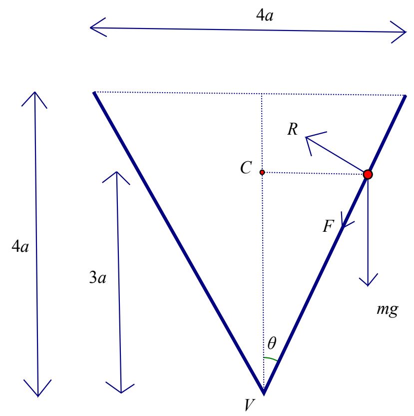
## Paper 4F: Further Mechanics 2 Mark Scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 1(a) | Total mass $= \int_0^{15} 10\left(1-\frac{x}{25}\right)\mathrm{d}x$ | M1 | 2.1 |
| | $=\left[10x-\frac{x^2}{5}\right]_0^{15}$ | A1 | 1.1b |
| | $=150-\frac{225}{5}=105\,(\text{kg})$ * | A1* | 1.1b |
| | | **(3)** | |
| (b) | Taking moments about the base: $\int_0^{15} 10x\left(1-\frac{x}{25}\right)\mathrm{d}x$ | M1 | 3.4 |
| | $=\left[5x^2-\frac{2}{15}x^3\right]_0^{15}\ (=675)$ | A1 | 1.1b |
| | $\Rightarrow 105d = 675$ | M1 | 3.4 |
| | $d = 6.43$ (m) $6\frac{3}{7}$ (m) | A1 | 1.1b |
| | | **(4)** | |
| | | | **(7 marks)** |

**Notes:**

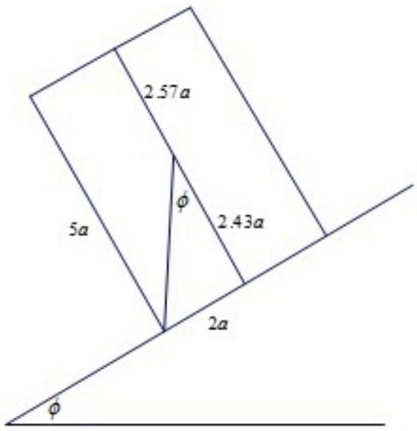
**(a)**
**M1:** Use integration (usual rules)
**A1:** Correct integration
**A1*:** Use limits and show sufficient working to justify given answer

**(b)**
**M1:** Use the model to find the moment about the base (usual rules for integration)
**A1:** Correct integration
**M1:** Use the model to complete the moments equation
Require 105 and their 675 used correctly
**A1:** 6.43 or better

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 2 |  | | |
| | Complete overall strategy | M1 | 3.1b |
| | Resolve vertically | M1 | 3.3 |
| | $mg + F\cos\theta = R\sin\theta$ | A1 | 1.1b |
| | Horizontal equation of motion | M1 | 3.3 |
| | $mr\omega^2 = R\cos\theta + F\sin\theta$ | A1 | 1.1b |
| | Use of limiting friction since maximum $\omega$ | M1 | 3.3 |
| | Substitute for trig ratios: $\frac{3a\omega^2}{2g} = \frac{9}{2}$ | M1 | 1.1b |
| | Maximum $\omega = \sqrt{\frac{3g}{a}}$ | A1 | 1.1b |
| | | | **(8 marks)** |

**Notes:**

**M1:** Overall strategy to form equation in $\omega$ only e.g. consider vertical and horizontal motrion and limiting friction
**M1:** Needs all 3 terms. Condone sign errors and sin/cos confusion
**A1:** Correct unsimplified equation
**M1:** Needs all 3 terms. Condone sign errors and sin/cos confusion
**A1:** Correct unsimplified equation
**M1:** Seen or implied
**M1:** Substitute to achieve equation in $a$, $\omega$ and $g$ only
**A1:** Or equivalent exact form

