A-LEVEL
MATHEMATICS

7357/1
Report on the Examination

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General

Compared with the last exam series, the average score on this paper was slightly higher and students found many questions more accessible. All of the multiple-choice questions performed well and enabled students to settle well at the start of the paper. There were, however, still some students who did not select an option for one or more of these questions.

Many of the questions on this paper were structured with the intention of guiding students through the question. For example, questions 5, 9, 13, 14 and 16 were all broken into parts that built towards later answers. A significant number of students did not see or use these pointers.

There were some instances of students not understanding the phrase “exact value”. Students should be aware that if an exact value is required then it usually means the answer should be left in terms of π, e or given as a fraction or surd. A long decimal is not an exact value.

There was a slight improvement in responses to “show that” questions. Nevertheless, students should be aware of the following points in these types of question:

- Often a required concluding statement gets missed out (eg questions 6 and 12(b)).
- Do not omit steps or take shortcuts, for example, when substituting limits into an integrated expression; it is important to convince the examiner that you know how to do it and not assume they will fill in the gaps for you (eg question 8).

While some explanations given by students were very clear and concise, many “explain” answers could have been improved by avoiding words like “it” or “they”. These words are usually too vague and the examiner cannot tell for sure what is being referred to. For example, in question 13(b)(ii) a response which did not score the mark such as “They all meet at a point” could easily have been improved by writing “The three graphs drawn in part (b)(i) all meet at a point.” “They” in the first response could be referring to $x - \cos x = 0$ and $\cos x = \arccos x$, or anything else that has come before.

As always, the strongest students made efficient and appropriate use of their calculators. For example, in question 9 where the centre of the circle could be found by solving two simultaneous equations, simply writing down the solution was acceptable.

Question 1

This question was answered correctly by over 70% of students, with the most frequently chosen incorrect answer being –2187.

Question 2

Students were extremely successful on this question with over 95% selecting the correct answer.

Question 3

The majority of students selected the correct equation for the given transformation, with almost 70% scoring the mark.

Question 4

Even though this proved to be the most challenging of the multiple-choice questions, over 60% of students selected the correct answer.

Question 5

Part (a) was answered well, with over half of the students scoring full marks. It was clear that the trapezium rule was understood by the majority. The correct value for h was usually seen, with $\frac{1}{2}$ being the most common incorrect value, which may have been caused by mixing up the number of strips and the number of ordinates.

By far the most common error was poor use of brackets. Students often wrote expressions like

$$\frac{0.6}{2}(2.90988+0.09329)+2(1.26485+0.62305+0.32374+0.17263)$$

which gave an incorrect answer.

The method mark in this part was for "substituting the given y-values" into the formula. Students who took shortcuts, shortening the y-values or missing out values or digits and replacing with three dots in their written work often got the wrong answer and lost both the M1 and A1 marks.

Part (b) was successfully answered by over 70% of students. Even with an incorrect answer for part (a), most realised they just had to multiply their answer by four.

The most common mistakes seen were multiplying by 20 or writing down an accurate answer from the calculator.

Question 6

Approximately 90% of students made some progress with this question by applying at least one correct law of logs, often using $2\log_{10} x = \log_{10} x^2$ correctly. Unfortunately, the manipulation of the right-hand side of the equation was often poorly understood.

Very common errors included "multiplying out a log" and writing

$$\log_{10}(x+8) \quad \text{as} \quad \log_{10} x + \log_{10} 8$$

or incorrect removal of logs, jumping from

$$\log_{10} x^2 = \log_{10} 4 + \log_{10}(x+8) \quad \text{to} \quad x^2 = 4 + x + 8$$

Most students scored the M mark for obtaining and solving a quadratic equation, even if they had not scored both B marks.

Approximately 30% of students scored full marks on this question. Those who dropped the final mark often made incomplete or imprecise statements. The most common incorrect explanation seen was "logs cannot be negative," rather than "the log of a negative number is undefined."

Question 7

Part (a) proved to be very accessible, with around 80% of students scoring at least one mark.

Two main approaches were seen:
- considering the two fractions separately and rationalising the denominators
- obtaining a common denominator and writing as a single fraction

of which the second option was more popular.

The most common errors in this question were sign errors and incorrect use of brackets.

Part (b) proved much more challenging, with just over 10% of students explicitly stating that 42 and $25n - 9$ are both integers in order to conclude that the expression is a rational number.

Common errors included explaining that the numerator or denominator was odd/even or positive/negative.

Question 8

Almost 80% of students recognised that they had to use integration by parts and thus made some progress with this question and most scored at least 5 marks.

The most common reason for lost marks was for not including enough detail in the solution. In particular, for the final mark, limits had to be explicitly substituted into the integrated expression to show how $-\frac{\pi}{8}$ was obtained.

Sign errors were also seen frequently when integrating sine and cosine or when substituting into the integration by parts formula.

