A2 Further Mathematics Unit 5: Further Statistics B
General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations
The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only
MR = misread
PA = premature approximation
bod = benefit of doubt
oe = or equivalent
si = seen or implied
ISW = ignore subsequent working

F.T. = follow through ($\surd$ indicates correct working following an error and $\not\surd$ indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation
A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads
When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.
This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

5. Marking codes

- 'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- 'm' marks are dependant method marks. They are only given if the relevant previous 'M' mark has been earned.
- 'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- 'S' marks are awarded for strategy
- 'E' marks are awarded for explanation
- 'U' marks are awarded for units
- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

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Solutions and Mark Scheme

Qu. No.	Solution	Mark	AO	Notes
1(a)(i)	Upper quartile = $\mu + 0.6745\sigma$ = 32 + 0.6745 × 4 = 34.7 This is the time that is exceeded on 25% of the days.	M1 A1 E1	AO3 AO1 AO2	
(ii)	Let $T = X_1 + X_2 + X_3 + X_4 + X_5$ Then $E(T) = 160$ $\text{Var}(T) = 5\text{Var}(X)$ $\text{Var}(T) = 80$ $P(T > 170) = 0.132$	 B1 M1 A1 B1	 AO3 AO3 AO1 AO1	
(b)	Consider $U = X - 2Y$ $E(U) = -4$ $\text{Var}(U) = \text{Var}(X) + 4\text{Var}(Y)$ = 32 We require P(U > 0) = 0.240	M1 A1 M1 A1 M1 A1 **[13]**	AO3 AO1 AO3 AO1 AO3 AO1	
2(a)	$\Sigma x = 691,\ \Sigma x^2 = 47762.32$ $\hat{\mu} = 69.1$ $s^2 = \sum \frac{x^2}{n-1} - \frac{(\sum x)^2}{n(n-1)}$ = 1.58 DF = 9 t value = 2.262 Standard error = $\frac{s}{\sqrt{n}} = \frac{\sqrt{1.58}}{\sqrt{10}}$ Confidence limits = $\bar{x} \pm t \times \frac{s}{\sqrt{n}}$ $= 69.1 \pm 2.262 \times \frac{\sqrt{1.58}}{\sqrt{10}}$ leading to [68.2,70.0]	 B1 M1 A1 B1 B1 B1 M1 A1 A1	 AO1 AO3 AO1 AO1 AO1 AO1 AO3 AO1 AO1	
(b)	The value of μ either lies in the interval or it does not, there is no question of a probability being involved. EITHER The confidence interval is an observed value of a random interval which contains μ with probability 0.95. OR If the process is carried out a large number of times, we would expect 95% of the confidence intervals obtained to contain μ. **[11]**	E1 E1 **[11]**	AO2 AO2 **[11]**	

Qu. No.	Solution	Mark	AO	Notes
3(a)	H_0 : The petrol consumptions of models A and B are the same	B1	AO3	B0 for saying that the **mean** petrol consumption is the same
	H_1 : The petrol consumptions of models A and B are not the same	B1	AO3	For correctly identifying the alternative hypothesis as two-sided
(b)	From tables upper crit value = 31 Therefore lower crit value = 36 – 31 = 5 The critical region is $(U \geq 31) \cup (U \leq 5)$	B1 B1 B1	AO1 AO2 AO2	
(c)	Use of the formula $U = \sum \sum z_{ij}$ $U = 1 + 6 + 2 + 6 + 6 + 3$ $= 24$	M1 A1	AO3 AO1	
	The conclusion is that there is no difference in petrol consumption of the two models because 24 is not in the critical region.	B1 B1 **[9]**	AO3 AO2	
4(a)	$\hat{p} = \frac{1242}{1800} = 0.69$	B1	AO3	
	ESE $= \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $= \sqrt{\frac{0.69 \times 0.31}{1800}}$ $= 0.0109(0107\ldots)$	M1 A1	AO1 AO1	
	95% confidence limits are $\hat{p} \pm z \times \text{ESE}$ $0.69 \pm 1.96 \times 0.0109..$ giving [0.669,0.711]	M1 A1 A1	AO3 AO2 AO1	
(b)(i)	$\hat{p} = \frac{0.672 + 0.732}{2} = 0.702$	B1	AO3	
	Number of people = 0.702 × 1000 = 702	B1	AO1	
(ii)	$0.732 - 0.672 = 2z\sqrt{\frac{0.702 \times 0.298}{1000}}$	M1	AO3	
	$z = 2.07417..$ Prob from tables = 0.98077 or 0.98097 from calc Confidence level = 96.2%	A1 A1 A1 **[12]**	AO1 AO1 AO2	

