $\frac{2}{9}$, , , $\frac{31}{81}$	B1 [1]	1.1		
1	(iii)		DR $E(X)=3\times\frac{2}{9}+4\times\frac{2}{9}+5\times\frac{14}{81}+6\times\frac{31}{81}$ $=\frac{382}{81}=4.716$ $E(X^2)=9\times\frac{2}{9}+16\times\frac{2}{9}+25\times\frac{14}{81}+36\times\frac{31}{81}$ $=\frac{1916}{81}=23.564$ $Var(X)=\frac{1916}{81}-\left(\frac{382}{81}\right)^2$ $=1.413$	M1 A1 M1 M1 A1 [5]	1.2 1.1 1.1 1.1 1.1	For Σrp (at least 3 terms correct) FT their part **(ii)** cao For Σr^2p (at least 3 terms correct) **M1**dep for – their $E(X)^2$ **A1** FT their $E(X)$ provided $Var(X) > 0$	

Question			Answer	Marks	AOs	Guidance	
2	(i)	(*A*)		**B1** **B1** **[2]**	**1.1** **1.1**	Shape of each part separately, domain correct All correct, including *y*-intercept, which may be labelled *a* or $\frac{1}{3}$	
2	(i)	(*B*)	Total area = 1 Area = $a+\int_0^1\left(a+x^2\right)\mathrm{d}x$ So $2a+\frac{1}{3}=1 \Rightarrow a=\frac{1}{3}$ AG	**B1** **M1** **A1** **[3]**	**1.2** **2.1** **2.2a**	Use of this principle somewhere in solution Attempt at two (or more) areas including a correct integral	
2	(ii)	(*A*)	$\frac{1}{3}+\int_0^{\frac{1}{2}}\left(\frac{1}{3}+x^2\right)\mathrm{d}x$ $=\frac{13}{24}=0.5417$	**M1** **A1** **[2]**	**1.1a** **1.1**	Attempt to find area from –1 to $\frac{1}{2}$ Must be seen BC As fraction, or given correct to 3 or 4 d.p.	
		(*B*)	$\mathrm{E}(X)=\int_{-1}^0 \frac{1}{3}x\mathrm{d}x+\int_0^1\left(\frac{1}{3}x+x^3\right)\mathrm{d}x$ $=\frac{1}{4}$	**M1** **A1** **[2]**	**1.1a** **1.1**	Must be seen BC	

Question			Answer	Marks	AOs	Guidance	
2	(iii)		area from –1 to 0 is $\frac{1}{3}$, so require				
			$\int_0^m \left(\frac{1}{3}+x^2\right)\mathrm{d}x = \frac{1}{6}$	E1	2.1		
			$\left[\frac{1}{3}x+\frac{1}{3}x^3\right]_0^m = \frac{1}{6}$	M1	1.1a		
			$\frac{1}{3}m+\frac{1}{3}m^3 = \frac{1}{6} \Rightarrow 2m^3+2m-1=0$ AG	A1	1.1		
				[3]			
3	(i)		At (24,11)	B1	1.1a		
			Residual	M1	1.1	Subtraction other way round scores **M1** only	
			$=11-(17.138-0.3727\times 24)=11-8.1932$				
			= 2.81	A1	1.1		
				[3]			
3	(ii)	(*A*)	x = 26 y = 7.45	B1	1.1		
			Interpolation and points lie fairly close to the line so probably a good estimate	E1	3.5a	Must mention both	
				[2]			
		(*B*)	x = 16 y = 11.17	B1	1.1		
			Extrapolation so probably not reliable	E1	3.5b		
				[2]			
3	(iii)		The only factor with a large effect size when correlated with hours of sleep is danger	E1	2.2b		
			It seems that the more dangerous the animal's situation, the less time it spends asleep	E1	2.2b	Or any other relevant comment, e.g. stating that the data do not demonstrate causality, or saying something relevant about the other factors	
				[2]			