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3(a)** | <table><tr><td></td><td>mass</td><td>c of m from $O$</td></tr><tr><td>cylinder</td><td>$4\pi a^2 h$</td><td>$\frac{h}{2}$</td></tr><tr><td>hemisphere</td><td>$\frac{2}{3}\pi a^3$</td><td>$\frac{3}{8}a$</td></tr><tr><td>$V$</td><td>$4\pi a^2 h - \frac{2}{3}\pi a^3$</td><td>$d$</td></tr></table> | | |
| | Mass ratios | B1 | 1.2 |
| | Correct distances | B1 | 1.2 |
| | Moments about a diameter through $O$ | M1 | 2.1 |
| | $4\pi a^2 h \times \frac{h}{2} - \frac{2}{3}\pi a^3 \times \frac{3}{8}a = 2\pi a^2\left(2h - \frac{1}{3}a\right) \times d$ | A1 | 1.1b |
| | $d = \dfrac{h^2 - \frac{a^2}{8}}{2h - \frac{a}{3}} = \dfrac{3\left(8h^2 - a^2\right)}{8\left(6h - a\right)}$ * | A1* | 2.2a |
| | | **(5)** | |
| **(b)** |   | | |
| | $h = 5a \Rightarrow d = 2.573...a$ | B1 | 1.1b |
| | About to topple so c of m above tipping point | M1 | 2.2a |
| | $\Rightarrow \tan\phi = \dfrac{2a}{5a - 2.573a}$ | A1ft | 1.1b |
| | $\phi = 39.5°$ **or** 0.689 rads | A1 | 1.1b |
| | | **(4)** | |
| | | | **(9 marks)** |

| **Question 3 notes:** |
|---|
| **(a)**<br>**B1:** Correct mass ratios<br>**B1:** Correct distances<br>**M1:** All three terms & dimensionally correct. Could use a parallel axis but final answer must be for the distance from $O$<br>**A1:** Correct unsimplified equation<br>**A1*:** Deduce the given answer. Their working must make it clear how they reached their answer |
| **(b)**<br>**B1:** Distance of com from base<br>**M1:** Condone tan the wrong way up<br>**A1ft:** Correct unsimplified expression for trig ratio for $\phi$ following their $d$<br>**A1:** $39.5°$ or 0.689 rads |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4(a)** | Equation of motion: $1800 - 2v^2 = 500a$ (when seen) | B1 | 2.1 |
| | Select form for $a$: $=500\dfrac{\mathrm{d}v}{\mathrm{d}t}$ | M1 | 2.5 |
| | $\int \dfrac{2}{500}\mathrm{d}t = \int \dfrac{1}{900-v^2}\mathrm{d}v = \dfrac{1}{60}\int \dfrac{1}{30+v} + \dfrac{1}{30-v}\mathrm{d}v$ | M1 | 2.1 |
| | $\dfrac{t}{250} = \dfrac{1}{60}\ln(30+v) - \dfrac{1}{60}\ln(30-v)\ (+C)$ | A1 | 1.1b |
| | $T = \dfrac{25}{6}\ln\left(\dfrac{30+10}{30-10}\right) = \dfrac{25}{6}\ln 2$ * | M1<br>A1* | 2.1<br>2.2a |
| | | **(6)** | |
| **(b)** | Equation of motion: $500v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 1800 - 2v^2$ | M1 | 2.5 |
| | $\int \dfrac{500v}{1800-2v^2}\mathrm{d}v = \int 1\mathrm{d}x$ | M1 | 2.1 |
| | $-125\ln\left(1800-2v^2\right) = x\ (+C)$ | A1 | 1.1b |
| | Use boundary conditions: $x = -125 \ln 1600 + 125 \ln 1800$ | M1 | 2.1 |
| | $x = 125\ln\dfrac{9}{8}$ (m) * | A1* | 2.2a |
| | | **(5)** | |
| | | | **(11 marks)** |

**Notes:**

**(a)**

**B1:** All three terms & dimensionally correct

**M1:** Use of correct form for acceleration to give equation in $v$, $t$ only

**M1:** Separate variables and integrate

**A1:** Condone missing $C$

**M1:** Use boundary conditions correctly

**A1*:** Show sufficient working to justify given answer and a 'statement' that the required result has been achieved

**(b)**

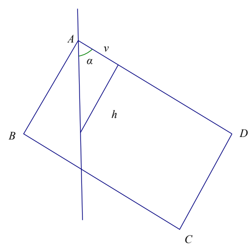
**M1:** Correct form of acceleration in the equation of motion to give equation in $v$, $x$ only

