## A2 Further Mathematics Unit 6: Pure Mechanics B
## General instructions for marking GCE Mathematics

**1.** The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

**2.** Marking Abbreviations
The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only
MR = misread
PA = premature approximation
bod = benefit of doubt
oe = or equivalent
si = seen or implied
ISW = ignore subsequent working

F.T. = follow through ( ✓ indicates correct working following an error and ✗ indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

**3.** Premature Approximation
A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

**4.** Misreads
When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.
This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

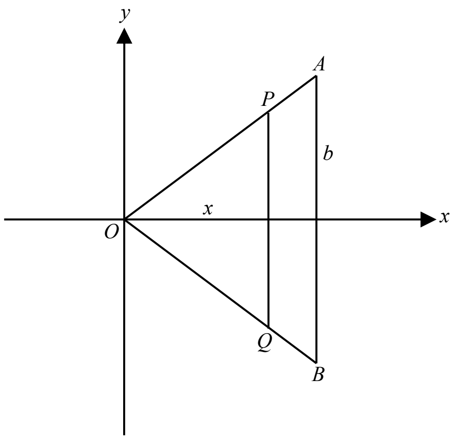
**5.** Marking codes

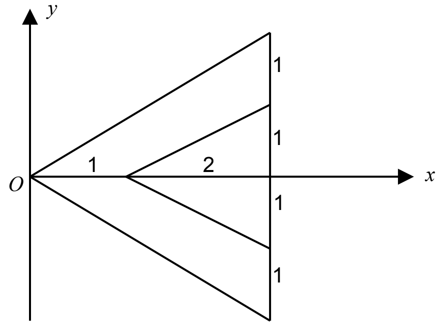
- 'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- 'm' marks are dependant method marks. They are only given if the relevant previous 'M' mark has been earned.
- 'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- 'S' marks are awarded for strategy
- 'E' marks are awarded for explanation
- 'U' marks are awarded for units
- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

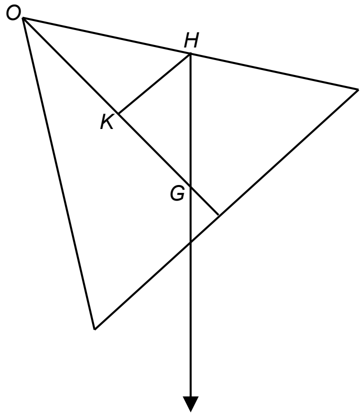
# A2 Further Mathematics Unit 6: Further Mechanics B

## Solutions and Mark Scheme

| Question Number | Solution | Mark | AO | Notes |
|---|---|---|---|---|
| 1. (a) | N2L on ball, upwards positive<br>$-0.01v^2 - 0.4g = 0.4a$ | M1<br>A1 | AO3<br>AO2 | dim correct<br>correct equation |
| | $0.4v\frac{\mathrm{d}v}{\mathrm{d}x} = -3.92 - 0.01v^2$ | | | |
| | $40v\frac{\mathrm{d}v}{\mathrm{d}x} = -(392 + v^2)$ | A1 | AO2 | convincing |
| (b) | $40\int \frac{v}{392+v^2}\,\mathrm{d}v = -\int \mathrm{d}x$ | M1 | AO2 | separate variables |
| | $20\ln(392 + v^2) = -x + C$ | A1<br>A1 | AO1<br>AO1 | $\ln(392 + v^2)$<br>everything correct |
| | When $t = 0$, $v = 17$, $x = 0$<br>$20\ln(392 + 17^2) = C$<br>$C = 20\ln(681)$ | m1<br>A1 | AO2<br>AO1 | use of initial conditions |
| | $x = 20\ln(681) - 20\ln(392 + v^2)$<br>$x = 20\ln\left(\frac{681}{392+v^2}\right)$<br>$\frac{x}{20} = \ln\left(\frac{681}{392+v^2}\right)$ | | | |
| | $\left(\frac{681}{392+v^2}\right) = e^{0.05x}$<br>$681 = (392 + v^2)e^{0.05x}$ | m1 | AO1 | |
| | $v^2 = 681e^{-0.05x} - 392$ | | | |
| | $v = \sqrt{681e^{-0.05x} - 392}$ | A1 | AO1 | |
| (c) | At greatest height $v = 0$ | M1 | AO2 | |
| | $x = 20\ln\left(\frac{681}{392}\right) = 11.05$ | A1 | AO1 | cao |
| (d) | Speed of ball when it returns to *O* is less than 17 $\mathrm{ms}^{-1}$. | B1 | AO2 | |
| | This is because energy is lost in overcoming air resistance. | E1 | AO2 | |
| | | **[14]** | | |