Question			Answer	Marks	AOs	Guidance	
3	(iv)		There are outliers which affect the size of the pmcc … A linear model may well be suitable for the data with these outliers removed	E1 E1 [2]	3.5b 3.5c	 Accept 'is suitable'. Or any other comment, e.g. redraw scatter diagram (or recalculate pmcc) without outliers	
4	(i)		$\left(\frac{5}{6}\right)^3 \times \frac{1}{6} = 0.0965$	B1 [1]	1.1		
4	(ii)		$1-\left(\frac{5}{6}\right)^4$ =0.518	M1 A1 [2]	1.1a 1.1		
4	(iii)		$\frac{5}{6}\times\left(\frac{1}{6}\right)^2 + \frac{1}{6}\times\frac{5}{6}\times\frac{1}{6}$ $= \frac{10}{216} = \frac{5}{108} = 0.0463$	M1 A1 [2]	3.1b 1.1		

Question			Answer	Marks	AOs	Guidance	
4	(iv)		$\frac{5}{108}+\left(\frac{1}{6}\right)^2$ $=\frac{8}{108}=\frac{2}{27}=0.0741$	M1 A1	3.1b 1.1		
			Alternative Method $3\times\left(\frac{1}{6}\right)^2\times\frac{5}{6}+\left(\frac{1}{6}\right)^3$ $=\frac{15}{216}+\frac{1}{216}=\frac{2}{27}=0.0741$	M1 A1			
				[2]			
4	(v)		Expected value for one five = 6 Because geometric So for two fives expected value = 6 + 6 = 12	E1 E1 B1 [3]	1.1 2.4 2.1	soi	
5	(i)		Mass of 5 small bags $\sim N(5\times508,5\times3.3^2)$ $\sim N(2540,54.45)$ $P(X<2550)=0.9123$	B2 B1 [3]	3.3 1.1 3.4	B1 For Normal and mean, B1 For variance BC FT their mean and variance	Distribution must be stated
5	(ii)		Mean of $L-3S=1515-3\times508=-9$ Variance of $L-3S=4.7^2+9\times3.3^2$ $L-3S\sim N(-9,120.1)$ $P(L-3S>0)=0.2058$	M1 M1 A1 A1 [4]	3.3 1.1 1.1 3.4	Mean Method for variance Correct variance BC	

Question			Answer	Marks	AOs	Guidance	
6	(i)		Shape of scatter diagram not approx. elliptical so no evidence of bivariate Normal required for test using pmcc to be valid	**B1** **B1** **[2]**	**3.5a** **2.4**	OR **B1** There does not appear to be a linear relationship OR **B1** so test using pmcc not appropriate [because that is for linear relationships]	
6	(ii)		One-tail because evidence of a positive relationship is sought	**B1** **[1]**	**2.4**	o.e.	
7	(i)		Kolmogorov-Smirnov *p*-value [greater than 0.15] indicates that the data could be from a Normal distribution Sample small with unknown population variance *t*-test	**E1** **E1** **B1** **[3]**	**3.5a** **2.4** **3.1b**		**SC2** For Wilcoxon test with • Cannot be sure the data are from a Normal distribution • Mean ≈ median indicates distribution is fairly symmetrical OR **SC1** For Wilcoxon with one of the above bullet points

