A-LEVEL
MATHEMATICS

7357/3 Paper 3
Report on the Examination

7357
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Version: 1.0

General

Students performed well across the paper compared with previous examination series. There were many opportunities for all students to score a reasonable number of marks. There was an improvement in the quality of written responses and this was reflected in a stronger overall performance on the challenging questions. On average achievement was better in the statistics section than in the pure section.

Overview of Entry

Topics that were done well included:

Section A (Pure)

- Laws of indices (Q4)
- Gradient of an exponential function (Q5)
- Cubic graphs and the factor theorem (Q6)
- Integration by substitution (Q8)

Section B (Statistics)

- Binomial distribution (Q12)
- Probability distribution and calculation (Q13(a), Q15(b))
- Hypothesis testing for the mean of a normal distribution (Q14(b))
- Normal distribution (Q16)
- Hypothesis testing for the proportion using a binomial distribution (Q17)

Topics which students found challenging included:

Section A (Pure)

- Area of a sector (Q7)
- Determining the nature of a stationary point (Q7)

Section B (Statistics)

- Conditional probability (Q13(b))
- The Large Data Set (Q15(c)(i))

Comments on Individual Questions

Section A (Pure)

Question 1

This question was answered correctly by a high proportion of students. The most common incorrect response was $y = |x+2| - 3$

Question 2

This question was answered correctly by a high proportion of students. The most common incorrect response was the third option.

Question 3

Just under a half of students answered this question correctly. The most common incorrect response was $y = \frac{7}{3}x$.

Question 4

Students responded well to this question, applying the laws of indices correctly. A few students wrote $\frac{5}{x^2}$ as $5x^{\frac{1}{2}}$ and others subtracted 2 from $\frac{1}{3}$ to give $q = -\frac{2}{3}$. Some students attempted to multiply the expressions in the numerator by x^2 rather than x^{-2} and did not score any marks.

Question 5

Most students were able to gain the first mark for finding the gradient function of the curve, although a common error was to give this as $6xe^{2x}$ Many students were able to find the exact value of x by substituting y = 10 in the equation of the curve. Very few students recognised that the gradient could also be expressed as 2y, but those who did typically scored full marks. Some students rounded the value of x too early and rounded their value for the gradient to 20 which did not gain the final accuracy mark.

Question 6

Students recognised that the curve was cubic in part (a), but some did not know the correct orientation for a positive cubic function. Many deduced the turning point at (0,0), but some students did not score the final mark as they did not label the other root. Curves were often so inaccurate that full marks could not be awarded, for example, the minimum point not reaching the x-axis or not being at the origin.

In part (b)(i), most students used the factor theorem correctly to show that $a = 2$ and completed a reasoned argument to achieve the required result. However, some could not gain the final mark as they did not equate $\mathrm{p}(-3)$ to 0.

Many students stated the correct transformation, which is translation, in part (b)(ii). However, few could state the required vector. A statement such as 'move it 36 units upwards' rather than a vector gained no mark.

Part (b)(iii) was not answered well. Very few scored the first mark for explaining that the translated graph only had one real root. The second mark was more accessible as students could deduce that the discriminant must be negative and then show the required result.

Question 7

This question was challenging for most students. The expression for the area of the rhombus was more often correct than the area of the sectors in part (a). Students had difficulty in working with angles in radians, often using 180 in place of π. It was common to see the angle in each sector as θ instead of $\pi - \theta$. To score full marks, there had to be a clear reasoned argument, showing how the rhombus and sector areas could be added together to achieve the required result.

Most students recognised the need to use differentiation in part (b)(i) and found the correct first derivative. Some lost marks by substituting $\theta = \frac{\pi}{3}$ to verify that $\frac{\mathrm{d}A}{\mathrm{d}\theta} = 0$, rather than proving the given result by forming and solving an equation. It was common to see $\frac{\mathrm{d}y}{\mathrm{d}x}$ instead of $\frac{\mathrm{d}A}{\mathrm{d}\theta}$, but this was condoned except for the E1 mark. Many students either did not explain that $\frac{\mathrm{d}A}{\mathrm{d}\theta} = 0$ at a maximum, or did not use the second derivative (or another appropriate method) to confirm that a maximum had been found.

Many students were able to find the exact maximum value for the area in part (b)(ii) by substituting $\theta = \frac{\pi}{3}$ into the given expression for A, but a significant number then gave a decimal value despite the instruction in the question to give an exact value. This sometimes happened because students used a calculator in degree mode to find $\sin\frac{\pi}{3}$.

