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Examiners' report

MATHEMATICS A

H240
For first teaching in 2017

H240/03 Summer 2019 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 3 series overview

This is the third examination component for the new revised A-Level examination for GCE Mathematics A. It is a two-hour paper consisting of 100 marks which tests content from Pure Mathematics (Section A, 50 marks) and Mechanics (Section B, 50 marks). Pure Mathematics content is tested on all three papers, and any topic could be tested on any of the three papers.

Inevitably, the report that follows will concentrate on aspects of the candidates' performance where improvement is possible to assist centres on preparing candidates for future series. However, this should not obscure the fact that a significant number of candidates who sat this paper in this reformed A Level qualification produced solutions which were a pleasure for examiners to assess. Many candidates demonstrated a most impressive level of mathematical ability and insight which enabled them to meet the various challenges posed by this paper on both the pure and mechanics content; precision, command of correct mathematical notation and excellent presentational skills were evident in many scripts.

The specification includes some guidance about the level of written evidence required in assessment question; these were provided to reflect the increased functionality of the available calculators and the changes in assessment objectives, since there is a significant change from when the equivalent legacy qualifications were designed. There are a number of questions on this paper which began with the demand 'In this question you must show detailed reasoning'; to quote the specification that 'when a question includes this instruction candidates must give a solution which leads to a conclusion showing a detailed and complete analytical method. Their solution should contain enough detail to allow the line of their argument to be followed. This is not a restriction on a candidate's use of a calculator when tackling a question…but it is a restriction on what will be accepted as evidence of a complete method.' The specification then considers several examples which centres should consider so that future candidates understand exactly what is required when this request appears in future series. This command phrase features in questions 3 and 5.

The word 'determine' in a question does not simply imply that candidates should find the answer but, to quote the specification, 'this command word indicates that justification should be given for any results found, including working where appropriate.' This command word features in question 2(b) and 4(d)(ii).

The phrase 'Show that' generally indicates that the answer has been given, and that candidates should provide an explanation that has enough detail to cover every step of their working. This command phrase features in questions 4(a), 4(c), 11(a) and 11(b).

Whilst there is no specific level of working needed to justify answers to questions which use the command word 'find …', method marks may still be available for valid attempts that do not result in a correct answer, and standard advice (included in the specification) that candidates should state explicitly any expressions, integrals, parameters and variables that they use a calculator to evaluate (using correct mathematical notation rather than model specific calculator notation).

i	**OCR support**	A poster detailing the different command words and what they mean is available here: https://teach.ocr.org.uk/italladdsup

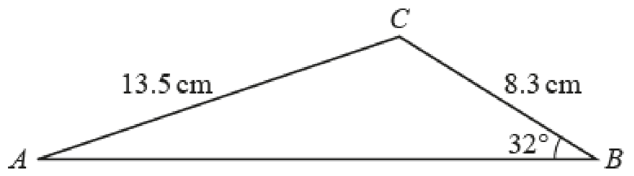
There are several examples where the question specifically asks for an exact value or in surd form, an approximate decimal equivalent will not gain full credit, examples are questions 4(c) and 6. Conversely question 4(b) specifies the level of accuracy to give the final answer. Regardless of the final required accuracy, candidates should be careful of not rounding prematurely, but also take care to avoid over specifying rounded answers where the context does not support that level of accuracy.

Section A overview

The Section A covers pure mathematics content of the H240 specification. Teachers using this (and subsequent papers) may want to remind candidates that the level of demand ramps up to the final few questions in Section A, which are designed to be challenging and that the Section B will start with less demanding questions before ramping up a second time. Candidates may benefit from leaving the more challenging Section A questions until after answering the more straightforward mechanics questions in Section B before returning to those missed questions.

Question 1

1



The diagram shows triangle ABC, with $AC = 13.5\text{cm}$, $BC = 8.3\text{cm}$ and angle $ABC = 32°$.

Find angle CAB. **[2]**

Nearly all candidates correctly applied the sine rule to calculate angle *CAB* – the most common error was those who believed that angle *CAB* was in fact angle *ACB* and instead gave an answer of 129 degrees.

Question 2 (a) (i)

2 A circle with centre C has equation $x^2 + y^2 - 6x + 4y + 4 = 0$.

(a) Find

(i) the coordinates of C, **[2]**

This proved to be a good question for nearly all candidates with the vast majority correctly completing the square (twice) to find the coordinates of the centre of the circle. When errors occurred, these were nearly always down to sign errors inside the two brackets.

Question 2 (a) (ii)

(ii) the radius of the circle. **[1]**

Nearly all candidates stated the radius of the circle correctly in either part (i) or part (ii) of (a).

Question 2 (b)

(b) Determine the set of values of k for which the line $y = kx - 3$ does not intersect or touch the circle. **[5]**

Although examiners did note that some candidates left this part blank, many candidates did start this problem correctly by eliminating y and obtaining $(1+k^2)x^2 + (-6-2k)x + 1 = 0$, but then many did not realise that if the line and circle did not intersect (or touch) then the discriminant of this quadratic in x would be negative. Of those that did realise this fact it was only arithmetical or algebraic errors that meant that the candidates could not determine that $k < -\frac{4}{3}$.

Question 3 (a)

3 (a) In this question you must show detailed reasoning.

Solve the inequality $|x-2| \leqslant |2x-6|$. **[4]**

This question was answered extremely well with nearly all candidates correctly solving this inequality involving the modulus function. Of the two main methods for solving this type of inequality the first, which involved re-writing as $(x-2)^2 \leq (2x-6)^2$ was far more successful than those candidates who decided to re-write as two linear equations/inequalities (one of which should have had signs of x and $2x$ the same and the other with signs different) as many made sign errors even though most started from the correct two equations/inequalities. Although most candidates obtain the correct two critical values, and so therefore obtained the first three values, the only answers accepted by examiners for consistency were:

$x \geq 4$ or $x \leq \frac{8}{3}$

$\{x : x \geq 4\} \cup \{x : x \leq \frac{8}{3}\}$

$\left(-\infty, \frac{8}{3}\right] \cup [4, \infty)$

Question 3 (b)

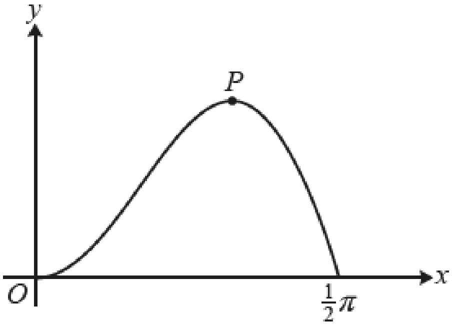
(b) Give full details of a sequence of two transformations needed to transform the graph of $y=|x-2|$ to the graph of $y=|2x-6|$. **[3]**

While it was clear that most candidates appreciated that the most common way of transforming the first graph to the other was by the application of a translation and a stretch the mathematical precision of language and understanding of the required order of transformations was not done well.