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th colspan="2">Guidance</th></tr>
<tr><td>7</td><td>(ii)</td><td></td><td>H_0: $\mu = 110.2$ H_1: $\mu \neq 110.2$</td><td>B1</td><td>1.1a</td><td>Both hypotheses</td><td></td></tr>
<tr><td></td><td></td><td></td><td>Where μ is the population mean petrol price for Yorkshire</td><td>B1</td><td>2.5</td><td>Correct verbal definition</td><td>Allow hypotheses based on median if Wilcoxon chosen in (i)</td></tr>
<tr><td></td><td></td><td></td><td>Test statistic is $\dfrac{111.6733 - 110.2}{6.4048/\sqrt{15}}$</td><td>M1</td><td>3.3</td><td></td><td>FT method for calculation of Wilcoxon statistic
<table><tr><td>Price</td><td>-110.2</td><td>rank</td></tr><tr><td>116.9</td><td>6.7</td><td>11</td></tr><tr><td>117.9</td><td>7.7</td><td>13</td></tr><tr><td>123.9</td><td>13.7</td><td>15</td></tr><tr><td>114.9</td><td>4.7</td><td>8</td></tr><tr><td>112.9</td><td>2.7</td><td>4</td></tr><tr><td>105.7</td><td>-4.5</td><td>6</td></tr><tr><td>110.9</td><td>0.7</td><td>1</td></tr><tr><td>99.9</td><td>-10.3</td><td>14</td></tr><tr><td>108.9</td><td>-1.3</td><td>2</td></tr><tr><td>113.9</td><td>3.7</td><td>5</td></tr><tr><td>114.9</td><td>4.7</td><td>8</td></tr><tr><td>102.9</td><td>-7.3</td><td>12</td></tr><tr><td>114.9</td><td>4.7</td><td>8</td></tr><tr><td>103.9</td><td>-6.3</td><td>10</td></tr><tr><td>112.7</td><td>2.5</td><td>3</td></tr></table></td></tr>
<tr><td></td><td></td><td></td><td>= 0.8909</td><td>A1</td><td>1.1</td><td></td><td>FT test statistic = 44</td></tr>
<tr><td></td><td></td><td></td><td>Use of t_{14}</td><td>M1</td><td>3.4</td><td></td><td>FT use of n = 15 row of Wilcoxon tables</td></tr>
<tr><td></td><td></td><td></td><td>Critical value = 2.145</td><td>A1</td><td>1.1</td><td></td><td>FT Critical value = 25</td></tr>
<tr><td></td><td></td><td></td><td>0.8089 < 2.145 so do not reject H_0</td><td>M1</td><td>2.2b</td><td></td><td>FT 44 > 25 so do not reject H_0</td></tr>
<tr><td></td><td></td><td></td><td>There is insufficient evidence that the average price in Yorkshire is different from that in the UK</td><td>A1
[8]</td><td>3.5a</td><td>Conclusion in context</td><td></td></tr>
</table>

Question			Answer	Marks	AOs	Guidance	
8	(i)		The neutrons that are detected must occur randomly, independently and at a constant average rate.	**E1** **E1** **[2]**	**3.3** **3.3**	For randomly, independently For constant average rate or uniform rate but not constant rate	
8	(ii)	(*A*) (*B*)	P(0) = 0.333 $\lambda = 66$ P(at least 60) = 1 – 0.214 = 0.786	**B1** **M1** **A1** **[3]**	**1.1** **3.3** **3.4**	BC BC	
8	(iii)		P(more than 8 neutrons) = 1 – 0.999997573 = 0.000002427 Expected number = 1000 × 0.000002427 = 0.00243	 **B1** **M1** **A1** **[3]**	 **3.4** **1.1a** **1.1**	 BC	
8	(iv)		New $\lambda = 3.4 + 1.1 = 4.5$ P(No alarm triggered in 1 second) = 0.95974 P(At least one in 10 pds) = 1 – $(0.95974)^{10}$ = 0.337	**B1** **B1** **M1** **A1** **[4]**	**3.1b** **3.4** **1.1a** **1.1**	 BC FT from here if $\lambda = 3.4$ used	

Question			Answer	Marks	AOs	Guidance	
9	(i)	(*A*)	Sample size = 450	**B1** **[1]**	**2.2a**		
9	(i)	(*B*)	Chi-squared test [for a contingency table]	**B1** **[1]**	**1.2**		
9	(i)	(*C*)	H_0: no association between age and news source H_1: some association between age and news source	**B1** **[1]**	**2.5**		
9	(ii)		D11= $\frac{113}{450}\times 100$ =25.11 C18 = $\frac{(8-11.84)^2}{11.84} = 1.25$ D17 = $\frac{(22-25.11)^2}{25.11} = 0.39$	**M1** **A1** **M1** **A1** **[4]**	**3.4** **1.1** **1.1** **1.1**	**M1** for $\frac{(O-E)^2}{E}$ applied at least once **A1** for both correct: accept 1.245, 0.385 (NB one can be calculated by subtraction)	
9	(iii)		Degrees of freedom = 9 Critical value = 16.92 Test statistic = 25.45 25.45 > 16.92 so reject H_0 There is sufficient evidence to suggest that there is some association between age and primary news source	**B1** **B1** **M1** **A1** **[4]**	**3.3** **1.1** **2.2b** **3.5a**		

