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Examiners' report

MATHEMATICS A

H240

For first teaching in 2017

H240/03 Summer 2022 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate responses is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

Paper 3 series overview

This is the third examination component for the A Level examination for GCE Mathematics A. It is a two-hour paper consisting of 100 marks which tests content from Pure Mathematics (Section A, 50 marks) and Mechanics (Section B, 50 marks). Pure Mathematics content is tested on all three papers, and any topic could be tested on any of the three papers. Inevitably, the report that follows will concentrate on aspects of the candidates' performance where improvement is possible to assist centres on preparing candidates for future series. However, this should not obscure the fact that a significant number of candidates who sat this paper in this reformed A Level qualification produced solutions which were a pleasure for examiners to assess. Many candidates demonstrated a most impressive level of mathematical ability and insight which enabled them to meet the various challenges posed by this paper on both the pure and mechanics content; precision, command of correct mathematical notation and excellent presentational skills were evident in many scripts.

The specification includes some guidance about the level of written evidence required in assessment question; these were provided to reflect the increased functionality of the available calculators and the changes in assessment objectives, since there is a significant change from when the equivalent legacy qualifications were designed. There are several questions on this paper which began with the demand 'In this question you must show detailed reasoning'; to quote the specification that 'when a question includes this instruction candidates must give a solution which leads to a conclusion showing a detailed and complete analytical method. Their solution should contain enough detail to allow the line of their argument to be followed. This is not a restriction on a candidate's use of a calculator when tackling a question…but it is a restriction on what will be accepted as evidence of a complete method.' The specification then considers several examples which centres should consider so that future candidates understand exactly what is required when this request appears in future series. This command phrase features in Questions 5, 6 and 7.

The word 'determine' in a question does not simply imply that candidates should find the answer but, to quote the specification, 'this command word indicates that justification should be given for any results found, including working where appd phrase features in Questions 4 (a), 5 (a), 5 (b), 5 (c), 7 (a), 12 (a) and 13 (a). While there is no specific level of working needed to justify answers to questions which use the command word 'find …', method marks may still be available for valid attempts that do not result in a correct response, and standard advice (included in the specification) that candidates should state explicitly any expressions, integrals, parameters and variables that they use a calculator to evaluate (using correct mathematical notation rather than model specific calculator notation).

OCR support

A poster detailing the different command words and what they mean is available on the OCR website.

Candidates who did well on this paper generally did the following:	Candidates who did less well on this paper generally did the following:
• made efficient use of calculators • understood the level of response required for command words used in the questions • read questions carefully and provided the responses that were requested • used formal mathematical notation and language correctly.	• made careless mistakes in algebraic manipulation • used imprecise notation or language • provided mathematical working that was correct but did not answer the specific question that was being asked • did not give sufficient evidence in 'Show that' and 'Determine' questions.

Section A overview

Content from the pure section of the specification may be assessed on any of the three papers of H240. Most candidates appeared well prepared for the pure content, with method marks being given, however, there were a number of places where a more concise use of algebraic notation and language may have led to more of the corresponding accuracy marks being available.

Question 1

1 Solve the equation $|2x-3| = 9$. **[3]**

This question was answered extremely well with nearly all candidates correctly solving $2x-3=\pm 9$, although some candidates only solved for one of these two possible cases. The two most common errors seen by examiners were those candidates who re-wrote the equation as either $2x+3=9$ or $(2x-3)^2=9$.

Question 2 (a)

2 **(a)** Give full details of the single transformation that transforms the graph of $y = x^3$ to the graph of $y = x^3 - 8$. **[2]**

While it was clear that most candidates appreciated that the transformation required was a translation, the mathematical precision of language could have been better.

Assessment for learning

Transformations at A Level require a more formal use of language to describe how to go from the object function to the image function. Translations are best expressed as a single column vector. Informal terms such as 'shift', 'move' or 'along' will not score full marks.

Question 2 (b)

The function f is defined by $\mathrm{f}(x) = x^3 - 8$.

(b) Find an expression for $\mathrm{f}^{-1}(x)$. **[2]**

While a few candidates either differentiated or found the reciprocal of $x^3 - 8$, most knew the correct process of finding the inverse of the given function. The most common error was from those candidates who left their response in terms of *y*.

Question 2 (c)

(c) State how the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ are related geometrically. **[1]**

Most candidates scored the mark for stating that one graph was a reflection of the other graph in the line $y = x$. The most common errors were those who gave a non-geometrically reason (for example, simply stating that one was the inverse of the other) or did not state the line of reflection.