Question 9

Nearly 90% of students obtained the correct midpoint in part (a)(i), which is an improvement on similar questions in previous series.

The most common incorrect answer was $(9, 2)$, which may have come from subtracting the coordinates rather than adding.

Part (a)(ii) was routine and around 90% of students scored at least two marks and almost 60% scored full marks.

Some students found an equation of the line through *P* or *Q* and provided their working was clear they were still able to score half of the marks.

A significant number of students who found the correct equation then dropped the final mark for not rearranging into the required form.

Part (b)(i) was a good example of a question where students needed to be mindful of the work they had already been asked to do in part (a). The question used the fact that the centre of the circle must lie on the line found in part (a) and on the line given in part (b). Both equations were in a form which could be entered directly into a calculator in order to solve simultaneously and find the centre. Using a calculator efficiently to simply state the coordinates of the centre, as in the typical solution on the mark scheme, was the method students were expected to use.

Common mistakes in this part included using *PQ* as a diameter of the circle.

To achieve the mark in part (b)(ii) students needed to state the correct number of intersections which had to come from obtaining the correct radius and centre in part (b)(i). A few students gave this one mark, “State” question rather more attention than it deserved, giving detailed and unnecessary explanations as to why the number of intersections was four.

Question 10

Over 90% of students made some progress with part (a)(i), with over 65% scoring full marks. This question tested knowledge of the symmetries and periodicity of the sine function. The given graph was often underused by students and labelling standard values on the x-axis, ± 90, ± 180, ± 270, would have helped to identify the correct value for a and to eliminate positive values for a, which were often seen.

Approximately 75% of students correctly answered part (a)(ii). This was testing the standard result $\sin(180 - x) = \sin x$, of which many students seemed unaware, often resulting in unnecessary work to obtain the correct answer.

Part (b)(i) was more challenging. All of the approaches listed in the mark scheme were seen with approximately 70% of students achieving at least one mark. Students could make some progress working in decimals, but a significant number did not realise the significance of the phrase “Find the exact value…” and did not achieve the second mark.

As the answer to part (b)(ii) required a surd, students who worked in decimals were unable to make any progress. Only about 30% of students realised what was required and chose a valid approach. Of those who scored two marks for getting at least as far as

$$\cos^2 b = \frac{40}{49}$$

only the strongest students went on to consider the position of *B* and deduce the correct exact negative value.

Question 11

Over 85% of students correctly answered part (a). A commonly seen incorrect answer was $u_2 = p400 + 70$ which does not use standard algebraic notation.

Part (b)(i) was generally very well answered with over 75% of students scoring full marks. Some students solved the given quadratic and checked the values worked, which was not creditworthy.

Over 80% of students made some progress in part (b)(ii) with over 65% scoring full marks. The key to this question was realising which value of p resulted in a decreasing sequence and most students correctly identified the right value. Errors in this part often came down to incorrect rounding, which could have easily been avoided through efficient calculator use.

This was the first time the limit of a sequence of this type had been tested in this specification and it was evident that a significant number of students were unfamiliar with the concept. Some students who did not score the mark for part (c)(i) were able to score the mark for part (c)(ii) by continuing to find further iterations on their calculators.

Question 12

A common mistake in part (a) was to find only the initial value of c with no explicit consideration of the value of f and hence the initial distance between the floor and ceiling.

In part (b) most students had the right idea and knew to use the harmonic form, $R\cos(x+\alpha)$, often obtaining the correct value of $R=\sqrt{17}$. However, marks tended to be lost in this “Show that” question through a lack of attention to detail.

Common errors included:

- not starting the argument by modelling the required distance, for example

$$\begin{aligned} d &= c - f \\ &= 8 - 4\sin t - (1 - \cos t) \\ &= 7 + \cos t - 4\sin t \end{aligned}$$

- careless sign errors when comparing coefficients of sine and cosine
- leaving the argument unfinished, failing to conclude with $d = 7 + \sqrt{17}\cos(t + 1.33)$.

The most efficient approach to part (c) was generally well understood by those who attempted it. As always, with this sort of question, there were some attempts to differentiate to find the minimum, which significantly overcomplicated matters.

The last mark was often lost through careless rounding or incorrect use of units and the incorrect answer of 3 cm from the decimal 2.88 was often seen.

Question 13

In part (a) over 70% of students stated the correct value of a.

Commonly seen incorrect values included 0 and π, which were not consistent with the position of a shown on the diagram.

Over 60% of students scored at least three marks for their graph sketching in part (b)(i). Most students drew a concave arc, but a significant minority had their curve intersecting the y-axis at $\frac{\pi}{2}$ or even higher. Most students correctly labelled the y-intercept as 1 or a. The straight line was usually drawn within acceptable limits. Students often, very sensibly, used the labels of $\frac{\pi}{2}$ on the two axes to draw horizontal and vertical dotted lines to help with the positioning of the line $y = x$.