**M1:** Separate variables and integrate

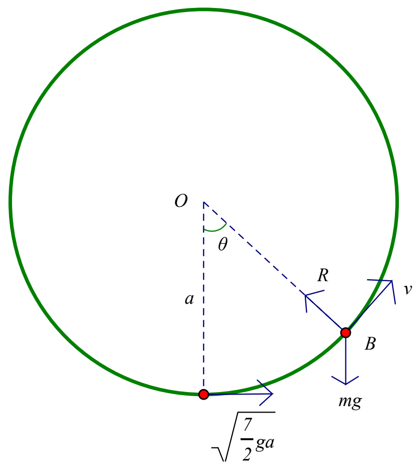
**A1:** Condone missing $C$

**M1:** Extract and use boundary conditions

**A1*:** Show sufficient working to justify given answer and a 'statement' that the required result has been achieved

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>5(a)</td><td><table><tr><td></td><td>Mass</td><td>From $AD$</td></tr><tr><td>Rectangle</td><td>$8a^2$</td><td>$a$</td></tr><tr><td>Semicircle</td><td>$\frac{1}{2}\pi a^2$</td><td>$\frac{4a}{3\pi}$</td></tr><tr><td>Sign</td><td>$a^2\left(8-\frac{\pi}{2}\right)$</td><td>$h$</td></tr></table></td><td></td><td></td></tr>
<tr><td></td><td>Mass ratios</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td>Moments about $AD$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$a^2\left(8-\frac{\pi}{2}\right)h = 8a^2\times a - \frac{1}{2}\pi a^2\times\frac{4a}{3\pi}\left(=8a^3-\frac{2}{3}a^3=\frac{22}{3}a^3\right)$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>$\Rightarrow h=\frac{22}{3}a\div\left(8-\frac{\pi}{2}\right)=\frac{44a}{3(16-\pi)}$ *</td><td>A1*</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(b)</td><td>Moments about $A$<br>$2aT=\frac{44a}{3(16-\pi)}W$</td><td>M1</td><td>3.1b</td></tr>
<tr><td></td><td>$T=\frac{hW}{2a}=\frac{22W}{3(16-\pi)}$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td>
</td><td></td><td></td></tr>
<tr><td></td><td>Take moments about $AB$ to find distance of com from $AB$</td><td>M1</td><td>3.1b</td></tr>
<tr><td></td><td>$8a^2\times 2a-\frac{1}{2}\pi a^2\times d=\left(8-\frac{1}{2}\pi\right)a^2\times v$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>$v=\frac{32a-\pi d}{16-\pi}$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>Correct trig for the given angle</td><td>M1</td><td>3.1b</td></tr>
<tr><td></td><td>$\tan\alpha=\frac{11}{18}=\frac{h}{v}=\frac{44a}{3(32a-\pi d)}$</td><td>A1ft</td><td>1.1b</td></tr>
<tr><td></td><td>$(24a=32a-\pi d,\quad 8a=\pi d)\qquad d=\frac{8a}{\pi}$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(6)</td><td></td></tr>
<tr><td colspan="4">(12 marks)</td></tr>
</table>

| Question 5 notes: |
|---|
| **(a)**<br>**B1:** Correct mass ratios<br>**M1:** Need all three terms, must be dimensionally correct<br>**A1:** Correct unsimplified equation<br>**A1*:** Show sufficient working to justify the given answer and a 'statement' that the required result has been achieved |
| **(b)**<br>**M1:** Could also take moments about B **or** about the c.o.m. and use<br>**A1:** cso |
| **(c)**<br>**M1:** All terms and dimensionally correct<br>**A1:** Correct unsimplified equation<br>**A1:** Or equivalent<br>**M1:** Condone tan the wrong way up<br>**A1:** Equation in a and d; follow through on their v<br>**A1:** cao |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 6(a) |  | | |
| | Conservation of energy | M1 | 2.1 |
| | $\frac{1}{2}mv^2 + mga(1-\cos\theta) = \frac{1}{2}m\left(\frac{7}{2}ga\right)$ | A1 | 1.1b |
| | $v^2 = ga\left(\frac{3}{2}+2\cos\theta\right)$* | A1* | 2.2a |
| | | **(3)** | |
| **(b)** | Resolve parallel to *OB* and use $\frac{mv^2}{a}$ | M1 | 3.1b |
| | $R - mg\cos\theta = \frac{mv^2}{a}$ | A1 | 1.1b |
| | Use R= 0 $g\cos\theta = -\frac{v^2}{a}$ | M1 | 3.1b |
| | Solve for $\theta$ $\Rightarrow$ $g\cos\theta = -g\left(\frac{3}{2}+2\cos\theta\right)$ | M1 | 1.1b |
| | $\theta = 120°$ | A1 | 1.1b |
| | | **(5)** | |
| **(c)** | Any appropriate comment e.g. the hoop is unlikely to be smooth | B1 | 3.5b |
| | | **(1)** | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **6(d)** | At rest $\Rightarrow \quad v = 0$ | M1 | 3.1b |
| | $\Rightarrow \quad \cos\theta = -\frac{3}{4}$ | A1 | 1.1b |
| | Acceleration is tangential | M1 | 3.1b |
| | Magnitude $\lvert g\cos(\theta - 90)\rvert = 6.48 \text{ m s}^{-2}$ or $\frac{\sqrt{7}}{4}g$ | A1 | 1.1b |
| | At $\left(\cos^{-1}\left(-\frac{3}{4}\right) - 90 =\right) 48.6°$ to the downward vertical | A1 | 1.1b |
| | | **(5)** | |
| | | | **(14 marks)** |

