GCE A LEVEL MARKING SCHEME

SUMMER 2023

A LEVEL
FURTHER MATHEMATICS
UNIT 6 FURTHER MECHANICS B
1305U60-1

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INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

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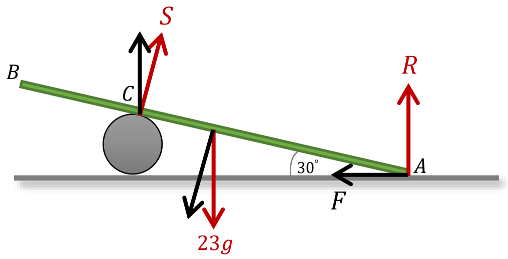
WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 6 FURTHER MECHANICS B

SUMMER 2023 MARK SCHEME

Q1	Solution	Mark	Notes
(a)			
	Resolve vertically	M1	Dim. correct equation with 3 terms
	$S\cos 30° + R = 23g$ $\frac{\sqrt{3}}{2}S + R = 23g$	A1	$23g = 225 \cdot 4 = \frac{1127}{5}$
	Resolve horizontally	M1	Dim. correct equation
	$F = S\sin 30°$ $F = \frac{1}{2}S$	A1	
	$F = \frac{2}{3}R$ $\frac{2}{3}R = S\sin 30°$ $\frac{2}{3}R = \frac{1}{2}S$ $R = \frac{3}{4}S$	B1	Used
	$\frac{\sqrt{3}}{2}S + \frac{3}{4}S = 23g$	m1	Elimination of one variable Both M's needed above
	$S = 139 \cdot 47(80054)$ (N) $R = 104 \cdot 60(85041)$ (N)	A1 A1	cao cao
	Note exact forms $$S = \frac{92g}{3}(2\sqrt{3} - 3) = \frac{92g}{2\sqrt{3} + 3}$$ $$R = 23g(2\sqrt{3} - 3) = \frac{69g}{2\sqrt{3} + 3}$$	**[8]**	
(b)	Moments about A	M1	Dim. correct equation, no extra terms
	$AC \times S = 23g \times 4\cos 30°$	A1 A1	LHS RHS
	$AC \times 139 \cdot 478 ... = 225 \cdot 4 \times 2\sqrt{3}$		
	$AC = \frac{23g \times 4\cos 30°}{139 \cdot 4780054}$ $\frac{225 \cdot 4 \times 2\sqrt{3}}{139 \cdot 4780054}$		FT S from (a)
	$AC = 5 \cdot 598076211$ (m) or $= \frac{6+3\sqrt{3}}{2}$ (m)	A1	cao
		[4]	

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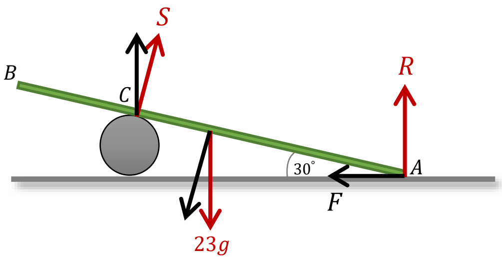
(c)	No, both reactions remain the same **and** (one of the following) • Calculations in (a) are **independent** of the location of the centre of mass • **Moments** were **not used** in part (a) and so distances were not considered	E1 **[1]**	
	Total for Question 1	**13**	

Q1	Alternative Solution	Mark	Notes
(a)			
	Resolve parallel to AB	M1	Dim. correct equation with 3 terms
	$23g\sin 30° = R\sin 30° + F\cos 30°$	A1	$23g\sin 30° = \frac{23}{2}g = \frac{1127}{10} = 112 \cdot 7$
	$\frac{23}{2}g = \frac{1}{2}R + \frac{\sqrt{3}}{2}F \quad \Leftrightarrow \quad 23g = R + \sqrt{3}F$		
	Resolve perpendicular to AB	M1	Dim. correct equation with 4 terms
	$S + R\cos 30° = 23g\cos 30° + F\sin 30°$	A1	
	$S + \frac{\sqrt{3}}{2}R = \frac{23\sqrt{3}}{2}g + \frac{1}{2}F \Leftrightarrow 2S + \sqrt{3}R = 23\sqrt{3}g + F$		
	$F = \frac{2}{3}R$	B1	Used
	Elimination of one variable (in either equation)	m1	
	$23g = R + \sqrt{3}\left(\frac{2}{3}R\right)$		
	$S = 139 \cdot 47(80054)$ (N) $R = 104 \cdot 60(85041)$ (N)	A1 A1 **[8]**	cao cao
	Note exact forms $$S = \frac{92g}{3}(2\sqrt{3} - 3) = \frac{92g}{2\sqrt{3} + 3}$$ $$R = 23g(2\sqrt{3} - 3) = \frac{69g}{2\sqrt{3} + 3}$$		