	AfL	When describing transformations in A level Mathematics and Further Maths, the mathematical terms that should be used are: translation, reflection, stretch, rotation, shear and enlargement and NOT shift, move, squash, etc. When describing a translation e.g. $y=\|x-2\|$ to $y=\|x-6\|$ then it should be given either in terms of the column vector $\begin{pmatrix}4\\0\end{pmatrix}$ or in words e.g. '…in the positive *x*-direction by 4 units' or '..by 4 units parallel to the *x*-axis'. Note that 'in/on/across/up/along' are not correct, 'to the right' is not sufficiently accurate and it is incorrect to define a translation with a 'scale factor' of 4. Note that the word 'translation' must be stated (and not 'transformation') too. With regards to describing a stretch e.g. $y=\|x-6\|$ to $y=\|2x-6\|$ then either '…by a scale factor of 0.5 in the *x*-direction' or '…parallel to the *x*-axis' is required. Once again 'in/on/across/up/along the *x* axis' are incorrect and so is 'in the positive *x*-direction', it is also incorrect to say '…a scale factor of 0.5 units'. Note that the word 'stretch' must be stated.

Question 4 (a)

4



The diagram shows the part of the curve $y = 3x\sin 2x$ for which $0 \leqslant x \leqslant \frac{1}{2}\pi$.

The maximum point on the curve is denoted by P.

(a) Show that the x-coordinate of P satisfies the equation $\tan 2x + 2x = 0$. **[3]**

While most candidates differentiated *y* correctly (using the product rule) and obtained $3\sin 2x + 6x\cos 2x = 0$ many candidates did not show enough working to convince examiners that they had derived and not simply stated the given result.

For example, $\sin\ 2x + 2x\cos\ 2x = 0 \Rightarrow \tan\ 2x + 2x = 0$ scored A0.

But $\sin 2x + 2x\cos 2x = 0 \Rightarrow \dfrac{\sin 2x}{\cos 2x} + 2x = 0 \Rightarrow \tan 2x + 2x = 0$ scored A1.

Question 4 (b)

(b) Use the Newton-Raphson method, with a suitable initial value, to find the x-coordinate of P, giving your answer correct to 4 decimal places. Show the result of each iteration. **[4]**

It was extremely disappointing that so many candidates did not realise that this part was asking candidates to apply the Newton-Raphson method to find a solution to the equation given in (a) with many thinking (by their choice of $\mathrm{f}(x)$) that instead it was asking them to solve $3x\sin 2x = 0$. Of those candidates that correctly realised that $\mathrm{f}(x) = \tan 2x + 2x$ and hence $\mathrm{f}'(x) = 2\sec^2 2x + 2$ most went on to correctly apply the Newton-Raphson formula with a suitable starting value and obtained the *x*-coordinate of *P* as 1.0144.

Question 4 (c)

(c) The trapezium rule, with four strips of equal width, is used to find an approximation to

$$\int_0^{\frac{1}{2}\pi} 3x \sin 2x \, dx.$$

Show that the result can be expressed as $k\pi^2(\sqrt{2}+1)$, where k is a rational number to be determined. **[4]**

It is vital in questions like this that candidates read the entire question first before starting their attempt as it was clear to examiners that while most candidates were familiar with the trapezium rule, and began by correctly stated the value of h as $\frac{\pi}{8}$, many incorrectly used approximate *y*-values instead of exact ones (which were required due to the nature of the given answer). Of those that did use exact values in surd form most went on to obtain the correct value of *k*.

Question 4 (d) (i)

(d) **(i)** Evaluate $\int_0^{\frac{1}{2}\pi} 3x \sin 2x \, dx.$ **[1]**

It was expected that candidates would evaluate this definite integral on their calculator and therefore write down the correct answer of $\frac{3\pi}{4}$, far too many candidates used integration by parts or a substitution even though only 1 mark was available.

Question 4 (d) (ii)

(ii) Hence determine whether using the trapezium rule with four strips of equal width gives an under- or over-estimate for the area of the region enclosed by the curve $y = 3x \sin 2x$ and the *x*-axis for $0 \leqslant x \leqslant \frac{1}{2}\pi$. **[1]**

It is clear that many candidates are unfamiliar with the command words, 'hence' and 'determine', in this case it was expected that candidates would use the values found in parts (c) and (d)(i) to determine that the trapezium rule (which had a numerical value of 2.233..) gives an under-estimate of the area when compared to the value of 2.356… (which was the numerical value of the definite integral from part (d)(i)).

Question 4 (d) (iii)

(iii) Explain briefly why it is not easy to tell from the diagram alone whether the trapezium rule with four strips of equal width gives an under- or over-estimate for the area of the region in this case. **[1]**

It was pleasing to note that many candidates realised that as the curve changed from being convex to concave it was not possible to tell from the diagram alone whether the trapezium rule would give an under- or over-estimate for the area of the region. Although a number of candidates gave spurious reasons such as there was no scaling on the *x*-axis or that the diagram was not sufficiently large enough to tell.

Question 5 (a)

5 In this question you must show detailed reasoning.

(a) Prove that $(\cot\theta + \operatorname{cosec}\theta)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$. **[4]**

There were many ways of tackling this proof, but the most efficient method was to realise that the right-hand side only contained cosines and so therefore it would be prudent to re-write the left-hand side as

$(\cot\theta + \operatorname{cosec}\theta)^2 = \left(\dfrac{\cos\theta}{\sin\theta} + \dfrac{1}{\sin\theta}\right)^2$, combine the two fractions and then use the fact that $\sin^2\theta = 1-\cos^2\theta$ to obtain $\dfrac{(1+\cos\theta)^2}{1-\cos^2\theta}$. For many candidates who got to this stage some did not show sufficient working and simply wrote down that $\dfrac{(1+\cos\theta)^2}{1-\cos^2\theta} = \dfrac{1+\cos\theta}{1-\cos\theta}$ without showing the intermediate step of $\dfrac{(1+\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$. Those candidates who started by trying to work from the right to left-hand side or began by expanding $(\cot\theta + \operatorname{cosec}\theta)^2$ as $\cot^2\theta + 2\cot\theta\operatorname{cosec}\theta + \operatorname{cosec}^2\theta$ were not usually successful.

Question 5 (b)

