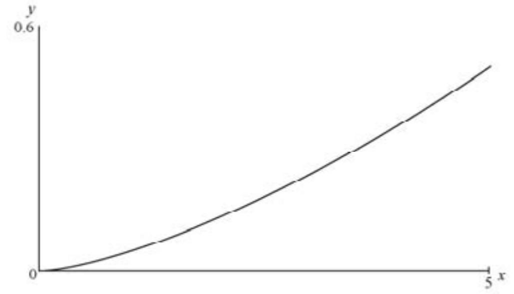
Paper 4E: Further Statistics 2 Mark Scheme

Question	Scheme	Marks	AOs
1(a)	$\mathrm{P}(A > 3) = \frac{2}{5}$	B1	1.1b
	$\left(\frac{2}{5}\right)^3 = \frac{8}{125}$	M1 A1	1.1a 1.1b
		(3)	
(b)	$\mathrm{f}(y) = \frac{3y^2}{125}$	M1	2.1
	$\mathrm{E}(Y) = \int_0^5 \frac{3y^3}{125}\,\mathrm{d}y$ $= \left[\frac{3y^4}{500}\right]_0^5 \qquad \left[= \frac{15}{4}\right]$	M1	1.1b
	$\mathrm{Var}(Y) = \int_0^5 \left(\frac{3y^4}{125}\right)\mathrm{d}y - \left(\frac{15}{4}\right)^2$	M1	1.1b
	$= 0.9375$*	A1*cso	1.1b
		(4)	
(c)	Mode = 5	B1	1.2
	 Or reason based on $\frac{\mathrm{df}(y)}{\mathrm{d}y} > 0$	B1	2.4
		(2)	
(d)	From a sketch or mode > mean therefore it has negative skew	B1ft	2.4
		(1)	
(e)	$\frac{(2k)^3}{125} - \frac{k^3}{125} = 0.189$	M1	3.1a
	$\frac{7k^3}{125} = 0.189$	A1	1.1b
	$k = 1.5$	A1	1.1b
		(3)	
			(13 marks)

Question 1 notes:

(a)

B1: $\frac{2}{5}$ o.e. may be implied by a correct answer

M1: $\left(\text{"their}\left(\frac{2}{5}\right)\text{"}\right)^3$ may be implied by a correct answer

A1: $\frac{8}{125}$ o.e.

(b)

M1: Realising that firstly need to find pdf f(y) and attempt to differentiate F(y)

M1: Continuing the argument with an attempt to integrate y × "their f(y)"
$y^n \to y^{n+1}$

M1: Integrating y^2 × "their f(y)" - ["their E(Y)"]^2 $y^n \to y^{n+1}$

A1*: Complete correct solution no errors

(c)

B1: 5 only

B1: Explain their reason by either an accurate sketch or $\frac{\mathrm{df}(y)}{\mathrm{d}y} > 0$ therefore an increasing function o.e.

(d)

B1ft: Explaining the reason for their answer. Follow through their part(b) or mean from(d) and mode from(c). A correct sketch of "their f(y)" – may be seen anywhere in question or ft their mean and mode plus a correct conclusion

NB: Watch for gaming. A student who writes both negative skew with a reason and positive skew with a reason. Please send these to your Team Leader

(e)

M1: Attempting to translate the problem into an equation using $2k$ and k. Allow if the brackets are missing e.g. $\frac{2k^3}{125} - \frac{k^3}{125}$. No need for the 0.189

A1: A correct equation in any form

A1: A correct answer only

Question	Scheme	Marks	AOs
2(a)	$\mathrm{H}_0 : \rho = 0, \mathrm{H}_1 : \rho > 0$	B1	2.5
	Critical value at 1% level is 0.8929	B1	1.1b
	r_s < 0.8929 so not significant evidence to reject H_0	M1	2.1
	The researcher's claim is not correct (at 1% level) **or** insufficient evidence for researcher's claim **or** there is insufficient evidence that water gets deeper further from inner bank **or** no (positive) correlation between depth of water and distance from inner bank	A1ft	2.2b
		(4)	
(b)(i)	The **ranks will remain the same** therefore there will be **no change** to the spearman's rank correlation coefficient	**B1**	2.4
(ii)	Spearman's rank correlation coefficient will **increase** since	**B1**	2.2a
	The **ranks are the same** for both distance and depth therefore ***d* = 0** however, ***n* has increased or** the new position follows the pattern that large *b* is associated with large *s* and so r_s will increase	**B1**	2.4
		(3)	
(c)	The mean of the tied ranks is given to each…	B1	2.4
	… then use PMCC	B1	2.4
		(2)	
		(9 marks)	