Question 3

3 The points P and Q have coordinates $(2, -5)$ and $(3, 1)$ respectively.

Determine the equation of the circle that has PQ as a diameter. Give your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are integers. **[4]**

Almost all candidates attempted to find the equation of the required circle by first finding the corresponding centre and radius. However, many candidates clearly thought that the distance between the two given points was the radius and not the diameter and, although less common, several candidates thought that the centre of the circle was given by the formula $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$. Even when candidates correctly had the circle in the form $\left(x - \frac{5}{2}\right)^2 + (y+2)^2 = \frac{37}{4}$, many struggled to simplify to the required form.

Question 4 (a)

4 The positive integers x, y and z are the first, second and third terms, respectively, of an arithmetic progression with common difference -4.

Also, x, $\frac{15}{y}$ and z are the first, second and third terms, respectively, of a geometric progression.

(a) Show that y satisfies the equation $y^4 - 16y^2 - 225 = 0$. **[4]**

Candidates found this first part very demanding with many struggling to make any meaningful progress. The most successful responses were those that recognised that the information regarding the GP implied that $\frac{15}{yx} = \frac{zy}{15}$ therefore $xy^2z = 225$, and that the information regarding the AP implied that $x = y + 4$ and $z = y - 4$. Those that did usually went on to derive the required quartic equation in y.

Question 4 (b)

(b) Hence determine the sum to infinity of the geometric progression. **[4]**

Even though this question said 'hence' many candidates did not use their calculators to solve the given quartic equation from part (a). Of those that did many simply stated that $y = \pm 5$ even though the question explicitly stated that y was a positive integer (with many candidates only realising that $y \neq -5$ much later in the question when this led to a value for the common ratio that was greater than 1). Of those candidates who successfully found a common ratio for the infinite geometric progression most correctly applied the correct formula for the corresponding sum to infinity.

Misconception

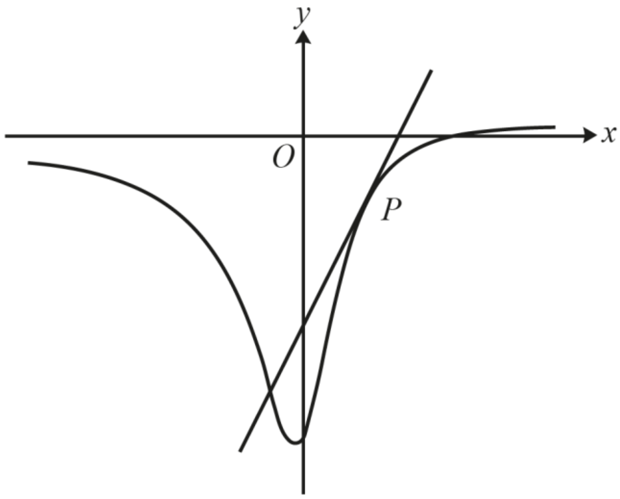
?

Many candidates did not hesitate to use the formula $S_\infty = \dfrac{a}{1-r}$ with values of $r > 1$ or $r < -1$.

Candidates are reminded that for a finite sum to infinity for a GP to exist then $|r| < 1$.

Question 5 (a)

5 In this question you must show detailed reasoning.



The diagram shows the curve with equation $y = \dfrac{2x-3}{4x^2+1}$. The tangent to the curve at the point P has gradient 2.

(a) Show that the x-coordinate of P satisfies the equation

$$4x^3 + 3x - 3 = 0.$$ **[5]**

It was very pleasing that most candidates correctly applied the quotient rule to the given equation and put the resulting derivative equal to 2 (although examiners did note that on occasion the derivative was set equal to zero). Most candidates then removed the fraction(s) and bracket(s) successfully and derived the equation $16x^4 + 12x^2 - 12x = 0$ (or a multiple of this). However, the question required 'detailed reasoning', and it was evident that a large number of candidates simply cancelled an x from each term of their equation believing that this was sufficient to show the required result. Only the most successful responses made any reference to the given diagram which showed that the *x*-coordinate of point *P* could not be equal to zero.