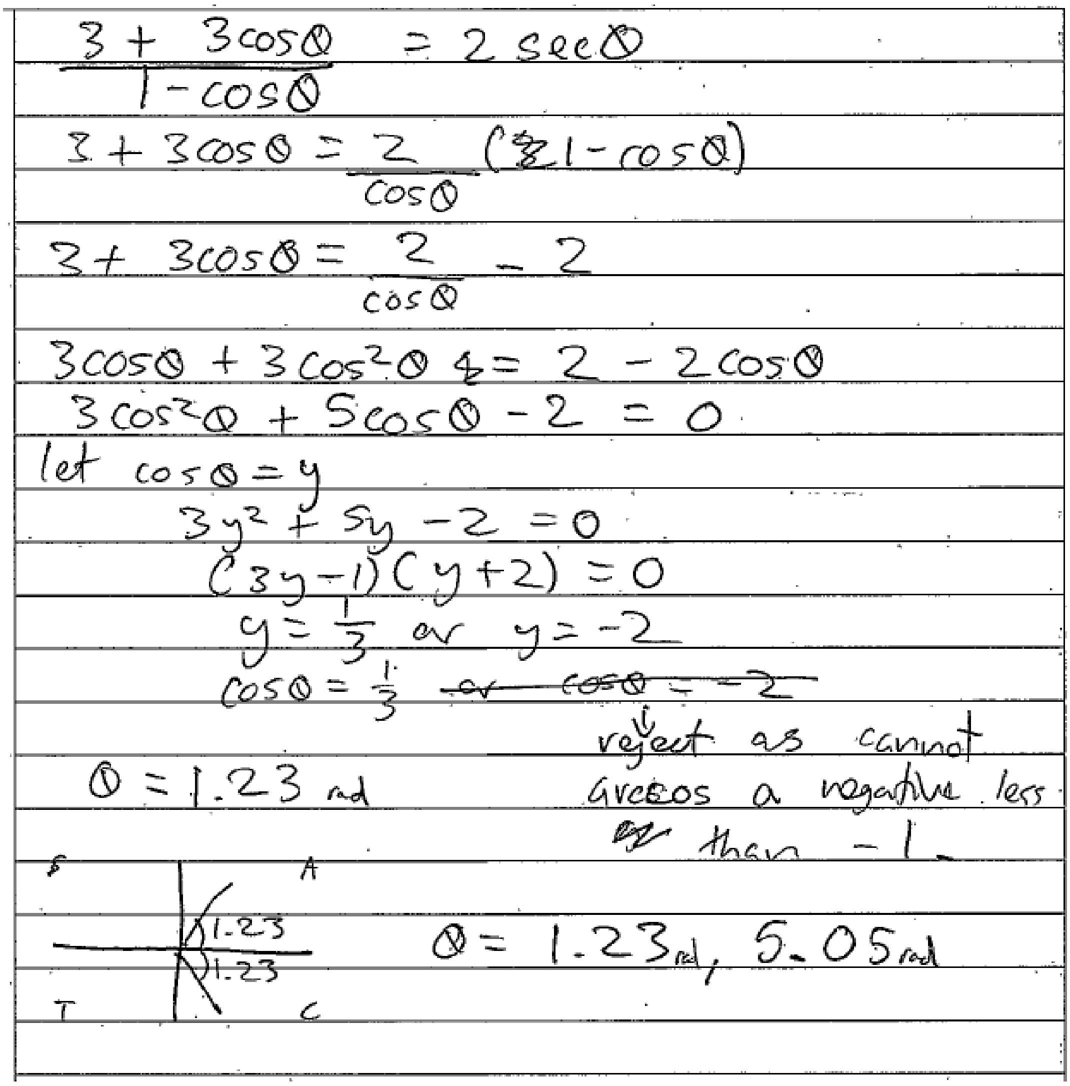
(b) Hence solve, for $0 < \theta < 2\pi$, $3(\cot\theta + \operatorname{cosec}\theta)^2 = 2\sec\theta$. **[5]**

It was pleasing to note that most candidates used the result in part (a) to write down the required equation

$3\left(\dfrac{1+\cos\theta}{1-\cos\theta}\right) = \dfrac{2}{\cos\theta}$. It was, however, disappointing that many candidates either misplaced the 2 or the 3 or believed that $3\left(\dfrac{1+\cos\theta}{1-\cos\theta}\right) = \dfrac{3+3\cos\theta}{3-3\cos\theta}$. Most candidates removed the fractions and obtained a three-term quadratic in cosine, although mistakes with signs when collecting together the like terms was common. As this question explicitly asked candidates to 'show detailed reasoning' it was extremely disappointing that many went directly from the correct equation $3\cos^2\theta + 5\cos\theta - 2 = 0$ to $\cos\theta = -2$ and $\cos\theta = \dfrac{1}{3}$ without any evidence of a correct analytical method. Examiners expected to see some indication of the rejection of $\cos\theta = -2$ and although most candidates correctly stated $\theta = 1.23$ some forgot about the other value in the interval (which was 5.05).

	AfL	A review of GCSE work on manipulation of algebraic expressions and solving quadratic equations may help reduce careless mistakes. Available on the subject webpage: Student guide to bridging the gap between GCSE and AS/A Level

Exemplar 1



$\dfrac{3+3\cos\theta}{1-\cos\theta} = 2\sec\theta$

$3+3\cos\theta = \dfrac{2}{\cos\theta}(1-\cos\theta)$

$3+3\cos\theta = \dfrac{2}{\cos\theta} - 2$

$3\cos\theta + 3\cos^2\theta = 2 - 2\cos\theta$

$3\cos^2\theta + 5\cos\theta - 2 = 0$

let $\cos\theta = y$

$3y^2 + 5y - 2 = 0$

$(3y-1)(y+2) = 0$

$y = \frac{1}{3}$ or $y = -2$

$\cos\theta = \frac{1}{3}$ ~~or $\cos\theta = -2$~~

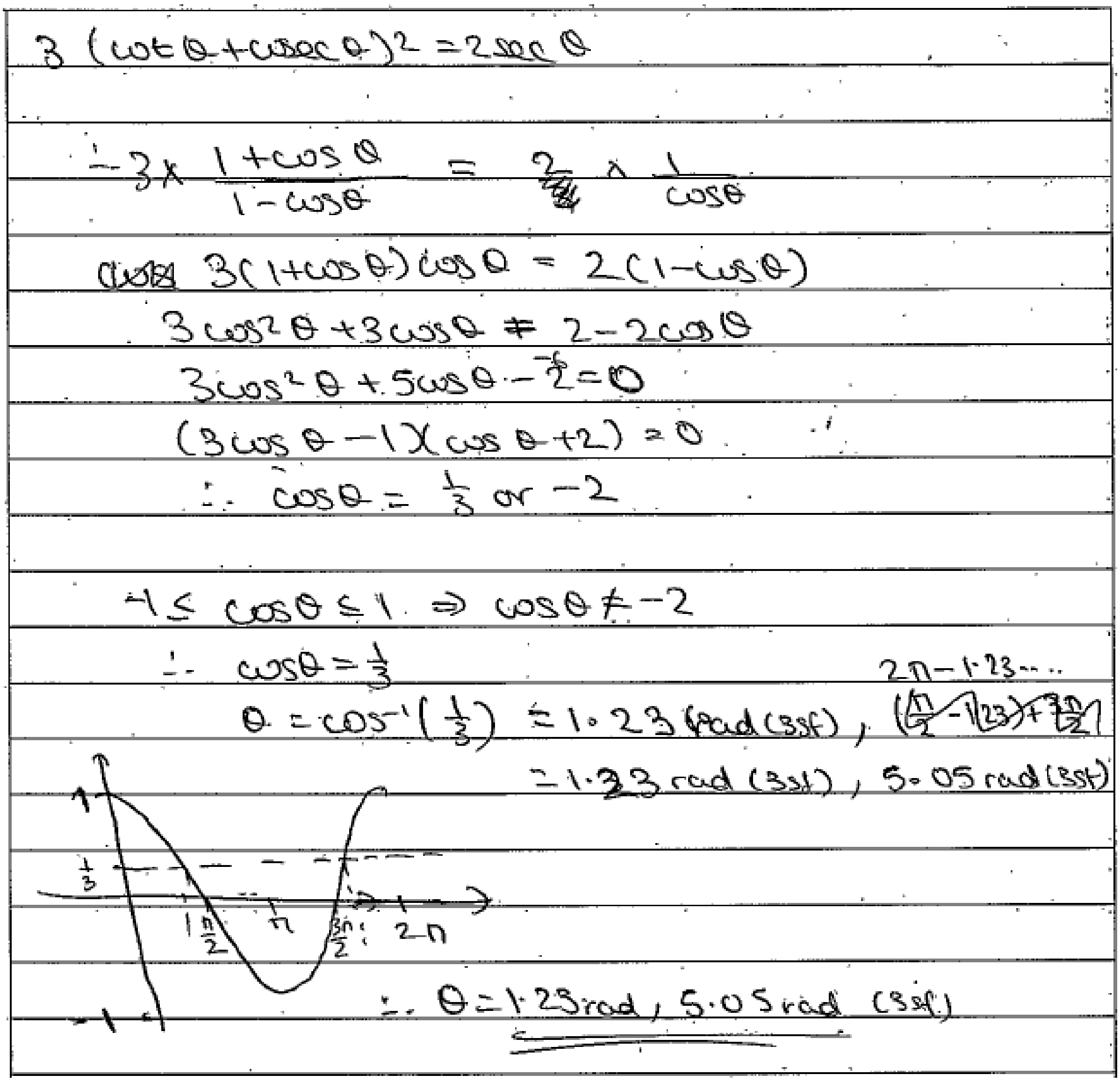
reject as cannot arccos a negative less than −1