Qu. No.	Solution	Mark	AO	Notes
5(a)	$H_0 : \mu_M = \mu_F; H_1 : \mu_M \neq \mu_F$	B1	AO3	
(b)	Let X= male weight, Y=female weight			
	$(\sum x = 39.2; \sum y = 46.6)$			
	$\bar{x} = 4.9;$	B1	AO1	
	$\bar{y} = 4.66$	B1	AO1	
	SE of diff of means= $\sqrt{\frac{0.5^2}{8} + \frac{0.5^2}{10}}$	M1	AO2	
	$= 0.237\ldots$	A1	AO1	Award m0 if no working seen
	Test statistic = $\frac{4.9 - 4.66}{0.237...}$	m1	AO1	
	$= 1.01$	A1	AO1	From calculator, prob = 0.1558 FT 'their' test statistic
	Prob from tables = 0.1562	A1	AO1	From calculator, p-value = 0.3116
	p-value = 0.3124	B1	AO2	FT 'their' p-value
	Insufficient evidence to conclude that there is a difference in mean weight between males and females.	B1	AO3	
		[10]		
6(a)	The differences are 5 –2 8 10 –6 12 –4 7 9 1	B1	AO3	
	The signs may be omitted at this stage. The ranks are 4 2 7 9 5 10 3 6 8 1	M1 A1	AO3 AO1	Attempting to rank absolute values All correct
	W = Sum of positive ranks = 4 + 7 + 9 +10 + 6 + 8 + 1 = 45 The critical value is 44.	M1 A1 B1	AO3 AO1 AO1	
(b)	The conclusion at this significance level is that Method B gives on average a higher reading than Method A because 45 > 44	B1 E1 **[8]**	AO3 AO2	

Qu. No.	Solution	Mark	AO	Notes
7(a)	$E(X) = \theta + 3(1-3\theta) + 5\times 2\theta$ $= 2\theta + 3$	M1 A1	AO1 AO1	
	$\text{Var}(X) = \theta + 9(1-3\theta) + 25\times 2\theta - (2\theta+3)^2$ $= \theta + 9 - 27\theta + 50\theta - 4\theta^2 - 12\theta - 9$ $= 4\theta(3-\theta)$	M1 A1	AO2 AO2	
(b)(i)	Consider $E(V) = \dfrac{E(\overline{X}) - 3}{2}$ $= \dfrac{2\theta + 3 - 3}{2} = \theta$ (Therefore V is unbiased)	M1 A1	AO2 AO2	
(ii)	$\text{Var}(V) = \dfrac{\text{Var}(\overline{X})}{4}$ $= \dfrac{\theta(3-\theta)}{n}$	M1 A1	AO3 AO1	
(c)	Y is B(n,θ) So $E(Y) = n\theta$ $E(W) = E\left(\dfrac{Y}{n}\right) = \theta$ (Therefore W is unbiased)	M1 A1 A1	AO3 AO2 AO2	
	$\text{Var}(W) = \dfrac{\text{Var}(Y)}{n^2}$ $= \dfrac{\theta(1-\theta)}{n}$	M1 A1	AO2 AO1	
(d)	$\dfrac{\text{Var}(V)}{\text{Var}(W)} = \dfrac{\theta(3-\theta)}{n} \div \dfrac{\theta(1-\theta)}{n}$ $= \dfrac{(3-\theta)}{(1-\theta)}$ It follows that W is the better estimator since it has the smaller variance	M1 A1 B1 B1 **[17]**	AO3 AO1 AO2 AO2	