GCE

FURTHER MATHEMATICS

UNIT 6: FURTHER MECHANICS B

SAMPLE ASSESSMENT MATERIALS

(1 hour 45 minutes)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Take g as 9.8 ms^{-2}.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. A ball of mass 0.4 kg is thrown vertically upwards from a point O with initial speed 17 ms^{-1}. When the ball is at a height of x m above O and its speed is v ms^{-1}, the air resistance acting on the ball has magnitude $0.01v^2$ N.

   (a) Show that, as the ball is ascending, v satisfies the differential equation

   $$40v\frac{\mathrm{d}v}{\mathrm{d}x} = -(392 + v^2)\,.$$ [3]

   (b) Find an expression for v in terms of x. [7]

   (c) Calculate, correct to two decimal places, the greatest height of the ball. [2]

   (d) State, with a reason, whether the speed of the ball when it returns to O is greater than 17 ms^{-1}, less than 17 ms^{-1} or equal to 17 ms^{-1}. [2]

2. (a) Prove that the centre of mass of a uniform solid cone of height h and base radius b is at a height of $\frac{1}{4}h$ above its base. [4]

   (b) A uniform solid cone C_1 has height 3 m and base radius 2 m. A smaller cone C_2 of height 2 m and base radius 1 m is contained symmetrically inside C_1. The bases of C_1 and C_2 have a common centre and the axis of C_2 is part of the axis of C_1. If C_2 is removed from C_1, show that the centre of mass of the remaining solid is at a distance of $\frac{11}{5}$ m from the vertex of C_1. [6]

   (c) The remaining solid is suspended from a string which is attached to a point on the outer curved surface at a distance of $\frac{1}{3}\sqrt{13}$ m from the vertex of C_1. Given that the axis of symmetry is inclined at an angle of α to the vertical, find $\tan\alpha$. [5]

3. A body, of mass 9 kg, is projected along a straight horizontal track with an initial speed of 20 ms^{-1}. At time t s the body experiences a resistance of magnitude $(0.2 + 0.03v)$ N where v ms^{-1} is its speed.

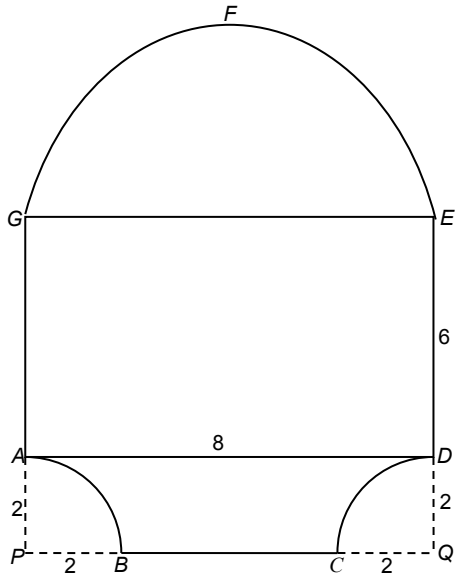
   (a) Show that v satisfies the differential equation

   $$900\frac{\mathrm{d}v}{\mathrm{d}t} = -(20 + 3v)\,.$$ [3]

   (b) Find an expression for t in terms of v. [5]

   (c) Calculate, to the nearest second, the time taken for the body to come to rest. [2]

4. The diagram shows a uniform lamina consisting of a rectangular section *GPQE* with a semi-circular section *EFG* of radius 4 cm. Quadrants *APB* and *CQD* each with radius 2 cm are removed. Dimensions in cm are as shown in the diagram.



(a) Write down the distance of the centre of mass of the lamina *ABCDEFG* from *AG*. [1]

(b) Determine the distance of the centre of mass of the lamina *ABCDEFG* from *BC*. [7]

(c) The lamina *ABCDEFG* is suspended freely from the point *E* and hangs in equilibrium. Calculate the angle *EG* makes with the vertical. [3]

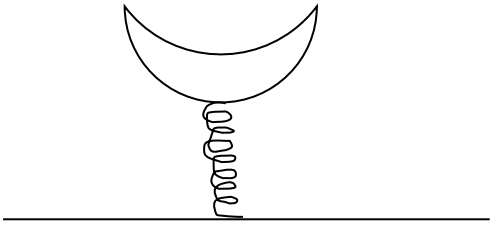
5. A particle *A*, of mass m kg, has position vector $11\mathbf{i} + 6\mathbf{j}$ and a velocity $2\mathbf{i} + 7\mathbf{j}$. At the same moment, second particle *B*, of mass $2m$ kg, has position vector $7\mathbf{i} + 10\mathbf{j}$ and a velocity $5\mathbf{i} + 4\mathbf{j}$.

   (a) If the particles continue to move with these velocities, prove that the particles will collide. Given that the particles coalesce after collision, find the common velocity of the particles after collision. [9]

   (b) Determine the impulse exerted by *A* on *B*. [2]

   (c) Calculate the loss of kinetic energy caused by the collision. [2]

6. The diagram shows a playground ride consisting of a seat *P*, of mass 12 kg, attached to a vertical spring, which is fixed to a horizontal board. When the ride is at rest with nobody on it, the compression of the spring is 0.05 m.



   The spring is of natural length 0.75 m and modulus of elasticity λ.

   (a) Find the value of λ. [2]

   The seat *P* is now pushed vertically downwards a further 0.05 m and is then released from rest.

   (b) Show that *P* makes Simple Harmonic oscillations of period $\frac{\pi}{7}$ and write down the amplitude of the motion. [5]

   (c) Find the maximum speed of *P*. [2]

   (d) Calculate the speed of *P* when it is at a distance 0.03 m from the equilibrium position. [3]

   (e) Find the distance of *P* from the equilibrium position 1.6 s after it is released. [3]

   (f) State one modelling assumption you have made about the seat and one modelling assumption you have made about the spring. [2]