$\theta = 1.23$ rad

$\theta = 1.23$ rad, 5.05 rad

Exemplar 1 shows a detailed and complete analytical method. The candidate obtained the correct quadratic equation in cosine so scored the first two marks. There is then a clear substitution method shown for how the 'disguised quadratic' has be evaluated. There is a mark for explicitly rejecting the -2 root. The sketch is not specifically needed, but hopefully has helped the candidate avoid a careless mistake in determining all the answers within the required range.

Exemplar 2



$3(\cot\theta + \text{cosec}\,\theta)^2 = 2\sec\theta$

$\therefore 3 \times \dfrac{1+\cos\theta}{1-\cos\theta} = 2 \times \dfrac{1}{\cos\theta}$

$3(1+\cos\theta)\cos\theta = 2(1-\cos\theta)$

$3\cos^2\theta + 3\cos\theta = 2 - 2\cos\theta$

$3\cos^2\theta + 5\cos\theta - 2 = 0$

$(3\cos\theta - 1)(\cos\theta + 2) = 0$

$\therefore \cos\theta = \frac{1}{3}$ or -2

$-1 \le \cos\theta \le 1 \Rightarrow \cos\theta \ne -2$

$\therefore \cos\theta = \frac{1}{3}$ $2\pi - 1.23\ldots$

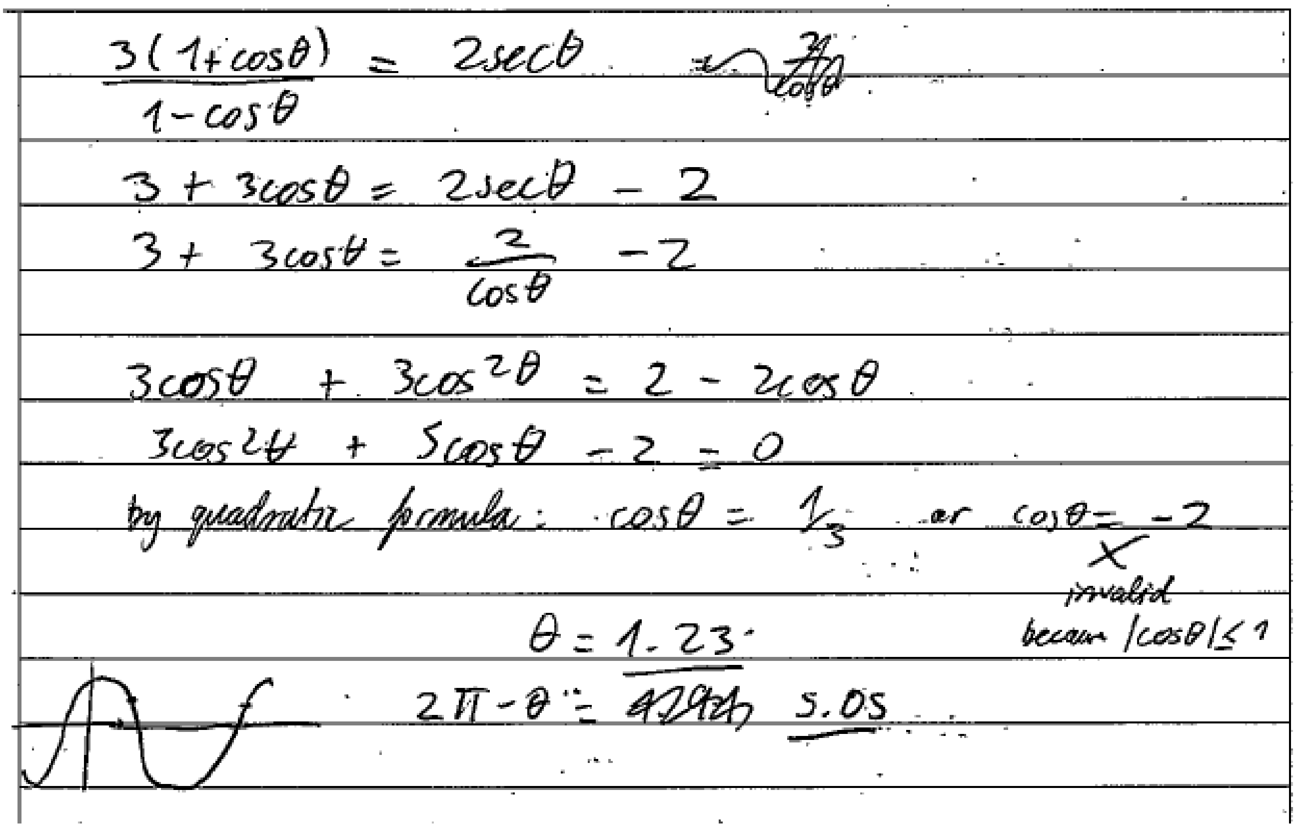
$\theta = \cos^{-1}(\frac{1}{3}) \approx 1.23$ rad (3sf), $(\frac{\pi}{2} - 1.23) + \frac{3\pi}{2}$

$= 1.23$ rad (3sf), 5.05 rad (3sf)

$\therefore \theta = 1.23$ rad, 5.05 rad (3sf)

Exemplar 2 also shows a detailed and complete analytical method for full credit. This candidate has factorised the quadratic directly in terms of $\cos\theta$. Again a nice sketch has be utilised to help identify the second solution.

Exemplar 3


$\frac{3(1+\cos\theta)}{1-\cos\theta} = 2\sec\theta$

$3 + 3\cos\theta = 2\sec\theta - 2$

$3 + 3\cos\theta = \frac{2}{\cos\theta} - 2$

$3\cos\theta + 3\cos^2\theta = 2 - 2\cos\theta$

$3\cos^2\theta + 5\cos\theta - 2 = 0$

by quadratic formula: $\cos\theta = \frac{1}{3}$ or $\cos\theta = -2$ ✗ invalid because $|\cos\theta| \leq 1$

$\theta = 1.23$

$2\pi - \theta = 5.05$

This response scored three of the five marks. The candidate obtained the correct quadratic equation in cosine so scored the first two marks. However, the candidate did not show detailed reasoning in how they have used the quadratic formula to obtain the values of $\frac{1}{3}$ and -2; the third method mark and therefore the next accuracy mark could not be awarded.