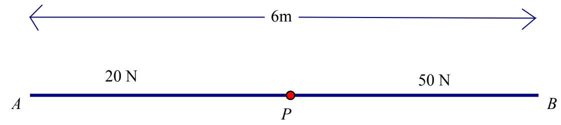
**Question 6 notes:**

**(a)**
**M1:** All terms required. Must be dimensionally correct
**A1:** Correct unsimplified equation
**A1*:** Show sufficient working to justify the given answer and a 'statement' that the required result has been achieved

**(b)**
**M1:** Resolve parallel to *OB*
**A1:** Correct equation
**M1:** Use $R = 0$ seen or implied
**M1:** Solve for $\theta$
**A1:** Accept $\frac{2\pi}{3}$

**(c)**
**B1:** Any appropriate comment e.g.
- hoop may not be smooth;
- air resistance could affect the motion

**(d)**
**M1:** $v = 0$ seen or implied
**A1:** Correct equation in $\theta$
**M1:** Correct direction for acceleration
**A1:** Accept 6.48, 6.5 or exact in $g$
**A1:** Accept 0.848 (radians)

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **7(a)** |  | | |
| | $T_A = \frac{20e}{2},\ T_B = \frac{50(2-e)}{2}\ e$ | M1 | 3.1a |
| | In equilibrium $T_A = T_B$, $10e = 25(2-e)$ | M1 | 3.1a |
| | $(35e = 50),\quad e = \frac{10}{7}$ | A1 | 1.1b |
| | Equation of motion for $P$ when distance $x$ from equilibrium position towards $B$: | M1 | 3.1a |
| | $3.5\ddot{x} = T_B - T_A = \frac{50(2-e-x)}{2} - \frac{20(e+x)}{2}$ | A1<br>A1 | 1.1b<br>1.1b |
| | $= \frac{50\left(\frac{4}{7}-x\right)}{2} - \frac{20\left(\frac{10}{7}+x\right)}{2}$ | | |
| | $\Rightarrow 3.5\ddot{x} = -35x,\quad \ddot{x} = -10x$<br>and hence SHM about the equilibrium position | A1 | 3.2a |
| | | **(7)** | |
| **(b)** | Amplitude $= 2 - \frac{10}{7} = \frac{4}{7}$ | B1 ft | 2.2a |
| | Use of max speed $= a\omega$ | M1 | 1.1b |
| | $= \frac{4}{7}\sqrt{10} = 1.81$ (m s$^{-1}$) | A1 ft | 1.1b |
| | | **(3)** | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 7(c) | Nearer to *A* than to *B*: $x < -\frac{3}{7}$ | B1 | 3.1a |
| | Solve for $\sqrt{10}t$ : $\cos\sqrt{10}t = -\frac{3}{4}$ , $\sqrt{10}t = 2.418$............ | M1 | 3.1a |
| | Length of time: $\frac{2}{\sqrt{10}}(\pi - 2.418...)$ | M1 | 1.1b |
| | 0.457 (seconds) | A1 | 1.1b |
| | Alternative: $\frac{3.864 - 2.419}{\sqrt{10}} = 0.457$ | | |
| | Alternative:<br>$x = \frac{4}{7}\sin\sqrt{10}t = \frac{3}{7} \Rightarrow \sqrt{10}t = 0.8481$ or $\sqrt{10}t = 2.29353$<br>$t_1 = 0.2682,\ t_2 = 0.72527$<br>$\Rightarrow$ time = 0.457 (seconds) | | |
| | | **(4)** | |
| | | **(14 marks)** | |

**Notes:**

**(a)**

**M1:** Use of $T = \frac{\lambda x}{a}$

**M1:** Dependent on the preceding M1. Equate their tensions

**A1:** cao

**M1:** Condone sign error

**A1:** Correct unsimplified equation in *e* and *x* A1A1
Equation with one error A1A0

**A1:** Full working to justify conclusion that it is SHM about the equilibrium position

**(b)**

**B1ft:** Seen or implied. Follow their *e*

**M1:** Correct method for max. speed

**A1ft:** 1.81 or better. Follow their $a, \omega$

**(c)**

**B1:** Seen or implied

**M1:** Use of $x = a\cos wt$

**M1:** Correct strategy for the required interval

**A1:** 0.457 or better