FURTHER MATHEMATICS

General Certificate of Education

Summer 2023

Advanced Subsidiary/Advanced

FURTHER PURE MATHEMATICS B – A2 UNIT 4

Overview of the Unit

The candidates performed very well on a high number of occasions and there were some excellent scripts. However, some candidates encountered difficulties with the requirements of some questions and poor algebraic skills were often seen, leading to low marks being awarded.

Comments on individual questions/sections

Q.1 This question was more poorly answered than expected. In part (a), some candidates seemed unfamiliar with composite functions, which was disappointing, as this topic is in the GCE Mathematics Unit 3 subject content. Some candidates gave the correct domain for fg, but were not precise enough with the range: $(1, \infty)$ was often seen rather than $[1, \infty)$. In part (b), most candidates were able to find the value of x to 3 decimal places; however, some candidates failed to take note of the domain and wrote ±1.662 as their final answer, losing the final accuracy mark. Some candidates decided to work with the exponential or logarithmic forms of $\cosh x$, which could lead to the correct answer, but candidates' responses often contained arithmetic errors.

Q.2 Both parts of this question were well answered, with the only errors in part (a) being arithmetic when calculating the cofactors. In part (b), there was an equal proportion of candidates either completing the square, calculating the discriminant, or solving the equation, but not all candidates were able to explain why the matrix was singular following this.

Q.3 Part (a) was well-answered, although some candidates should remember to show sufficient working in their responses, particularly when the answer is given in the question. In part (b), most candidates were able to expand $\left(z + \frac{1}{z}\right)^6$, but not all appreciated the need to halve their answer. Some candidates began by expanding $(\cos\theta + \mathrm{i}\sin\theta)^6$, and this approach could have led to the required form with extensive working, but few made progress.

Q.4 This question was well answered, but arithmetic errors meant some candidates lost accuracy marks unnecessarily. More than half the candidates opted to begin with row reductions, with some continuing to echelon form, whilst others formed simultaneous equations in two variables, before solving the equations. A significant proportion of candidates used the inverse matrix approach. A high level of success was seen with each of the methods.

Q.5 In part (a), the majority of candidates used the Maclaurin expansion of $\sin x$, replacing x with $2x$, but many candidates also chose to use the expansion of $f(x)$ – both methods proved successful for candidates. Part (b) was poorly answered. Many candidates tried to use the double-angle formulae for $\sin 2x$ to obtain an expression for $\cos x$, to be subsequently squared.

Few candidates seemed to be familiar with the idea of differentiating Maclaurin expansions to obtain other expansions; those who did, often earned full marks.

Q.6 All parts of this question were well answered. However, errors occurred in part (b) when differentiating using the product rule and simplifying $\sin\theta$ and $\cos\theta$ terms to give $\tan\theta$, which was disappointing, as these are GCE Mathematics Unit 3 skills. In part (c), some candidates made errors in manipulating the fractions to form an equation, whilst others divided by t, losing a set of coordinates.

Q.7 This question was well answered, with candidates choosing to write the complex number in either exponential form or trigonometric form. Disappointingly, errors occurred when calculating the modulus and argument of z, the required pre-requisite skills from Further Mathematics Unit 1.

Q.8 Most candidates answered this question very well. However, not all candidates were able to complete the square correctly. Other errors, although infrequent, were omitting the ½ in part (a), and π in part (b), or not giving their answers to the required level of accuracy.

Q.9 Although many candidates achieved full marks on this question, some candidates made careless algebraic errors, e.g. when dividing by $(x+1)^5$, candidates forgot to divide the constant of integration by this term. When little credit was given, this was often due to an incorrect integrating factor.

Q.10 This question proved to be a very challenging question for many candidates, with few fully correct solutions seen. In part (a), some candidates used the chain rule and the result in the Formula Booklet, rather than a proof of the derivative. There was an equal proportion of candidates using the different methods given in the mark scheme for part (a), but, in both instances, the final mark was often withheld, as very few candidates were able to justify why the positive square root was chosen. In part (b), candidates were not often successful in finding the required range of values. The most successful candidates realised that $2x+5$ must strictly lie between -1 and 1, whilst some candidates found the values of -2 and -3, but did not note the range correctly.

Q.11 In part (a), there was an equal proportion of candidates using the three methods given in the mark scheme, with each method proving successful for candidates. In part (b), most candidates spotted the need for partial fractions, but not all were able to find the correct values in their expression. Of those that did, not all could split the $\frac{ax+b}{x^2+9}$ term into two terms in order to proceed with the required integration. However, it was pleasing to see many fully correct solutions for this question.

Q.12 This question was well answered. Common errors were dividing throughout by $\cos\theta$ and thereby losing some solutions, or multiplying the general solutions of 3θ by 3, instead of dividing by 3.

Q.13 This question was answered better than expected, with full credit given on many occasions. In part (a) there was a variety of methods used to reach the required result. The general solution of x often followed correctly, although some candidates noted this as y in terms of x rather than x in terms of t, and consequently lost a mark. In part (b), most candidates realised that they needed to use the general solution of x and the differential equations given in the stem of the question, but some repeated the processes used in part (a)(i), only to arrive at a similar result as that in (a)(i). In part (c), errors were often in the arithmetic. Candidates who made errors in parts (a) and (b) often found that they had more complex expressions to deal with in part (c).