Either showing the line $\cos\theta = \frac{-5 \pm \sqrt{5^2 - 4(3)(-2)}}{2 \times 3}$, or using substitution in the form

Let $x = \cos\theta$

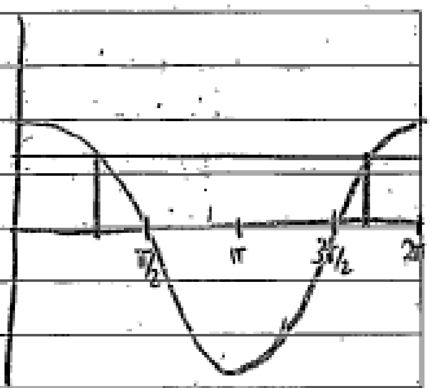
$\therefore 3x^2 + 5x - 2 = 0$

$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-2)}}{2 \times 3}$ etc would have supported the detailed reasoning.

This candidate has been awarded the final B mark for the correct answers of 1.23 and 5.05.

Exemplar 4

3. $\left(\frac{1+\cos\theta}{1-\cos\theta}\right) = 2\sec\theta$



$\frac{3+3\cos\theta}{3-3\cos\theta} = 2\sec\theta$

$3+3\cos\theta = \frac{2}{\cos\theta} \times (3-3\cos\theta)$

$3+3\cos\theta = \frac{2(3-3\cos\theta)}{\cos\theta}$

$3+3\cos\theta = \frac{6-6\cos\theta}{\cos\theta}$ $(\times\cos\theta)$

$3\cos\theta + 3\cos^2\theta = 6-6\cos\theta$ $(+6\cos\theta)$

$3\cos^2\theta + 9\cos\theta = 6$ (-6)

$3\cos^2\theta + 9\cos\theta - 6 = 0$ $(\div 3)$

$\cos^2\theta + 3\cos\theta - 3 = 0$

let $x = \cos\theta$

$x^2 + 3x - 3 = 0$

$a = 1$ $\quad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$b = 3$

$c = -3$ $\quad = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-3)}}{2}$

$\cos\theta = \frac{-3+\sqrt{21}}{2}$ $\quad = \frac{-3 \pm \sqrt{9+12}}{2}$

$\theta_1 = 0.658$

$\theta_2 = 5.37$ $\quad x = \frac{-3 \pm \sqrt{21}}{2}$

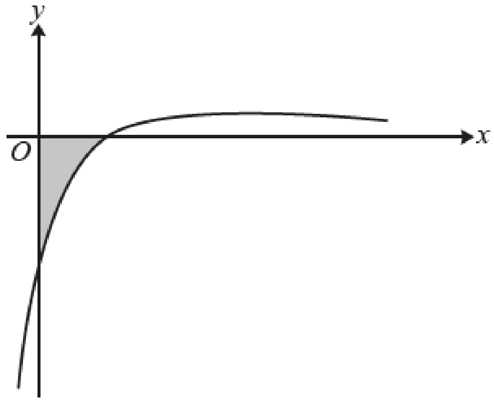
$\cos\theta \neq \frac{-3-\sqrt{21}}{2}$ $\quad$ Answer: $\theta = 0.658$ and $\theta = 5.37$

Na

This response scored all three of the method marks, but the initial error with multiplying the fraction by 3 meant that the accuracy marks were lost. In general, candidates that opted for the use of the quadratic formula rather than factorisation seemed to make mistakes in the initial algebraic manipulation.

Question 6

6



The diagram shows part of the curve $y = \dfrac{2x-1}{(2x+3)(x+1)^2}$.

Find the exact area of the shaded region, giving your answer in the form $p + q\ln r$, where p and q are positive integers and r is a positive rational number. **[10]**

Many candidates made a good attempt at this question with most realising that to integrate $\dfrac{2x-1}{(2x+3)(x+1)^2}$, partial fractions of the form $\dfrac{A}{2x+3} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$ were required, and many used a correct method to obtain an answer of $-\dfrac{16}{2x+3} + \dfrac{8}{x+1} - \dfrac{3}{(x+1)^2}$ (and so scored the first five marks in this part). It was the subsequent integration that most candidates found difficult with many believing that all three integrated expressions involved the natural log function or that $\int \dfrac{dx}{(x+1)^2} = -\dfrac{1}{(x+1)^3}$. Of those that integrated correctly and used the correct limits of 0.5 and 0 it was pleasing to note that many obtain a correct value for the integral as $8\ln\frac{9}{8} - 1$. However, most candidates who had got this far did not realise that this was not of the required form as stated in the question (and so therefore this was not an expression for the exact area of the shaded region). Only the most able of candidates realised that the area of the shaded region was in fact $1 + 8\ln\frac{8}{9}$.

Exemplar 5

intersection ⇒ $y = 0$.

$0 = 2x - 1$

$2x = 1$

$x = \frac{1}{2}$.

consider $\frac{2x-1}{(2x+3)(x+1)^2}$

$\frac{2x-1}{(2x+3)(x+1)^2} = \frac{A}{2x+3} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$2x - 1 \equiv A(x+1)^2 + B(x+1)(2x+3) + C(2x+3)$

when $x = -1$ when $x = -\frac{3}{2}$ $x^2: \; 0 = 1A + 2B + 0C$.

$-3 = 1C$ $-4 = \frac{1}{4}A$ $0 = -16 + 2B$

$C = -3$ $A = -16$. $B = 8$.

$\int_0^{\frac{1}{2}} \frac{-16}{2x+3} + \frac{8}{x+1} - \frac{3}{(x+1)^2} \, dx.$

$\int_0^{\frac{1}{2}} \frac{-16}{2x+3} + \frac{8}{x+1} - 3(x+1)^{-2} \, dx.$

$= \left[-8\ln(2x+3) + 8\ln(x+1) \frac{-3(x+1)^{-1}}{-1}\right]_0^{\frac{1}{2}}$

$= \left[8\ln\left(\frac{x+1}{2x+3}\right) + 3(x+1)^{-1}\right]_0^{\frac{1}{2}}$

$= \left[8\ln\left(\frac{3}{8}\right) + 2\right] - \left[8\ln\left(\frac{1}{3}\right) + 3\right]$

$= 8\ln\left(\frac{3}{8}\right) + 2 - 8\ln\left(\frac{1}{3}\right) - 3.$

$= 8\ln\left(\frac{9}{8}\right) - 1$

$= -1 + 8\ln\left(\frac{9}{8}\right)$

This response scored the first nine marks. The candidate provided an almost perfect solution to the problem but due to the nature of their exact answer the candidate has not realised that their answer is negative (and therefore not the exact area of the shaded region). Even though the question explicitly said that the answer should be given in the form $p + q\ln r$ with p as a positive integer many candidates failed to realise that the required area was in fact $1 + 8\ln\left(\frac{8}{9}\right)$.