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Q2	Solution	Mark	Notes
	Shape / **Volume/Mass** / **Distance of COM from base**		
	Large Cone: $\frac{1}{3}\pi(3x)^2(6y)\rho$ $(= 18\pi x^2 y)$; $\frac{1}{4}(6y)$ $\left(=\frac{3y}{2}\right)$	B1	Condone omission of ρ (mass per unit volume) Both volume and distance
	Small Cone: $\frac{1}{3}\pi(x)^2(2y)\rho$ $\left(=\frac{2}{3}\pi x^2 y\right)$; $4y + \frac{1}{4}(2y)$ $\left(=\frac{9y}{2}\right)$	B1	Identification of $2y$ in either
		B1	For both volume and distance
	Frustum: $\left(18\pi x^2 y - \frac{2}{3}\pi x^2 y\right)\rho$ $\left(=\frac{52}{3}\pi x^2 y\right)$; $\bar{y}$	B1	cao
	Moments about base	M1	Masses and moments consistent All terms, allow sign errors FT table throughout
	$\frac{52}{3}\pi x^2 y \times \bar{y} = 18\pi x^2 y \times \frac{3y}{2} - \frac{2}{3}\pi x^2 y \times \frac{9y}{2}$ $\frac{52}{3} \times \bar{y} = 18 \times \frac{3y}{2} - \frac{2}{3} \times \frac{9y}{2}$	A1	
	$\bar{y} = \frac{18}{13}y$	A1	Convincing (cao)
		[7]	
	Total for Question 2	7	

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Q2	Alternative Solution	Mark	Notes
(b)			Condone omission of ρ (mass per unit volume)

Shape	Volume/Mass	Distance of COM from vertex
Large Cone	$\frac{1}{3}\pi(3x)^2(6y)\rho$ $(=18\pi x^2 y)$	$\frac{3}{4}(6y)$ $\left(=\frac{9y}{2}\right)$
Small Cone	$\frac{1}{3}\pi(x)^2(2y)\rho$ $\left(=\frac{2}{3}\pi x^2 y\right)$	$\frac{3}{4}(2y)$ $\left(=\frac{3y}{2}\right)$
Frustum	$\left(18\pi x^2 y-\frac{2}{3}\pi x^2 y\right)\rho$ $\left(=\frac{52}{3}\pi x^2 y\right)$	$\bar{y}$

Mark	Notes
B1	Both volume and distance
B1	Identification of $2y$ in either
B1	Both volume and distance
B1	cao

Moments about vertex

$$\frac{52}{3}\pi x^2 y\times\bar{y}=18\pi x^2 y\times\frac{9y}{2}-\frac{2}{3}\pi x^2 y\times\frac{3y}{2}$$

$$\frac{52}{3}\times\bar{y}=18\times\frac{9y}{2}-\frac{2}{3}\times\frac{3y}{2}$$

$$\bar{y}=\frac{60}{13}y$$

∴ Distance from base $=6y-\frac{60}{13}y=\frac{18}{13}y$

Mark	Notes
M1	Masses and moments consistent All terms, allow sign errors
A1	FT table throughout
A1	Convincing (cao)
[7]	

			Condone omission of ρ (mass per unit volume)

Shape	Volume/Mass	Distance of COM from top of frustum
Large Cone	$\frac{1}{3}\pi(3x)^2(6y)\rho$ $(=18\pi x^2 y)$	$\frac{3}{4}(6y)-2y$ $\left(=\pm\frac{5y}{2}\right)$
Small Cone	$\frac{1}{3}\pi(x)^2(2y)\rho$ $\left(=\frac{2}{3}\pi x^2 y\right)$	$\frac{1}{4}(2y)$ $\left(=\pm\frac{y}{2}\right)$
Frustum	$\left(18\pi x^2 y-\frac{2}{3}\pi x^2 y\right)\rho$ $\left(=\frac{52}{3}\pi x^2 y\right)$	$\bar{y}$

Mark	Notes
B1	Sight of $2y$ in either
B1	Identification of $2y$ in either
B1	Both volume and distance
B1	cao