The most common reason for the loss of the final mark was for not having all three graphs intersecting at a common point.

In part (b)(ii) most students missed the connection with part (b)(i) and opted to try to justify the statement algebraically. Having written $x - \cos x = 0 \Rightarrow x = \cos x$ a frequently seen error was to jump to $\cos x = \arccos x$ without showing the intermediate step $x = \cos x \Rightarrow \arccos x = x$

Students who attempted to answer the question using the graphs of the equations from part (b)(i) often made statements that were too vague such as “they meet at a point” without making it clear what was meant by “they.”

Part (c) required the use of the Newton-Raphson method and it was pleasing to see students being more successful with this technique than in previous exam series. Approximately 60% of students scored at least two marks. $x - \cos x$ was usually correctly differentiated to obtain $1 + \sin x$. Common errors seemed to be caused by calculators set in degree mode or assuming the given value was for x_1 and then performing too few iterations. Students who did not write down their general Newton-Raphson formula, but instead substituted numbers immediately often lost the available method mark, because they obtained an incorrect value for the answer.

Question 14

Just over 60% of students correctly recalled the derivative of a^x in part (a)(i).

In part (a)(ii) only a minority of students recognised how to use the fundamental theorem of calculus with their answer to part (a)(i) to successfully integrate 2^x. A clue here was the word “Hence” which indicates the work already done should be helpful. Some students used a substitution and this was given credit if fully correct. As the question was a test of the fundamental theorem of calculus, we needed to see a constant of integration for full marks.

Part (b)(i) was answered correctly by almost 60% of students. Some students dropped marks for using –0.5 for the base of the triangle and then not explaining why their $-\frac{\sqrt{2}}{4}$ became positive. Other common errors included integrating under the curve, which did not lead to the required answer.

Almost 70% of students made some progress in part (b)(ii), usually scoring the first mark by setting up the sum to 8 terms of a geometric series using $n = 8$ and the given value for r. The A1 mark was then often lost for using $a = \frac{1}{32}$ instead of $a = \frac{\sqrt{2}}{4}$, found in part (b)(i). Those who listed the areas of the eight rectangles often scored full marks.

Part (b)(iii) proved to be the most demanding part of the question with less than 20% of students making any progress. The majority incorrectly attempted to use the sum to infinity of a geometric series and did not understand the link to integration. However, there were also some excellent, concise solutions.

Question 15

Almost 40% of students scored full marks in part (a) and over 80% recognised the need to use implicit differentiation and scored two of the first three marks.

Many attempted to use the product rule but errors were seen, often through a lack of correct bracketing, which led to sign errors.

While an improvement on previous exam series, a significant number of students still began their implicit differentiation with an incorrect $\frac{\mathrm{d}y}{\mathrm{d}x} =$. Some recovered and eventually obtained correct differentiated equations, but those who kept the extra $\frac{\mathrm{d}y}{\mathrm{d}x}$ term lost at least 3 marks.

Many students rearranged their differentiated equations to obtain an explicit expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. This was not required and it is worth noting that the simplest approach was to substitute $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ as soon as the differentiation was complete.

Over half of the students scored full marks in part (b). It was pleasing to see students persevering at this stage of the exam, using the given result from part (a), even if they had been unable to show it themselves.

A commonly seen error was that students found the value of y then x, and then wrote the coordinates in the wrong order $(2,4)$ instead of $(4,2)$. This was not usually penalised on this occasion, provided their intended values for y and x were clear.

Question 16

A routine for finding partial fractions was understood by the vast majority of students, with almost 75% scoring full marks for part (a). Careless errors were sometimes seen when rearranging, with $1 \equiv A(4-3x) + B(4+3x)$ used instead of $1 \equiv A(4+3x) + B(4-3x)$, or $A(4+3x) + B(4-3x) \equiv$ an expression in x^2. Incorrect values for A and B were sometimes seen after correctly obtaining $8A = 1$ or $8B = 1$.

Most students found part (b)(i) to be very challenging. Some very concise solutions were seen, but often the given result was arrived at through incorrect working, which was not creditworthy.
One mark was often achieved for obtaining the relationship $V = 1.25 \times 1.6d \Rightarrow d = \frac{V}{2}$.

Just under half of the students made some progress with part (b)(ii) and it was pleasing to see that some students who were unable to start part (b)(i) persevered and used the given result to attempt to solve the differential equation. Of those who made a start, most were able to separate the variables and integrated their constant term correctly.
A significant minority did not make the connection with the partial fractions found in part (a) and attempts to integrate $\frac{1}{16-9V^2}$ often led to incorrect answers of the form $k\ln(16-9V^2)$

Those who attempted to integrate their partial fractions often forgot to divide by 3 and –3.

Of those students who attempted part (b)(iii), most realised that $V = 1$ had to be substituted into their answer from part (b)(ii).

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.