| Question Number | Solution | Mark | AO | Notes |
|---|---|---|---|---|
| 2. (a) |   | | | |
| | Let $\rho$ be mass per unit volume.<br>By symmetry, c of m lies on $Ox$.<br>Divide cone into slices parallel to base.<br>Consider slice $PQ$, distance $x$ from $O$<br>and of thickness $\delta x$. | M1 | AO2 | |
| | By similar triangles, radius of slice is $\frac{bx}{h}$.<br>Mass of slice = $\frac{\pi b^2 x^2}{h^2}\rho\delta x$ acting $x$ from $O$.<br>Mass of cone = $\frac{\pi b^2 h}{3}\rho$ acting at $\bar{x}$ from $O$.<br>Take moments about $y$ axis | m1 | AO2 | |
| | $\frac{\pi b^2 h}{3}\rho\bar{x} = \int_0^h \frac{\pi b^2 x^2}{h^2}\times x\rho\,\mathrm{d}x$ | A1 | AO2 | |
| | $\frac{1}{3}h\bar{x} = \frac{1}{h^2}\left[\frac{1}{4}x^4\right]_0^h$<br>$\bar{x} = \frac{3}{h^3}\frac{h^4}{4}$<br>$\bar{x} = \frac{3h}{4}$ | A1 | AO2 | |

| Question Number | Solution | Mark | AO | Notes |
|---|---|---|---|---|
| 2 (b) |  | | | |
| | Shape mass distance | | | |
| | $C_1$ $\frac{\pi}{3}(2)^2\times 3\rho$ $\frac{3}{4}\times 3$ | B1 | AO1 | |
| | $C_2$ $\frac{\pi}{3}\times 1^2\times 2\rho$ $1+\frac{3}{4}\times 2$ | B1 | AO1 | |
| | Rem. $\frac{\pi}{3}\rho(12-2)$ $\bar{h}$ | B1 | AO1 | |
| | Take moments about $y$ axis | M1 | AO3 | |
| | $\frac{\pi}{3}\rho\times 10\times\bar{h} = \frac{\pi}{3}\times 12\times\rho\times\frac{9}{4}$ | | | |
| | $- \frac{\pi}{3}\times 2\rho\times\frac{5}{2}$ | A1 | AO1 | |
| | $\bar{h}=\frac{11}{5}$ | A1 | AO1 | |

| Question Number | Solution | Mark | AO | Notes |
|---|---|---|---|---|
| 2. (c) |  | | | |
| | Draw $HK$ perpendicular to $OG$. | | | |
| | $OH = \frac{\sqrt{13}}{3}$, $OG = \frac{11}{5}$ | | | |
| | Angle $HOK = \theta$, $\tan\theta = \frac{2}{3}$ | B1 | AO3 | |
| | $\sin\theta = \frac{2}{\sqrt{13}}$, $\cos\theta = \frac{3}{\sqrt{13}}$ | B1 | AO3 | |
| | $HK = OH\sin\theta = \frac{\sqrt{13}}{3} \times \frac{2}{\sqrt{13}} = \frac{2}{3}$ | B1 | AO3 | |
| | $KG = \frac{11}{5} - OH\cos\theta = \frac{11}{5} - \frac{\sqrt{13}}{3} \times \frac{3}{\sqrt{13}}$ | | | |
| | $KG = \frac{6}{5}$ | B1 | AO3 | |
| | $\tan\alpha = \frac{2}{3} \div \frac{6}{5} = \frac{2}{3} \times \frac{5}{6} = \frac{5}{9}$ | B1 | AO3 | |
| | | **[15]** | | |