Moments about smaller circular surface

$$\frac{52}{3}\pi x^2 y\times\bar{y}=18\pi x^2 y\times\pm\frac{5y}{2}-\frac{2}{3}\pi x^2 y\times\mp\frac{y}{2}$$

$$\frac{52}{3}\times\bar{y}=18\times\frac{5y}{2}-\frac{2}{3}\times-\frac{y}{2}$$

$$\bar{y}=\frac{34}{13}y$$

∴ Distance from base $=4y-\frac{34}{13}y=\frac{18}{13}y$

Mark	Notes
M1	Masses and moments consistent All terms, allow sign errors
A1	FT table throughout
A1	Convincing (cao)
[7]	

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Q3	Solution	Mark	Notes
(a)	period $= 12$ (hours) $\qquad T = 12$	B1	si
	amplitude $= 4$ (m) $\qquad a = 4$	B1	si
		[2]	
(b)	$\omega = \frac{\pi}{6}$ $\qquad \omega = \frac{\pi}{21600}$	B1	$\omega = \frac{2\pi}{\text{period}}$ FT period from (a)
	Using $x = \pm a\cos(\omega t)$	M1	Allow $\pm a\sin(\omega t)$
	$x = -4\cos\left(\frac{\pi}{6}t\right)$	A1	oe, cao
		[3]	
(c)	$-2 = -4\cos\left(\frac{\pi}{6}t\right)$	M1	FT x from (b)
	$t = 2$ (hours)	A1	cao
	$t = 10$ (hours)	A1	cao
	Earliest time: 7 a.m. and Latest time: 3 p.m.	A1	cao, both times
		[4]	
(d)	$v = \frac{\mathrm{d}x}{\mathrm{d}t}$	M1	FT x from (b)
	$v = 4\sin\left(\frac{\pi}{6}t\right) \times \frac{\pi}{6}$	A1	
	$v = \frac{2}{3}\pi\sin\left(\frac{\pi}{6}t\right)$		
	At $t = 9$, $v = \frac{2}{3}\pi\sin\left(\frac{\pi}{6} \times 9\right)$	M1	FT v
	$v = -\frac{2}{3}\pi$ $\qquad = -2 \cdot 0(94\ldots)$		
	Rate (at which the level of water is falling) $= \frac{2}{3}\pi$ (m/hour)	A1	
		[4]	
	Total for Question 3	13	

Further Notes	
(b)	Equivalent forms for A1
	$x = -4\cos\left(\frac{\pi}{6}t\right) \quad = 4\sin\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) \quad = -4\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right) \quad = 4\cos\left(\frac{\pi}{6}t \pm \pi\right)$
	Corresponding derivatives for part (d)
	$v = x' = 4\sin\left(\frac{\pi}{6}t\right) \times \frac{\pi}{6} \quad = 4\cos\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) \times \frac{\pi}{6} \quad = -4\cos\left(\frac{\pi}{6}t + \frac{\pi}{2}\right) \times \frac{\pi}{6} \quad = -4\sin\left(\frac{\pi}{6}t \pm \pi\right) \times \frac{\pi}{6}$

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Q3	Alternative Solution	Mark	Notes
	Using $t = 9$ (5 a.m. to 2 p.m.) to deduce that	M1	$x = -4\cos\left(\frac{\pi}{6} \times 9\right)$
	$x = 0$	A1	
	Using an expression for v with $x = 0$	m1	
	$v = \pm\begin{cases}\omega\sqrt{a^2 - 0^2} \\ \omega a\end{cases}$		
	$v = \pm\frac{2}{3}\pi$ $\quad = \pm 2\cdot 0(94\ldots)$		
	Rate (at which the level of water is falling) $= \frac{2}{3}\pi$ (m/hour)	A1	
		[4]	

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Q4	Solution	Mark	Notes
(a)			

Shape	Mass	Distance from AC	Distance from AB	Mark	Notes
	$\frac{\pi(12)^2}{4} \times 2m$ $(= 72m\pi)$	$\frac{16}{\pi}$	$\frac{16}{\pi}$	B1 B1	Correct mass Both distances correct
C	$50m$	0	28		
F	$30m$	$22 \cdot 5$	14	B2	Masses and distances for C, F, B
B	$20m$	45	0		
Lamina	$(72\pi + 100)m$	$\bar{x}$	$\bar{y}$	B1	cao