Question			Answer	Marks	AOs	Guidance	
9	(iv)		For age group 18-32 and 33-47, the contributions of 3.18 and 2.82 show that more than expected have primary source the internet	E1	3.5a		
			For age group 65+, the contributions of 7.54 and 4.53 show that fewer than expected have primary source the internet and more than expected have primary source newspapers.	E1	3.5a		
			For age group 48 - 64 the contributions show that primary sources are as expected	E1 [3]	3.5a	Allow other suitable answers. Max 2 out of 3 if numerical values of contributions to test statistic not mentioned	

Question			Answer	Marks	AOs	Guidance	
10	(i)		Estimate of population variance = $\frac{134.280-\frac{89.758^2}{60}}{59}$	M1	1.1		
			= 0.00008515	A1 [2]	1.1		
10	(ii)		1.49597	B1	1.1		
			$\pm 1.96 \times\sqrt{\frac{0.00008515}{60}}$	M1 M1	3.3 1.1		
			= 1.49597 ± 0.00233 or (1.4936,1.4983)	A1 [4]	3.4	Allow (1.494, 1.498)	
10	(iii)		It appears that the (population) mean content is not 1.5 litres as the calculated interval does not contain 1.5.	E1 [1]	3.5a		
10	(iv)		Each time a sample is taken it will be different, so e.g. will have a different mean hence different midpoint for confidence interval.	E1 E1 [2]	2.4 2.4	Samples vary …so confidence intervals vary	
10	(v)		$300\times 0.95 = 285$	B1 [1]	1.1		

Question			Answer	Marks	AOs	Guidance	
11	(i)		$\mathrm{P}(L=60)=\frac{1}{6}$	B1	1.1		
			$\mathrm{P}(H=60)=\left(\frac{1}{6}\right)^{10}$	M1	1.1		
			$=1.65\times10^{-8}$	A1	1.1		
				[3]			
11	(ii)		Lili's score because Hui's score uses more results so is likely to be closer to the mean	E1	2.2b	Any reasonable explanation e.g. Lili's score has greater standard deviation, as the extreme scores have higher probability	
				[1]			
11	(iii)		Discrete uniform	B1	3.3		
			$\mathrm{E}(X_1)=3.5$	B1	1.1		
			$\mathrm{Var}(X_1)=\frac{35}{12}[=2.917]$	B1	3.4		
				[3]			
11	(iv)	(*A*)	$\mathrm{E}(L)=35$			See (*C*)	
		(*B*)	$\mathrm{Var}(L)=10^2\times\frac{35}{12}$	M1	1.2		
			$=\frac{875}{3}=291.7$	A1	1.1		
		(*C*)	$\mathrm{E}(H)=35$	B1	1.1	Both expected values correct	
		(*D*)	$\mathrm{Var}(H)=10\times\frac{35}{12}$	M1	2.4		
			$\frac{175}{6}=29.17$	A1	1.1		
				[5]			

Question			Answer	Marks	AOs	Guidance	
11	(v)		Estimate of $P(L > 40) = \frac{11}{25}$ Estimate of $P(H > 40) = \frac{4}{25}$	B1 B1 [2]	2.2b 1.1		
11	(vi)	(*A*)	$P(L > 40) = \frac{1}{3}$	B1 [1]	1.1		
11	(vi)	(*B*)	Estimate 0.44, calculated value 0.33. Some way off but not totally unreasonable approximation with only 25 trials.	E1 [1]	3.2b	Any sensible relevant comment Ft their **(v)**	
11	(vii)	(*A*)	Produce a normal probability plot of the 25 values of Hui's scores (or of the scores $\div 10$)	B1 [1]	1.2		
11	(vii)	(*B*)	… if approximately a straight line then would appear to be from Normal distribution… … so Central Limit Theorem would seem to apply.	E1 E1 [2]	2.4 2.2b		

Question			Answer	Marks	AOs	Guidance	
11	(viii)	(*A*)	Mean $\sim N\left(35, \frac{350}{12}\right)$	M1	1.2		
			P(Mean > 40) = P(Normal >40.5)	B1	3.4	Continuity correction – with value of 40.5 as border (may have 40.5 included)	
			So $P(H > 40) \approx 0.154$	A1 [3]	1.1	BC	
11	(viii)	(*B*)	Agrees well with $\frac{4}{25} = 0.16$	E1 [1]	3.2b	FT their **(v)**	