Very few students scored both marks in part (c). Although some stated that the angle would be the same, most found it difficult to work out the effect of doubling the length *PQ* on the area.

Question 8

Most students showed good understanding of the processes required for integration by substitution and gained all or most of the marks. The answer being given meant that many students restarted their work at various points to get to the required solution. Many students set their work out clearly with correct mathematical notation.

Students who did not use the substitution gained no marks.

Some did not know to start by differentiating the substitution, and so substituted for the $(x^5 + 2)^3$ with u^3 but were not able to proceed further. There were two main approaches to the substitution, using $u = x^5 + 2$, or $x = (u - 2)^{\frac{1}{5}}$, with the first being seen much more often. Most students were able to differentiate correctly whichever route they chose, however, those that chose the first approach were far more successful in obtaining an integrand in terms of u, although some did not complete the process by simplifying the terms x^9 and x^4. The second approach normally resulted in the student not being able to complete the algebraic manipulation required to eliminate x fully.

Many issues with powers and roots were seen, such as $x = \sqrt[5]{u-2}$ leading to $x^9 = \sqrt[4]{u-2}$. However, some students were able to complete this successfully. Of the students that found an integral just in terms of u, most had all mathematical notation correct, including the du.

Most students used laws of indices to write the integrand as a multiple of $u^{-2} - 2u^{-3}$ which could be integrated easily. Others chose to integrate $(u-2)u^{-3}$ by parts which was often done correctly, but was more likely to lead to errors. Occasionally, mistakes were made with the constant such as $\frac{1}{5u^3}$ becoming $5u^{-3}$.
After integrating, a minority of students finished their solution in terms of x but had to be careful to use the appropriate limits. However, most substituted limits in correctly and gained full marks.

Question 9

Students performed well in this question, applying mathematical problem-solving techniques in the given context. Most found the y-values to give the heights of P and Q and subsequently found the difference between these values in part (a). Some also found the values of x at P and Q which were not required. Some did not understand how to answer a 'show that' question and marks were lost for not explicitly comparing 6.53 with 7 or not concluding with a statement that the slide meets the safety requirement.

In part (b)(i), most students were able to use the chain rule to find $\frac{dy}{dx}$ with correct unsimplified answers scoring full marks. Some students struggled to correctly use indices in this question including the differentiation of expressions with negative indices. Those that used the chain rule as $\frac{dy}{dt} \div \frac{dx}{dt}$ were more successful than those that used $\frac{dy}{dt} \times \frac{dt}{dx}$ as there were often algebraic errors in inverting $\frac{dx}{dt}$, such as stating $\frac{dt}{dx} = 1 + t^2$ from the correct expression for $\frac{dx}{dt}$.

Most students identified the value of t at the stationary point of the curve and substituted this value into the expression for y in part (b)(ii). They correctly set $\frac{dy}{dx}$ or $\frac{dy}{dt}$ to zero, although some students stopped at this stage and did not go on to find the length of RS. For full marks, the correct answer needed to come from a correct derivative so there were occasions when a correct answer came from incorrect working. A few students did not gain the final accuracy mark as they did not include the unit for the length.

Many students did not attempt part (b)(iii) and many others started incorrectly, suggesting little understanding of the connection between the gradient of the curve at Q and the angle asked for. Only a few recognised they needed to find the value of the gradient at $t = 3$ and only these students were able to proceed further to get the correct acute angle by finding the inverse tan of their gradient. However, most students wrongly believed that the gradient at Q was the same as the gradient of the straight-line PQ, which did not gain a mark.

Section B (Statistics)

Question 10

This question was answered correctly by a high proportion of students. The most common incorrect response was 0.

Question 11

This question was answered correctly by a high proportion of students. The most common incorrect response was the first option.

Question 12

Whilst most students could calculate correctly with a binomial distribution, many did not know how to apply this in context. Only a few students stated the assumption in context in part (a). The statement that probability is fixed was the most common response, but essential words such as ‘passing’ or ‘test’ were missing. There were other imprecise and thus incorrect statements such as the probability of passing being independent, or the drivers are independent of each other or there is a fixed number of tests or fixed driving conditions. A few students used ‘chance,’ ‘likelihood’ or ‘possibility’ instead of probability. The word ‘test’ was required in a statement that identified one of three situations: (i) the probability of passing the test is constant; (ii) the outcome of passing the test occurs independently; (iii) there are only two outcomes: passing the test or failing the test.