Section B overview

Two general points with regards to the answering of certain mechanics questions should be made in this overview. The first is that unless told otherwise the value that candidates should use for the acceleration due to gravity, g, is 9.8 and not 10 or 9.81 (and this value is stated explicitly on the front cover of the examination paper). Secondly, when applying Newton's second law in the context of connected particles, centres (when teaching) and candidates (when answering examination questions) are strongly encouraged to apply $F = ma$ to each particle separately rather than attempting to apply this equation to the whole system. These attempts generally result in either the incorrect number of forces on the left-hand side of the equation or errors with the mass/acceleration of the combined system on the right-hand side. Often these attempts score no marks (as was commonly seen in this paper in question 9(b)).

Question 7 (a)

7 A cyclist starting from rest accelerates uniformly at $1.5\,\text{m}\,\text{s}^{-2}$ for 4 s and then travels at constant speed.

(a) Sketch a velocity-time graph to represent the first 10 seconds of the cyclist's motion. **[2]**

The velocity-time graph sketch was nearly always completed correctly with the only error being a lack of correct values labelled on the axes.

Question 7 (b)

(b) Calculate the distance travelled by the cyclist in the first 10 seconds. **[2]**

This part also proved to be a good source of marks for most candidates with most correctly applying the formula for the area of a trapezium (or the areas of a triangle + rectangle) to obtain the correct distance travelled as 48 m.

Question 8 (a)

8 A particle P projected from a point O on horizontal ground hits the ground after 2.4 seconds.

The horizontal component of the initial velocity of P is $\frac{5}{3}d\,\text{m}\,\text{s}^{-1}$.

(a) Find, in terms of d, the horizontal distance of P from O when it hits the ground. **[1]**

Most candidates correctly found the horizontal distance of P from O when it hits the ground as $4d$.

Question 8 (b)

(b) Find the vertical component of the initial velocity of P. **[2]**

Candidates were evenly split between those who correctly used $s = ut + \frac{1}{2}at^2$ with $s = 0, a = -g$ and $t = 2.4$, and those who used $v = u + at$ with $v = 0, a = -g$ and $t = 1.2$. Where errors occurred it was usually in mixing up these two methods, for example, some candidates applied $v = u + at$ with $v = 0, a = -g$ and $t = 2.4$.

Question 8 (c)

P just clears a vertical wall which is situated at a horizontal distance *d* m from *O*.

(c) Find the height of the wall. **[3]**

Those candidate who realised that it took 0.6 seconds for *P* to reach the wall (which came from the fact that $d = \left(\frac{5}{3}d\right)t$) usually went on to apply $s = ut + \frac{1}{2}at^2$ correctly to find the height of the wall. The most common error was believing that the time it took *P* to reach the wall was either 1.2 or 2.4 seconds.

Question 8 (d)

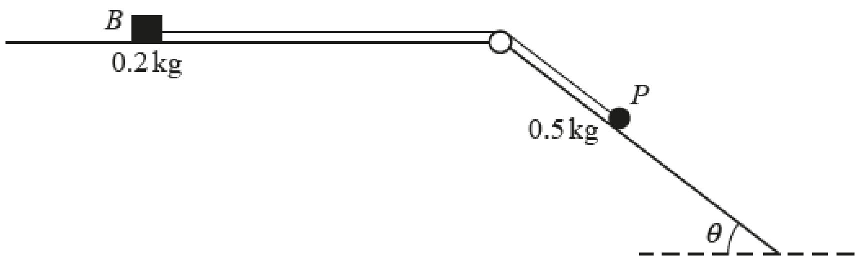
The speed of *P* as it passes over the wall is $16\,\mathrm{m\,s^{-1}}$.

(d) Find the value of *d* correct to 3 significant figures. **[4]**

The most common misconception here was to use the result from part (b) together with $v = 16$ in the equation $v^2 = u^2 + 2as$ instead of realising that the vertical component of the speed, v_1, at the top of the wall had to be worked out first before then using the equation $\sqrt{\left(\frac{5}{3}d\right)^2 + v_1^2} = 16$ and solving to find the value of *d*.

Question 9 (a)

9



The diagram shows a small block *B*, of mass 0.2 kg, and a particle *P*, of mass 0.5 kg, which are attached to the ends of a light inextensible string. The string is taut and passes over a small smooth pulley fixed at the intersection of a horizontal surface and an inclined plane.

The block can move on the horizontal surface, which is rough. The particle can move on the inclined plane, which is smooth and which makes an angle of θ with the horizontal where $\tan\theta = \frac{3}{4}$.

The system is released from rest. In the first 0.4 seconds of the motion *P* moves 0.3 m down the plane and *B* does not reach the pulley.

(a) Find the tension in the string during the first 0.4 seconds of the motion. **[4]**

Examiners noted that it was rather surprising that while many candidates could correctly apply Newton's second law for particle *P* and obtain a correct equation of the form $0.5g\sin\theta - T = 0.5a$, a significant proportion failed to calculate the acceleration correctly as 3.75 with many believing that the acceleration was 0.75 (which came from incorrectly dividing 0.3 by 0.4). It was pleasing though to note that many candidates used the correct exact value of $\sin\theta = \frac{3}{5}$ instead of needing to work out θ as 36.869…

Question 9 (b)

(b) Calculate the coefficient of friction between B and the horizontal surface. **[5]**

Some candidates only managed to score the mark for realising that the normal contact force *R*, at *B*, was equal to 0.2g. Many then incorrectly believed that the coefficient of friction was simply given by the value found in (a) divided by 0.2g. Of those that correctly applied Newton's second law for block *B* as $T - \mu R = 0.2a$ most went on to obtain a correct value for μ. As mentioned in the Section B overview candidates are advised to apply Newton's second law to each particle separately and not attempt to apply it to the whole system as very few obtained the correct equation of $0.5g\sin\theta - \mu R = 0.7a$.

Question 10 (a)