| Question Number | Solution | Mark | AO | Notes |
|---|---|---|---|---|
| 3. (a) | Using N2L | M1 | AO3 | |
| | $-0.2 - 0.03v = 9\dfrac{\mathrm{d}v}{\mathrm{d}t}$ | A1 | AO2 | |
| | $900\dfrac{\mathrm{d}v}{\mathrm{d}t} = -(20 + 3v)$ | A1 | AO2 | |
| (b) | $900\int \dfrac{\mathrm{d}v}{20+3v} = -\int \mathrm{d}t$ | M1 | AO2 | sep. var. |
| | $900 \times \dfrac{1}{3}\ln(20 + 3v) = -t\ (+ C)$ | A1<br>A1 | AO1<br>AO1 | $\ln(20 + 3v)$<br>all correct |
| | When $t = 0$, $v = 20$<br>$C = 300 \ln 80$ | m1 | AO2 | used |
| | Therefore<br>$t = 300\ln(80) - 300\ln(20 + 3v)$<br>$t = 300\ln\left(\dfrac{80}{20+3v}\right)$ | A1 | AO1 | |
| (c) | When body is at rest, $v=0$<br>$T = 300 \ln(80) - 300 \ln(20)$ | m1 | AO2 | used |
| | $T = 300\ln(4)$<br>$T = \underline{416\ \text{s}}$ | A1 | AO1 | cao |
| | | **[10]** | | |

| Question Number | Solution | Mark | AO | Notes |
|---|---|---|---|---|
| 4. (a) | $\bar{x}$ = 4 (cm) | B1 | AO1 | |
| (b) | Shape mass distance<br>*GPQE* 64 4 | B1 | AO1 | |
| | *EFG* $8\pi$ $8+\frac{16}{3\pi}$ | B1 | AO3 | |
| | *APB* $\pi$ $\frac{8}{3\pi}$ | | | |
| | *CQD* $\pi$ $\frac{8}{3\pi}$ | B1 | AO1 | either *APB* or *CQD* |
| | *ABCDEFG* $64+6\pi$ $\bar{y}$ | B1 | AO1 | areas |
| | Moments about *BC* | M1 | AO3 | |
| | $(64+6\pi)\bar{y} = 64\times 4 + 8\pi\times(8+\frac{16}{3\pi})$<br>$-2\pi\times\frac{8}{3\pi}$ | A1 | AO1 | |
| | $\bar{y}$ = 5.967 (cm) (correct to 3 d.p.) | A1 | AO1 | |
| (c) | If hanging in equilibrium, vertical passes through centre of mass. | M1 | AO3 | correct triangle |
| | $\theta=\tan^{-1}\left(\frac{8-5\cdot 967}{4}\right)$ | A1 | AO1 | |
| | $\theta$ = 26.94(1954…)° | A1 | AO1 | |
| | | **[11]** | | |