There was evidence of good calculator use to find the probabilities in parts (b) and (c).

In part (d) it was common to see P($X > 12$) incorrectly interpreted as 1 – P($X \leq 11$).

In part (e) most students used the correct formula for the mean to get 12.8. However, it was common to see students make incorrect assumptions that, as the context was the number of drivers per day who pass their test, that this meant their answer had to be rounded to an integer, which is incorrect.

Similarly, most students used the correct formula for variance in part (f). A few students thought this was the standard deviation or left the answer as $\sqrt{7.68}$ whilst others tried to use a formula for the standard deviation of a set of data.

Question 13

A large proportion of students scored full marks in part (a). A few students approached the question thinking the coins were not replaced in the money box.

Many students found working with conditional probability challenging in part (b), although some were able to calculate the probability of at least one bronze coin and score the first mark. Those who drew a tree diagram were mostly successful in getting both marks, but there seemed to be a lack of familiarity with the conditional probability formula, which is given in the formulae book.

Question 14

Two thirds of students answered part (a) correctly. However, others somehow calculated a probability, often obtaining an answer which they then rounded to 1, which did not gain the mark.

Most students were able to score some of the marks in part (b) although only a quarter of them got full marks. Most students clearly stated both hypotheses and correctly found the sample mean. A few students could not identify the correct distribution of the sample mean, most commonly using $N(24500, 5200^2)$ instead, although there were also numerous responses using 26730 in place of 24500 and erroneous methods for finding the standard error.

The most common method used to carry out the test was to calculate the appropriate probability to compare with the significance level, although some students found the test statistic for the standard normal distribution and others determined a critical region.
Common errors included using 0.05, rather than 0.025, in the comparison or not rejecting H_0 after correctly comparing the figures. Responses that referred only to H_1 lost the mark. To gain full marks a concluding statement in context was needed, following correct working. This conclusion was often much better than in previous years. However, some students used the term 'enough' rather than 'sufficient' or definite terms such as 'show' or 'prove' rather than 'suggest' or made no reference to the context.

Part (c) was the most challenging question on the paper. Very few appreciated the indefinite nature of hypothesis testing and the randomness of the sample mean. In many responses it was unclear whether students were referring to the sample mean or the population mean.

Question 15

90% of students scored full marks in part (a)(i). However, a few rounded their interval values before showing that there is one outlier and did not gain the final mark.

In part (a)(ii), 70% of students recalled the correct statistical name, with the most common incorrect answer being 'anomaly.'

In part (b), 80% of students knew the total probability was 1 and almost all could then find the correct value of k. A common error was to equate the total probability to 0, which gave a negative value for k and hence negative probabilities. Where k was found correctly, some students found it difficult to choose the correct probabilities from the table.

In part (c)(i), many students showed little knowledge of the Large Data Set. Some failed to get both marks as their answers lacked the necessary detail of how to collect the required sample. There were many explanations of how to collect a sample, some referring to stratified or simple random sampling, without identifying that the LDS contains cars from 2 years and 5 makes of car.

In part (c)(ii) some students stated the relevant disadvantage of quota sampling in the context of the LDS as being biased or not random. A common error was to state that it is not representative**.**

Question 16

Students used their calculators well to find the normal distribution probabilities in parts (a)(i), (a)(ii) and (a)(iii) with over 80% answering each of these correctly.

In part (b), most students knew how to set up simultaneous equations using the formula $z = \frac{x - \mu}{\sigma}$. In the best solutions students went straight to $5.9 = \mu + 0.2533\sigma$ and $6.1 = \mu + 0.8416\sigma$, solving these simultaneous equations using a calculator.

Sign errors for z-values resulted in the loss of accuracy marks but students who used 0.6 and 0.2 from the question as their z-values gained no marks.

Question 17

Most students stated the hypotheses correctly. There were a few notational errors, for example using μ rather than p for the parameter. Common errors were to use the proportion in the sample, 0.84, as their population proportion or use 21 instead of 25 for the value of n in their binomial distribution. Most students used the probability method and calculated P($X \geq 21$), although some incorrectly calculated, for example, P($X > 21$), P($X = 21$) or even P($X \leq 21$). A few students chose the critical region method. Students should know that in all hypothesis tests, they are required to decide either to reject or not reject H_0 before concluding in context. It is not correct to refer to H_1 and doing so was not given credit if there was no correct reference to H_0. On this occasion stating “Accept H_0” was given credit as well as the technically correct “Do not reject H_0.”

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.