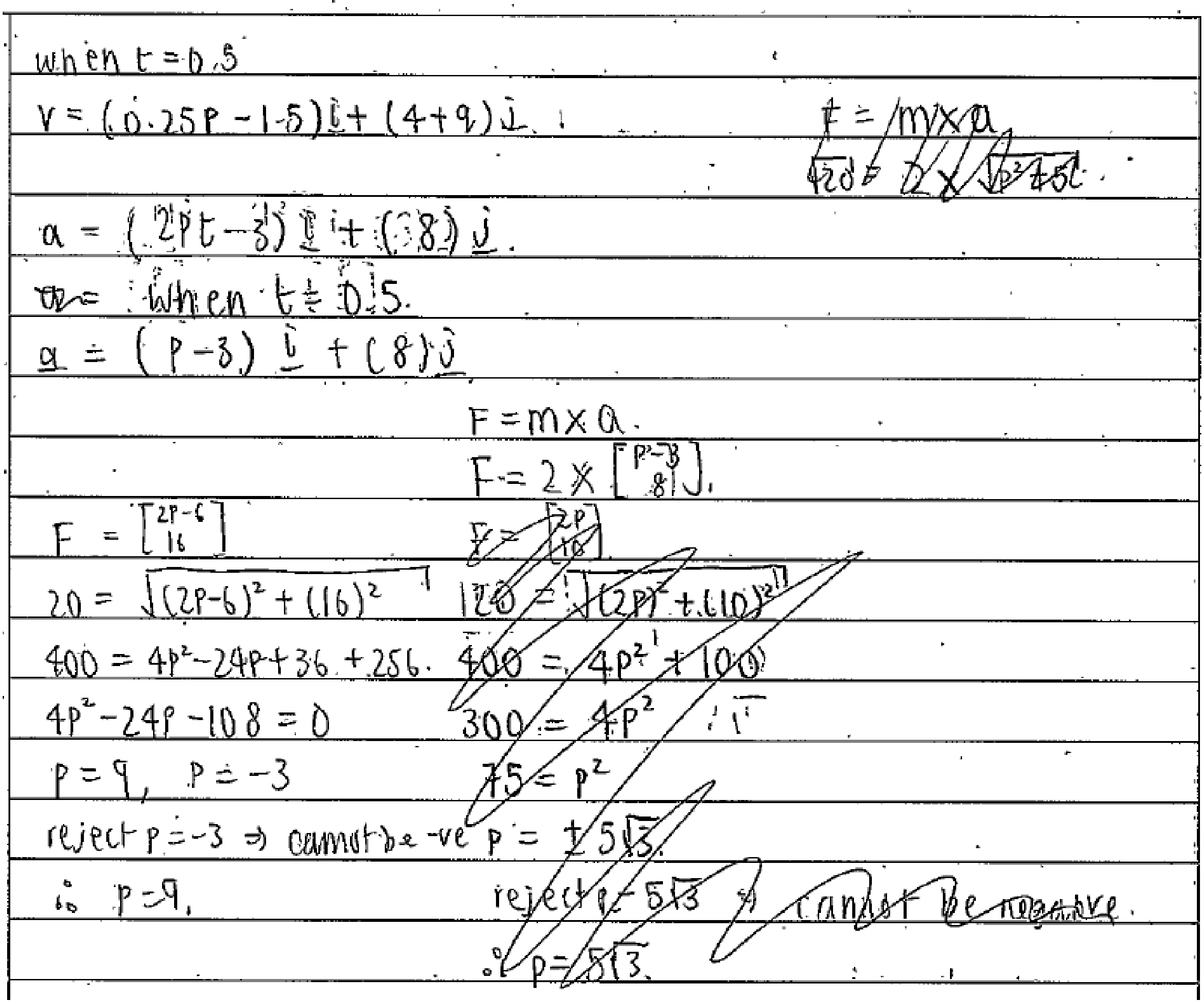
10 In this question the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the directions east and north respectively.

A particle R of mass $2\,\text{kg}$ is moving on a smooth horizontal surface under the action of a single horizontal force $\mathbf{F}\,\text{N}$. At time t seconds, the velocity $\mathbf{v}\,\text{m}\,\text{s}^{-1}$ of R, relative to a fixed origin O, is given by $\mathbf{v} = (pt^2 - 3t)\mathbf{i} + (8t + q)\mathbf{j}$, where p and q are constants and $p < 0$.

(a) Given that when $t = 0.5$ the magnitude of $\mathbf{F}$ is 20, find the value of p. **[6]**

Nearly all candidates correctly differentiated $\mathbf{v}$ and obtained $\mathbf{a} = (2pt - 3)\mathbf{i} + 8\mathbf{j}$. Unfortunately many candidates then made careless slips in applying $|\mathbf{F}| = m|\mathbf{a}|$ to obtain an equation in *p* only with the most common errors being forgetting the mass completely or only multiplying the **i** component by the value of *m*. Candidates are one again reminded to read the question carefully as while many had a correct equation in *p* e.g. $(p-3)^2 + 64 = 100$ many solved and gave the positive root even though the question specifically said that *p* was negative.

Exemplar 6



when $t = 0.5$

$v = (0.25p - 1.5)\mathbf{i} + (4+q)\mathbf{j}$

$a = (2pt - 3)\mathbf{i} + (8)\mathbf{j}$

when $t = 0.5$

$a = (p-3)\mathbf{i} + (8)\mathbf{j}$

$F = m \times a$

$F = 2 \times \begin{bmatrix} p-3 \\ 8 \end{bmatrix}$

$F = \begin{bmatrix} 2p-6 \\ 16 \end{bmatrix}$

$20 = \sqrt{(2p-6)^2 + (16)^2}$

$400 = 4p^2 - 24p + 36 + 256$

$4p^2 - 24p - 108 = 0$

$p = 9, \quad p = -3$

reject $p = -3 \Rightarrow$ cannot be -ve

$\therefore p = 9$

This response scored the first five marks. An almost perfect response let down by the fact that the candidate did not read the question carefully enough and assumed that the value of p could not be negative.

Question 10 (b)

When $t = 0$, R is at the point with position vector $(2\mathbf{i} - 3\mathbf{j})$ m.

(b) Find, in terms of q, an expression for the displacement vector of R at time t. **[4]**

This part was answered extremely well with many candidates correctly realising that they had to use integration to find an expression for the displacement in terms of t. However, many ignored the vector constant of integration that would arise from the corresponding indefinite integral and some incorrectly integrated q as $\frac{1}{2}q^2$ instead of as qt.

Question 10 (c)

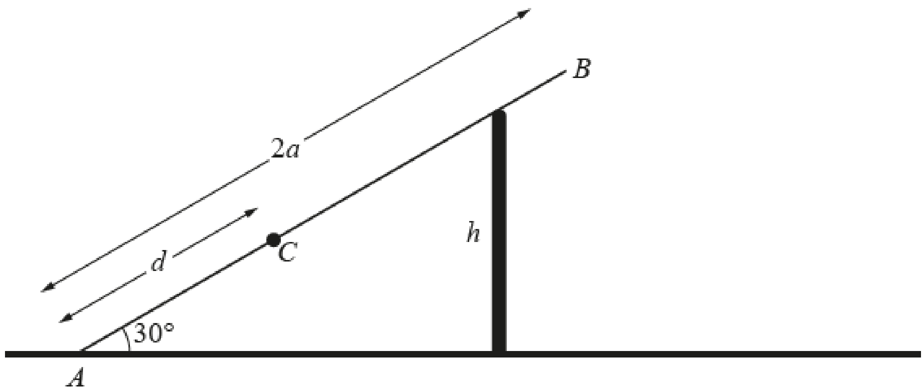
When $t = 1$, R is at a point on the line L, where L passes through O and the point with position vector $2\mathbf{i} - 8\mathbf{j}$.

(c) Find the value of q. **[3]**

This part discriminated well with many candidates making little progress apart from substituting $t = 1$ into their expression from part (b). Those candidates who realised that one way to find q was to equate a scalar multiple of their displacement vector (evaluated at $t = 1$) to $2\mathbf{i} - 8\mathbf{j}$ were usually successful in first finding the scalar multiple and hence the value of q.