| Question Number | Solution | Mark | AO | Notes |
|---|---|---|---|---|
| 5. (a) | $\mathbf{r}_A = 11\mathbf{i} + 6\mathbf{j} + (2\mathbf{i} + 7\mathbf{j})t$ | M1 | AO3 | |
| | $\mathbf{r}_B = 7\mathbf{i} + 10\mathbf{j} + (5\mathbf{i} + 4\mathbf{j})t$ | A1 | AO1 | |
| | If particles collide, $\mathbf{r}_A = \mathbf{r}_B$ for some value of *t.* | | | |
| | For $\mathbf{i}$ component | M1 | AO2 | |
| | $11 + 2t = 7 + 5t$ | | | |
| | $t = \frac{4}{3}$ | | | |
| | For $\mathbf{j}$ component | | | |
| | $6 + 7t = 10 + 4t$ | | | |
| | $t = \frac{4}{3}$ | A1 | AO2 | |
| | Since the value for *t* for both components are equal, the particles collide. | A1 | AO2 | |
| | Conservation of momentum | M1 | AO3 | |
| | $m(2\mathbf{i} + 7\mathbf{j}) + 2m(5\mathbf{i} + 4\mathbf{j}) = 3m(x\mathbf{i} + y\mathbf{j})$ | A1 | AO2 | |
| | $12\mathbf{i} + 15\mathbf{j} = 3x\mathbf{i} + 3y\mathbf{j}$ | | | |
| | $x = 4, y = 5$ | m1 | AO2 | |
| | $x\mathbf{i} + y\mathbf{j} = 4\mathbf{i} + 5\mathbf{j}$ (Ns) | A1 | AO1 | |
| (b) | $\mathbf{I}$ = change in momentum | M1 | AO3 | used |
| | $\mathbf{I} = 2m(4\mathbf{i} + 5\mathbf{j}) - 2m(5\mathbf{i} + 4\mathbf{j})$ | | | |
| | $\mathbf{I} = m(-2\mathbf{i} + 2\mathbf{j})$ | | | |
| | $\mathbf{I} = 2m(-\mathbf{i} + \mathbf{j})$ (Ns) | A1 | AO1 | |
| (c) | Loss in KE = $\frac{1}{2}m(4 + 49) + \frac{1}{2}2\text{m}(25 + 16)$ | | | |
| | $- \frac{1}{2} \times 3m(16 + 25)$ | M1 | AO3 | |
| | Loss in KE = $6m$ (J) | A1 | AO1 | |
| | | **[13]** | | |

| Question Number | Solution | Mark | AO | Notes |
|---|---|---|---|---|
| 6. (a) | At equilibrium | | | |
| | $12g = \dfrac{\lambda \times 0 \cdot 05}{0 \cdot 75}$ | M1 | AO3 | use of Hooke's Law |
| | $\lambda$ = <u>1764 (N)</u> | A1 | AO1 | |
| (b) | Consider a displacement $x$ from the equilibrium position. Apply N2L | | | |
| | $12g - T = 12\ddot{x}$ | M1 | AO3 | |
| | $12g - \dfrac{\lambda(0 \cdot 05 + x)}{0 \cdot 75} = 12\ \ddot{x}$ | A1 | AO3 | ft $\lambda$ |
| | $\ddot{x} = -(14)^2 x$ | | | |
| | Therefore is SHM (with $\omega$ = 14). | A1 | AO2 | |
| | Amplitude = <u>0.05 (m)</u> | B1 | AO1 | |
| | Period = $\dfrac{2\pi}{\omega} = \dfrac{\pi}{7}$ s | B1 | AO1 | |
| (c) | Maximum speed = $a\omega$ | M1 | AO3 | used |
| | = 0.05 × 14 | | | |
| | = <u>0.7 (ms$^{-1}$)</u> | A1 | AO1 | ft $a$ |
| (d) | Use of $v^2 = \omega^2(a^2 - x^2)$ | | | |
| | with $\omega = 14$, $a = 0.05$(c), $x = 0.03$ | M1 | AO3 | |
| | $v^2 = 14^2(0.05^2 - 0.03^2)$ | A1 | AO2 | ft $a$ |
| | $=14^2 \times 0.04^2$ | | | |
| | $v$ = <u>0.56 (ms$^{-1}$)</u> | A1 | AO1 | cao |
| (e) | Displacement from Origin = $x$ | | | |
| | $x = 0.05\cos(14t)$ | M1 | AO3 | (Accept ±) |
| | When $t = 1.6$ | | | |
| | $x = 0.05\cos(14 \times 1.6)$ | A1 | AO2 | ft $a$ (Accept ±) |
| | $x$ = (−)<u>0.046 (m)</u> | A1 | AO1 | cao |
| (f) | The seat is modelled as a particle. | B1 | AO3 | |
| | The spring is assumed to be light. | B1 | AO3 | |
| | | **[17]** | | |