Notes:

(a)
B1: Both hypotheses correct written using the notation ρ
B1: awrt 0.893
M1: Drawing a correct inference using their answer to part(a) and their CV
A1ft: Drawing a correct inference in context using their answer to part(a) and their CV

(b)(i)
B1: Stating **no change** and an explanation including **ranks remain unchanged** o.e. and **no change o.e.**

(b)(ii)
B1: Interpreted the outcome of adding a point as **increased** oe
B1: Explaining why. Need to mention the **ranks are the same for both oe** and ***n* has increased oe**

(c)
B1: Explaining that the mean of the values for the tied ranks is given to both values
B1: Explaining that the PMCC must be used

Question	Scheme	Marks	AOs
3(a)	95% CI for μ uses t value of **2.064**	B1	3.3
	$\frac{\hat{\sigma}}{\sqrt{25}} \times "2.064" = \frac{1}{2}(2.232 - 1.128)$ <u>or</u> $\frac{1}{2}(2.232 + 1.128) + "2.064" \times \frac{\hat{\sigma}}{\sqrt{25}} = 2.232$ (oe)	M1	2.1
	$\hat{\sigma} = \frac{2.76}{"2.064"}$ <u>or</u> 1.3372…	M1	1.1b
	$\hat{\sigma}^2 = 1.788...$ [=1.79 (3sf)] *	A1*cso	1.1b
		(4)	
(b)	12.401, $< \frac{24 \times 1.79}{\sigma^2} <$, 39.364	B1 M1	1.1b 1.1a
	<u>**1.09**</u> $< \sigma^2 <$ <u>**3.46**</u>	A1	1.1b
		(3)	
			(7 marks)

Notes:

(a)

B1: Realising that the t-distribution must be used as a model and finding the correct value awrt 2.06

M1: Using the correct formula with a t-value, $\frac{\hat{\sigma}}{\sqrt{25}} \times "t \text{ value}" = \frac{1}{2}(2.232 - 1.128)$ or

$$\frac{1}{2}(2.232 + 1.128) + "t \text{ value}" \times \frac{\hat{\sigma}}{\sqrt{25}} = 2.232 \quad \textbf{or}$$

$$\frac{1}{2}(2.232 + 1.128) - "t \text{ value}" \times \frac{\hat{\sigma}}{\sqrt{25}} = 1.128$$

M1: Rearranging one of these formula accurately to find a value of $\hat{\sigma}$

A1cso*: A correct solution only using awrt 1.79

(b)

B1: awrt 12.4 or 39.4 May be implied by a correct confidence interval

M1: $\frac{24 \times 1.79}{\sigma^2}$ May be implied by a correct confidence interval

A1: awrt 1.09 and awrt 3.46

Question	Scheme	Marks	AOs
4(a)	H_0: $\sigma_G^2 = \sigma_B^2$, H_1: $\sigma_G^2 \neq \sigma_B^2$,	B1	2.5
	$s_B^2 = \frac{1}{6}(56130 - 7 \times 88.9^2) = \dfrac{807.53}{6} = 134.6$	M1 A1	2.1 1.1b
	$s_G^2 = \frac{1}{7}(55746 - 8 \times 83.1^2) = \dfrac{501.12}{7} = 71.58$	A1	1.1b
	$\dfrac{s_B^2}{s_G^2} = 1.880\ldots$	M1	3.4
	Critical value $F_{6,7} = 3.87$	B1	1.1b
	Not significant, variances can be treated as the same	A1 ft	2.2b
		(7)	
(b)	H_0: $\mu_B = \mu_G$, H_1: $\mu_B > \mu_G$	B1	2.5
	Pooled estimate of variance $s^2 = \dfrac{6 \times 134.6 + 7 \times 71.58}{13} = 100.6653\ldots$	M1	3.1b
	Test statistic $t = \dfrac{88.9 - 83.1}{s\sqrt{\frac{1}{7} + \frac{1}{8}}}$ = awrt 1.12	M1 A1	1.1b 1.1b
	Critical value $t_{13}(5\%) = 1.771$	B1	1.1b
	Insufficient evidence to support mother's claim	A1 ft	2.2b
		(6)	
		(13 marks)	

Notes:

(a)

B1: Both hypotheses correct using the notation σ^2 **.** Allow σ rather than σ^2

M1: Using a correct Method for either s_B^2 or s_G^2 May be implied by a correct value