Q4	Solution	Mark	Notes
	(i) Moments about AC	M1	Masses and moments consistent All terms, allow sign errors FT table
	$\left((72\pi)\left(\frac{16}{\pi}\right) + (30)(22 \cdot 5) + (20)(45)\right)m = (72\pi + 100)m \times \bar{x}$	A1	
	$(1152 + 675 + 900)m = (72\pi + 100)m \times \bar{x}$		
	$\bar{x} = 8 \cdot 36(0038474)$ (cm)	A1	cao
	(ii) Moments about AB	M1	Masses and moments consistent All terms, allow sign errors FT table
	$\left((72\pi)\left(\frac{16}{\pi}\right) + (30)(14) + (50)(28)\right)m = (72\pi + 100)m \times \bar{y}$	A1	
	$(1152 + 420 + 1400)m = (72\pi + 100)m \times \bar{y}$		
	$\bar{y} = 9.11(1123705)$ (cm)	A1 **[11]**	cao
(b)	Length $AP = \bar{y}$	B1 **[1]**	FT $\bar{y}$ from (a)(ii)

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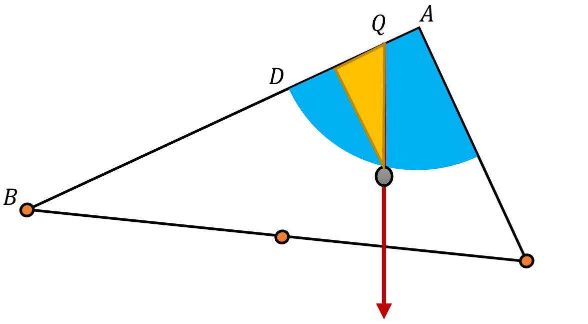
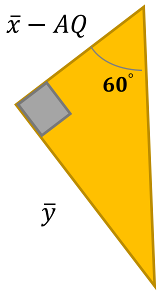
(c)			
	If hanging in equilibrium, vertical passes through centre of mass.	M1	Correct triangle identified FT $\bar{x}$ and $\bar{y}$ from (a)
	$\tan 60° = \dfrac{\bar{y}}{\bar{x} - AQ}$ $\quad\quad$ $\sqrt{3} = \dfrac{9.11 \ldots}{8 \cdot 36 \ldots - AQ}$	A1	FT $\bar{x}$ and $\bar{y}$ from (a) $\bar{x} - AQ = \dfrac{\bar{y}}{\tan 60°} = \bar{y} \cot 60°$ $= 5 \cdot 2603 \ldots$
	$AQ = \bar{x} - \frac{\bar{y}}{\sqrt{3}}$		
	$AQ = 3.0997 \ldots\ldots$ $\quad$ (cm)	A1	FT $\bar{x}$ and $\bar{y}$ from (a) provided $AQ < 12$, i.e. Q lies on AD
		[3]	
	Total for Question 4	**15**	

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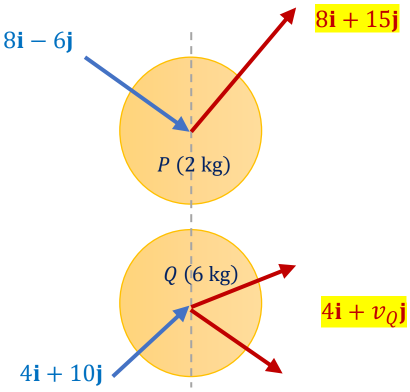
Q5	Solution	Mark	Notes
(a)	$\mathbf{r}_P = (8\mathbf{i} - 6\mathbf{j})t$	B1	
	$\mathbf{r}_Q = (12\mathbf{i} - 48\mathbf{j}) + (4\mathbf{i} + 10\mathbf{j})t$	B1	
	If spheres collide, then $\mathbf{r}_P = \mathbf{r}_Q$ for some value of t. Comparison of coefficients	M1	Both i and j coefficients compared
	$\mathbf{i}$ $\quad 8t = 12 + 4t$ $t = 3$		
	$\mathbf{j}$ $\quad -6t = -48 + 10t$ $t = 3$		
	Value of t for both components are equal, therefore spheres collide.	A1	
		[4]	
(b)			Before collision After collision
	Speed $= 5$ $\sqrt{4^2 + {v_Q}^2} = 5$	M1	Used
	$v_Q = \pm 3$	A1	At least one value
	Con. of momentum (along line of centres)	M1	Attempted
	$(2)(-6) + (6)(10) = 2v_P + 6v_Q$ $48 = 2v_P + (6)(\pm 3)$	A1	All correct. FT v_Q
	$v_Q = +3 \quad \Rightarrow \quad v_P = 15$ $v_Q = -3 \quad \Rightarrow \quad v_P = 33$	A1	Both values of v_P
	Restitution (along line of centres) $v_Q - v_P = -e(10 - -6)$	M1	FT v_Q and v_P
	$3 - 15 = -e(10 - -6)$ OR $-3 - 33 = -e(10 - -6)$ $e = \frac{3}{4}$ $\qquad e = \frac{9}{4}$	A1	All correct with their v_P corresponding to either v_Q
	$e = \frac{3}{4}$	A1	Both values of e with $e\left(= \frac{9}{4}\right) > 1$ clearly discarded
	Velocity of sphere P, $\mathbf{v}_P = 8\mathbf{i} + 15\mathbf{j}$ $\quad (\text{ms}^{-1})$	A1	cao
		[9]	

