GCE

FURTHER MATHEMATICS

UNIT 4: FURTHER PURE MATHEMATICS B

SAMPLE ASSESSMENT MATERIALS

(2 hour 30 minutes)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Evaluate the integral

$$\int_0^\infty \frac{\mathrm{d}x}{(1+x)^5}. \qquad [3]$$

(b) By putting $u = \ln x$, determine whether or not the following integral has a finite value.

$$\int_2^\infty \frac{\mathrm{d}x}{x\ln x}. \qquad [4]$$

2. Evaluate the integral

$$\int_0^1 \frac{\mathrm{d}x}{\sqrt{2x^2+4x+6}}. \qquad [6]$$

3. The curve C has polar equation $r = 3(2 + \cos\theta)$, $0 \le \theta \le \pi$. Determine the area enclosed between C and the initial line. Give your answer in the form $\frac{a}{b}\pi$, where a and b are positive integers whose values are to be found. [5]

4. Find the three cube roots of the complex number $2 + 3\mathrm{i}$, giving your answers in Cartesian form. [9]

5. Find all the roots of the equation

$$\cos\theta + \cos 3\theta + \cos 5\theta = 0$$

lying in the interval $[0, \pi]$. Give all the roots in radians in terms of π. [8]

6. The matrix $\mathbf{M}$ is given by

$$\mathbf{M}=\begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 5 \end{bmatrix}.$$

(a) Find

(i) the adjugate matrix of $\mathbf{M}$,

(ii) hence determine the inverse matrix $\mathbf{M}^{-1}$. [5]

(b) Use your result to solve the simultaneous equations

$$\begin{aligned} 2x + y + 3z &= 13 \\ x + 3y + 2z &= 13 \\ 3x + 2y + 5z &= 22 \end{aligned}$$

[2]

7. The function f is defined by

$$f(x) = \frac{8x^2 + x + 5}{(2x+1)(x^2+3)}.$$

(a) Express $f(x)$ in partial fractions. [4]

(b) Hence evaluate

$$\int_2^3 f(x)\mathrm{d}x\,,$$

giving your answer correct to three decimal places. [6]

8. The curve $y = 1 + x^3$ is denoted by C.

(a) A bowl is designed by rotating the arc of C joining the points (0,1) and (2,9) through four right angles about the y-axis. Calculate the capacity of the bowl. [5]

(b) Another bowl with capacity 25 is to be designed by rotating the arc of C joining the points with y coordinates 1 and a through four right angles about the y-axis. Calculate the value of a. [5]

9. (a) Use mathematical induction to prove de Moivre's Theorem, namely that

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta,$$

where n is a positive integer. [7]

(b) (i) Use this result to show that

$$\sin 5\theta = a\sin^5\theta - b\sin^3\theta + c\sin\theta,$$

where a, b and c are positive integers to be found.

(ii) Hence determine the value of $\lim_{\theta \to 0} \dfrac{\sin 5\theta}{\sin\theta}$ [7]

10. Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\tan x = \sin x, \qquad 0 < x < \frac{\pi}{2}.$$

(a) Find an integrating factor for this differential equation. [4]

(b) Solve the differential equation given that $y = 0$ when $x = \frac{\pi}{4}$, giving your answer in the form $y = f(x)$. [7]

11. (a) Show that

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), \qquad \text{where } -1 < x < 1.$$ [4]

(b) Given that

$$a\cosh x + b\sinh x \equiv r\cosh(x + \alpha), \qquad \text{where } a > b > 0,$$

show that

$$\alpha = \frac{1}{2}\ln\left(\frac{a+b}{a-b}\right)$$

and find an expression for r in terms of a and b. [7]

(c) Hence solve the equation

$$5\cosh x + 4\sinh x = 10,$$

giving your answers correct to three significant figures. [6]

12. The function f is given by

$$f(x) = e^x \cos x.$$

(a) Show that $f''(x) = -2e^x \sin x$. [2]

(b) Determine the Maclaurin series for $f(x)$ as far as the x^4 term. [6]

(c) Hence, by differentiating your series, determine the Maclaurin series for $e^x \sin x$ as far as the x^3 term. [4]

(d) The equation

$$10e^x \sin x - 11x = 0$$

has a small positive root. Determine its approximate value, giving your answer correct to three decimal places. [4]