A1: awrt 135

A1: awrt 71.6

M1: Using the F-distribution as the model e.g. $\dfrac{s_B^2}{s_G^2}$

B1: awrt 3.87

A1ft: Drawing a correct inference following through their CV and value for $\dfrac{s_B^2}{s_G^2}$

(b)

B1: Both hypotheses correct using the notation μ

M1: For realising the need to find the pooled estimate for the test require from a correct interpretation of the question

M1: Correct method for test statistic $t = \dfrac{88.9 - 83.1}{\text{"their } s\text{"}\sqrt{\frac{1}{7} + \frac{1}{8}}}$ May be implied by a correct awrt 1.12

A1: awrt 1.12

B1: awrt 1.77

A1ft: Drawing a correct inference following through their CV and test statistic

Question	Scheme	Marks	AOs
5(a)	Let $X = L - 4S$ then E(X) = 19.6 – 4 $\times$ 4.8	M1	2.3
	= 0.4	A1	1.1b
	Var(X) = $\text{Var}(L) + 4^2\text{Var}(S) = 0.6^2 + 16 \times 0.3^2$	M1	2.1
	= 1.8	A1	1.1b
	P(X > 0) = $\left[\text{P}\left(Z > \dfrac{0-0.4}{\sqrt{1.8}} = -0.298......\right)\right]$	M1	2.1
	= 0.617202... awrt **<u>0.617</u>**	A1	1.1b
		(6)	
(b)	$T = S_1 + S_2 + S_3 + S_4$ (May be implied by 0.36)	M1	3.3
	$T \sim$ N(19.2, 0.36) E(T) = 19.2	B1	1.1b
	Var(T) = 0.36 **or** 0.6^2	A1	1.1b
		(3)	
(c)	Let $Y = L - T$ E(Y) = E(L) – E(T) = [0.4]	M1	3.3
	Var(Y) = Var(L) + Var(T) = [0.72]	M1	1.1b
	Require P(– 0.2 < Y < 0.2)	M1	3.1a
	= 0.16708… awrt **<u>0.167</u>**	A1	1.1b
		(4)	
			(13 marks)

Notes:

(a)

M1: Selecting and using an appropriate model i.e. $\pm(L - 4S)$**.** May be implied by 0.4

A1: 0.4 oe

M1: For realising the need to use Var(L) + 4^2Var(S). Allow use of 0.6 for Var(L) instead of 0.6^2 and/or 0.3 for Var(S) instead of 0.3^2 may be implied by 1.8

A1: 1.8 only

M1: For realising P(X > 0) is required and an attempt to find it e.g. $\dfrac{0-0.4}{\sqrt{\text{"their Var}(X)\text{"}}}$ but do not allow a negative Var(X)

A1: awrt 0.617

(b)

M1: Selecting and using an appropriate model ie $S_1 + S_2 + S_3 + S_4$: may be implied by 0.36

B1: 19.2 only

A1: 0.36

(c)

M1: Setting up and using the model $Y = L - T$. May be implied by $E(Y) = E(L) - E(T)$

M1: Using Var(Y) = Var(L) + Var(T)

M1: Dealing with the modulus and realising they need to find P(– 0.2 < Y < 0.2)