Exemplar 1

5(a)

$y = \frac{2x-3}{4x^2+1}$ p has gradient 2

$\cancel{u = 4x^2+1}$

$\cancel{u' = 8x}$

$u = 2x-3 \qquad v = 4x^2+1$

$u' = 2 \qquad v' = 8x$

$dy/dx = \frac{u'v - v'u}{v^2}$

$= \frac{2(4x^2+1) - 8x(2x-3)}{(4x^2+1)^2}$

$= \frac{8x^2+2-16x^2+24x}{(4x^2+1)^2}$

$= \frac{-8x^2+24x+2}{(4x^2+1)^2}$

$= \cancel{-24x^2}$

$2 = \frac{-8x^2+24x+2}{(4x^2+1)^2}$

$2(4x^2+1)^2 = -2(4x^2 \mp 12x \mp 1)$

$2(16x^4+8x^2+1) = -8x^2+24x+2$

$32x^4+16x^2+2 = -8x^2+24x+2$

$32x^4+24x^2-24x = 0$

$4x^4+3x^2-3x = 0$

$4x^3+3x-3 = 0.$

This response scored 4 (of the 5) marks. This was a very common response seen by examiners. The candidate has correctly applied the quotient rule and set the derivative equal to 2. They have then removed the fraction, expanded and simplified to a correct quartic equation in x $\left(4x^4+3x^2-3x=0\right)$.

However, no justification is given by the candidate for why it is acceptable to cancel the x's when simplifying the quartic equation to the required cubic equation which is why they did not achieve full marks.

Question 5 (b)

(b) Show by calculation that the x-coordinate of P lies between 0.5 and 1. **[2]**

Although most candidates applied the sign change test to the equation from part (a) some used the original equation of the curve (which was an incorrect method which could not show that the *x*-coordinate of *P* was between 0.5 and 1). Of those that did use a correct expression (with the majority using $4x^3+3x-3$ although some did use $\frac{2(4x^2+1)-8x(2x-3)}{(4x^2+1)^2}$ or a correct equivalent and compared with the value of 2) most:

-stated the values of these expressions when $x=0.5$ and $x=1$,

-proceeded to give a correct explanation ('change of sign' or 'either side of 2'),

-followed by a correct conclusion ('hence the x-coordinate of *P* lies between 0.5 and 1').

Question 5 (c)

(c) Show that the iteration

$$x_{n+1}=\frac{3-4x_n^3}{3}$$

cannot converge to the x-coordinate of P whatever starting value is used. **[2]**

This was probably the least well answered question on the whole paper with most candidates scoring zero marks (with many just showing that the given iteration did not converge for a number of different starting values). Only the most able realised what was required in this part. Of those that did calculate $F'(x_n)$ where $x_{n+1}=F(x_n)$, very few could correctly articulate that as the *x*-coordinate of *P* lies in the interval (0.5, 1) and $F'(\alpha)<-1$ for all values of α where $\alpha\in(0.5,1)$, then this implies that the given iteration cannot converge to the *x*-coordinate of *P* whatever starting value is used.

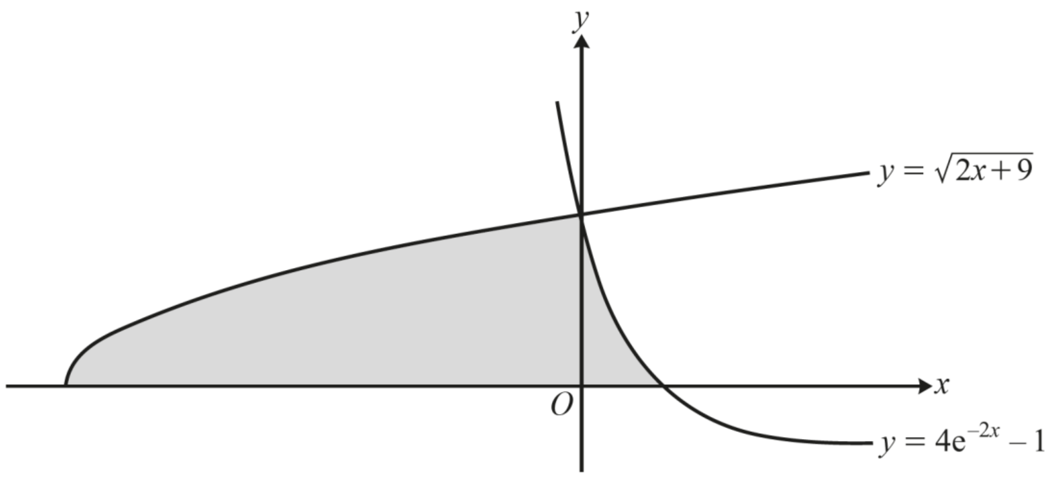
Question 5 (d)

(d) Use the Newton-Raphson method, with initial value 0.5, to determine the coordinates of P correct to **5** decimal places. **[5]**

Many candidates struggled on this question with many unsure which expression they should be applying Newton-Raphson to. Of those that did realise that $\mathrm{f}(x_n)=4x_n^{\ 3}+3x_n-3$ many did derive the *x*-coordinate of *P* but many then did not find the corresponding *y* coordinate.