Question 11 (a)

11



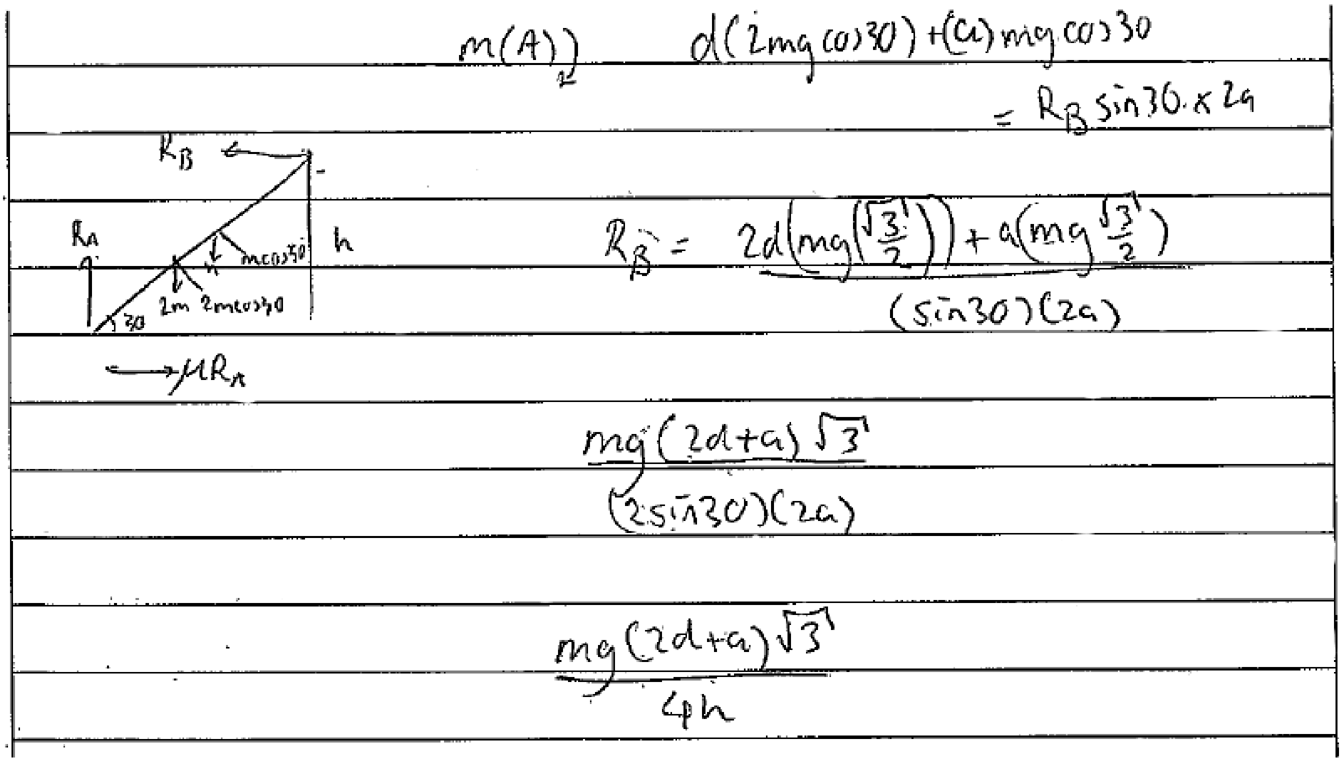
The diagram shows a ladder AB, of length $2a$ and mass m, resting in equilibrium on a vertical wall of height h. The ladder is inclined at an angle of 30° to the horizontal. The end A is in contact with horizontal ground. An object of mass $2m$ is placed on the ladder at a point C where $AC = d$.

The ladder is modelled as uniform, the ground is modelled as being rough, and the vertical wall is modelled as being smooth.

(a) Show that the normal contact force between the ladder and the wall is $\frac{mg(a+2d)\sqrt{3}}{4h}$. **[4]**

While many candidates correctly derived the given answer for the normal contact force between the ladder and the wall it was clear that many candidates were unclear how to proceed and many were hampered by assuming that this normal contact force was acting either horizontally or vertically at the wall instead of acting perpendicular to the ladder. It was evident to examiners that while many candidates did attempt to take moments about A and some correctly achieved the clockwise moment of $2mgd\cos 30 + mga\cos 30$ (for the two weights) many fudged the anticlockwise moment with very few convincingly realising that this anticlockwise moment was given by the expression $R_w\left(\frac{h}{\sin 30}\right)$.

Exemplar 7



This response scored no marks. The diagram, together with the fact that in their moments equation they have a component of the normal contact force at the wall, indicates that their normal contact force is not pointing in the correct direction for this problem. The fact that the candidates obtains the 'correct' answer does not change the fact this response scored no marks as in the penultimate line of working the denominator is clearly equivalent to 2*a* but conveniently appears as the correct 4*h* in the final line.

Question 11 (b)

It is given that the equilibrium is limiting and the coefficient of friction between the ladder and the ground is $\frac{1}{8}\sqrt{3}$.

(b) Show that $h = k(a+2d)$, where k is a constant to be determined. **[7]**

For those candidates who realised that resolving vertically and horizontally for the system and then applying $F = \mu R$ was the way of deriving the 'Show that' in this part most came unstuck due to the early error in part (a) of not having the correct direction for the normal contact force at the wall. Of those that did resolve correctly, a number then struggled with the corresponding algebra and very few achieved the correct answer of $k = \frac{11}{24}$.

Exemplar 8

$\mu = \frac{\sqrt{3}}{8}$

resolve(↕): $R_B \cos 30 + R_A = 2mg + mg$

$$\frac{\sqrt{3}}{2} R_B + R_A = 3mg$$

$$R_A = 3mg - \frac{\sqrt{3}}{2} R_B \left(\frac{\sqrt{3}\,mg(a+2d)}{4h}\right)$$

$$R_A = 3mg - \frac{3mg(a+2d)}{8h}$$

resolve(↔): $R_B \sin 30 = F_R$, $F_R = \mu R$

$$\frac{1}{2} R_B = \frac{\sqrt{3}}{8} R_A$$

$$\frac{1}{2} R_B = \frac{\sqrt{3}}{8}\left(3mg - \frac{\sqrt{3}}{2} R_B\right)$$

$$\frac{1}{2} R_B = \frac{3\sqrt{3}}{8} mg - \frac{3}{16} R_B$$

$$\frac{11}{16} R_B = \frac{3\sqrt{3}}{8} mg$$

$$\frac{11}{16}\left(\frac{\sqrt{3}\,mg(a+2d)}{4h}\right) = \frac{3\sqrt{3}}{8} mg$$

$$\frac{11}{2}\left(\frac{a+2d}{4h}\right) = 3$$

$$3h = \frac{11}{8}(a+2d)$$

$$h = \frac{11}{24}(a+2d)$$

A completely correct solution in which the candidate has clearly shown each stage of working from resolving horizontally and vertically to applying $F = \mu R$ leading to the correct expression for h in terms of a and d.

Question 11 (c)

(c) Hence find, in terms of a, the greatest possible value of d. **[2]**

Only the most able candidates realised that h could not exceed $2a\sin 30$ and therefore the greatest possible value of d was $\frac{13}{22}a$.

Question 11 (d)

(d) State one improvement that could be made to the model. **[1]**

Although many candidates gave a correct improvement (e.g. model the ladder as non-uniform, include a frictional component for the contact of the ladder with the wall) many either believed that the model could be improved by modelling the ground as smooth or using a more accurate value of g. Also, some candidates did not read this part carefully and instead gave a limitation of the model (e.g. the wall is unlikely to be smooth) and not an improvement.

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