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(c)	Impulse, $\mathbf{I}$ = change in momentum $\mathbf{I} = 2\big(0 - (8\mathbf{i} + 15\mathbf{j})\big)$ $\mathbf{I} = -(16\mathbf{i} + 30\mathbf{j})$ $\lvert\mathbf{I}\rvert = 34 \ \ (\text{Ns})$	M1 A1 A1 **[3]**	Used FT $\mathbf{v}_P$ cao
Total for Question 5		**16**	

Q5	Alternative Solutions	Mark	Notes
(a)	$\mathbf{r}_Q - \mathbf{r}_P = (12 - 4t)\mathbf{i} + (-48 + 16t)\mathbf{j}$ $\mathbf{r}_Q - \mathbf{r}_P = 0$ $\Rightarrow \quad 12 - 4t = 0 \quad \text{and} \quad -48 + 16t = 0$ $t = 3$ for both components are equal, therefore spheres collide.	B1 B1 M1 A1 **[4]**	B1 for each component
(a)	$\mathbf{r}_Q - \mathbf{r}_P = (12 - 4t)\mathbf{i} + (-48 + 16t)\mathbf{j}$ Forming and solving a quadratic equation $\lvert\mathbf{r}_Q - \mathbf{r}_P\rvert^2 = 272t^2 - 1632t + 2448 = 0$ $t = 3$ or $b^2 - 4ac = 0$, therefore spheres collide.	B1 B1 M1 A1 **[4]**	B1 for each component

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Q6	Solution	Mark	Notes
(a)	Application of N2L	M1	Dim. correct. $250\,000v$ and T in same direction
	$-T - 250\,000v = 50\,000a$	A1	
	$T = 312\,500x$	B1	$a = \frac{d^2x}{dt^2}$
	$-312\,500x - 250\,000v = 50\,000\frac{d^2x}{dt^2}$		$\div 12\,500$
	$4\frac{d^2x}{dt^2} + 20\frac{dx}{dt} + 25x = 0$	A1	Convincing
		[4]	
(b)	Auxiliary equation: $4r^2 + 20r + 25 = 0$ Roots: $r = -\frac{5}{2}$ (twice)	M1 A1	
	General solution: $x = e^{-\frac{5}{2}t}(At + B)$	B1	
	Initial conditions $t = 0,\ x = 0,\ (x' = U)$	M1	Used in general solution
	$B = 0$ $\qquad U = -\frac{5}{2}B + A$	A1	cao
	Differentiating $v = x' = -\frac{5}{2}e^{-\frac{5}{2}t}(At + B) + e^{-\frac{5}{2}t}A$	M1 A1	$U = -\frac{5}{2}B + A$ Initial conditions give $A = U$
	$\therefore\ x = Ue^{-\frac{5}{2}t}t$	A1	cao
		[8]	
(c)	$v = x' = Ue^{-\frac{5}{2}t}\left(1 - \frac{5}{2}t\right)$		
	When $v = 0$ $(\Rightarrow\ t = \frac{2}{5})$	M1	FT $v = x'$
	Using $x = Ue^{-\frac{5}{2}t}t$ at $t = \frac{2}{5}$ and $x = 0 \cdot 3$.	m1	FT $v = x'$ and $t > 0$
	$0 \cdot 3 = Ue^{-\frac{5}{2}\left(\frac{2}{5}\right)}\left(\frac{2}{5}\right)$		
	$U = \frac{3}{4}e = 2.0387\ \ldots$ $\qquad (\text{ms}^{-1})$	A1	cao
		[3]	
(d)	Critical damping Repeated root in (b)	E1 **[1]**	oe, $b^2 - 4ac = 0$ **or** $k^2 - \omega^2 = 0$ when written in the form $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega^2 x = 0$
	Total for Question 6	**16**	

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