A1: awrt 0.167

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>6(a)</td><td>$\left[b = \frac{S_{xm}}{S_{xx}} = -\ 0.0277576\right]$</td><td>M1</td><td>3.3</td></tr>
<tr><td></td><td>$[a = \bar{m} - b\bar{x} = 1.278 + 0.0277576 \times 8.5 = 1.5139]$</td><td></td><td></td></tr>
<tr><td></td><td>$m = 1.5139 - 0.02775\ldots x$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>$\text{RSS} = 0.12756 - \frac{(-2.29)^2}{82.5}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>= 0.06399*</td><td>A1*</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td>
<table>
<tr><td>x</td><td>m</td><td>$m = a + bx$</td><td>ε</td></tr>
<tr><td>4</td><td>1.50</td><td>1.4029</td><td>+0.0971</td></tr>
<tr><td>5</td><td>1.20</td><td>1.3752</td><td>– 0.1752</td></tr>
<tr><td>6</td><td>1.40</td><td>1.3474</td><td>+0.0526</td></tr>
<tr><td>7</td><td>1.40</td><td>1.3196</td><td>+0.0804</td></tr>
<tr><td>8</td><td>1.23</td><td>1.2919</td><td>– 0.0619</td></tr>
<tr><td>9</td><td>1.30</td><td>1.2641</td><td>+0.0359</td></tr>
<tr><td>10</td><td>1.20</td><td>1.2364</td><td>– 0.0364</td></tr>
<tr><td>11</td><td>1.15</td><td>1.2086</td><td>– 0.0586</td></tr>
<tr><td>12</td><td>1.25</td><td>1.1808</td><td>+0.0692</td></tr>
<tr><td>13</td><td>1.15</td><td>1.1531</td><td>– 0.0031</td></tr>
</table>
</td><td>M1
A1</td><td>3.4
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(d)</td><td>The point (5, 1.2) is an outlier</td><td>B1ft</td><td>2.2b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(e)(i)</td><td>It is a valid piece of data so should be used
or
It does not follow the pattern according to the residuals so may contain an error making the result invalid so should be removed</td><td>B1</td><td>2.4</td></tr>
<tr><td>(ii)</td><td>$a = \bar{m} - b\bar{x} = 1.28667 + 0.03765 \times 8.88889 = 1.6213$</td><td>M1</td><td>3.3</td></tr>
<tr><td></td><td>$m = 1.6213 - 0.03765x$</td><td>A1</td><td>1.1b</td></tr>
<tr><td>(iii)</td><td>$m = 1.6213 - 0.03765 \times 15$</td><td></td><td></td></tr>
<tr><td></td><td>= 1.056 or awrt 1.06</td><td>B1ft</td><td>3.4</td></tr>
<tr><td>(iv)</td><td>The model is only reliable if the values are limited to those in the given range so probably not reliable</td><td>B1</td><td>3.5b</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td colspan="4">(12 marks)</td></tr>
</table>

Question 6 notes:	
6(a)	
M1:	Realising the need to use $b = \dfrac{S_{xm}}{S_{xx}}$ and $a = \ \bar{m} - b\bar{x}$
A1:	m = awrt 1.51) – (awrt 0.0278) x. Award M1A1 for correct equation
(b)	
M1:	Using $S_{mm} - \dfrac{(S_{xm})^2}{S_{xx}}$
A1*:	awrt 0.064
(c)	
M1:	Using the model in part (a) i.e. m – (“1.5139” – “0.02775”x) implied by a correct value
A1:	All correct. Award M1A1 for a list of correct residuals
(d)	
B1:	Inferring from the residuals that the outlier is (5, 1.2) ft their residuals.
(e)(i)	
B1:	Explaining why the outlier should be removed or not.
(ii)	
M1:	Removing the outlier and refining the model by finding a new regression line.
A1:	m = (awrt 1.62) – (awrt 0.0377)x
(iii)	
B1ft:	using their model in e(i) with $x = 15$. awrt 1.06 or ft their e(ii)
(iv)	
B1:	Realising the limitations of the model by stating it is <u>not reliable</u> and giving the reason why i.e. extrapolation/out of range o.e.

Question	Scheme	Marks	AOs
7(a)	$S_{xx} = \sum(10s)^2 - \frac{\left(\sum 10s\right)^2}{10}$	M1	2.1
	$2658.9 = 100\sum(s)^2 - \frac{100\left(\sum s\right)^2}{10}$	M1	1.1b
	$2658.9 = 100\ S_{ss}$		
	$S_{ss} = 26.589$ *	A1*cso	1.1b
		(3)	
(b)	$64 = \sum_{1}^{10} 10(d_i - 9)$	M1	3.1a
	$64 = 10\sum_{1}^{10} d_i - 900$		
	$\sum_{1}^{10} d_i = 96.4$	A1	1.1b
	$S_{dd} = 1081.74 - \frac{("96.4")^2}{10}$	M1	1.1b
	$= 152.444$		
	$r = 0.935$	A1ft	1.1b
		(4)	
(c)	Linear correlation is significant but scatter diagram suggests a non-linear relationship between the level of serum magnesium, and the level of the disease protein	B1	3.5a
		(1)	
		(8 marks)	

Notes:

(a)

M1: Attempting to use $S_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{10}$ with $x = 10s$

M1: Substituting in 2658.9 and dealing with the 10 correctly

A1*: cso A complete solution with no errors leading to 26.589 only

(b)

M1: Realising that either $64 = \sum_{1}^{10} 10(d_i - 9)$ or $64 = 10\sum_{1}^{10} d_i - 900$ o.e. must be used. May be implied by seeing 96.4

A1: 96.4 only

M1: Attempting to use $S_{dd} = \sum d^2 - \frac{\left(\sum d\right)^2}{10}$ may be implied by 0.935

A1ft: awrt 0.935 ft "their 96.4"

(c)

B1: A correct comment comparing their value of r and the scatter diagram in context