Question 6

6 In this question you must show detailed reasoning.



The diagram shows the curves $y = \sqrt{2x+9}$ and $y = 4e^{-2x} - 1$ which intersect on the y-axis. The shaded region is bounded by the curves and the x-axis.

Determine the area of the shaded region, giving your answer in the form $p + q\ln 2$ where p and q are constants to be determined. **[8]**

This question was answered extremely well with many candidates scoring all 8 marks for correctly determining the required shaded area of the given region. As this was a DR question it was not acceptable to simply state that $\int_{-\frac{9}{2}}^{0}(2x+9)^{\frac{1}{2}}\,dx = 9$ or to apply the limits the wrong way round and then change a response of -9 to 9 without suitable justification. Some candidates struggled to solve the equation $4e^{-2x} - 1 = 0$ exactly or to find the required upper limit for the integral $\int 4e^{-2x} - 1\,dx$. Furthermore, some had difficulty with evaluating this indefinite integral (it was common for examiners to report that this was often integrated to $-8e^{-2x} - x$). Many candidates struggled to evaluate $\left[-2e^{-2x} - x\right]_0^{\ln 2}$ accurately even though they had access to calculators which would have enabled them to check their response.

Question 7 (a)

7 In this question you must show detailed reasoning.

(a) Show that the equation $m\sec\theta + 3\cos\theta = 4\sin\theta$ can be expressed in the form

$m\tan^2\theta - 4\tan\theta + (m+3) = 0.$ **[3]**

It was pleasing that many candidates seemed to answer this part with ease. The most successful responses started by dividing each term by $\cos\theta$ therefore obtaining $m\sec^2\theta + 3 = 4\tan\theta$. From here most candidates correctly applied the identity $1+\tan^2\theta \equiv \sec^2\theta$ and obtained the given response. The most common incorrect method seen by examiners was to take the given equation and replace it with $m^2\sec^2\theta + 9\cos^2\theta = 16\sin^2\theta$.

Question 7 (b)

(b) It is given that there is only one value of θ, for $0 < \theta < \pi$, satisfying the equation $m\sec\theta + 3\cos\theta = 4\sin\theta$.

Given also that m is a negative integer, find this value of θ, correct to **3** significant figures. **[5]**

While it was pleasing that most candidates realised that the equation derived in part (a) was a quadratic equation in $\tan\theta$ and hence they should consider the discriminant (of this equation) it was then disappointing how many candidates simplified the equation $(-4)^2 - 4m(m+3) = 0$ to $16 - 4m^2 + 12m = 0$. Of those that did obtain the correct quadratic in m and hence found the corresponding value of θ as 2.68 (correct to **3** significant figures) many did not give suitable 'detailed reasoning' and therefore very few explained why the discriminant should be set equal to zero in the first place.

Exemplar 2

7(b)

$m\sec\theta + 3\cos\theta = 4\sin\theta$

$m\tan^2\theta - 4\tan\theta + (m+3) = 0.$

$b^2 - 4ac = 0.$

$(-4)^2 - 4(m)(m+3) = 0$

$16 - 4m(m+3) = 0 \qquad m < 0$

$16 - 4m^2 - 12m = 0$

$4m^2 + 12m - 16 = 0$

$m^2 + 3m - 4 = 0$

$(m+4)(m-1) = 0$

$m = 4$ or $m = 1$

$m \neq 1$ as $m < 0$

$\therefore \cancel{m = 4}\ m = -4.$

$\cancel{4\tan^2\theta - 4\tan\theta + (4+3) = 0} \qquad -4\tan^2\theta - 4\tan\theta - 1 = 0$

$\cancel{4\tan^2\theta - 4\tan\theta + 7 = 0} \qquad -(4\tan^2\theta + 4\tan\theta + 1) = 0$

$\cancel{\frac{4 \pm \sqrt{0}}{8} = \tan\theta} \qquad -(2\tan\theta + 1) = 0$

$\cancel{\frac{1}{2} = \tan\theta} \qquad \tan\theta = -1/2$

$\theta = \cancel{-0.464}, 2.677945$

$\cancel{\theta = 2.678}$

$\theta = 2.68$

$0 < \theta < \pi$

This response scored 4 (of the 5) marks. Although the candidate has correctly found the value of θ they have not given the required 'detailed reasoning' of why they considered the discriminant of the quadratic equation in tan equal to zero and so did not score the final accuracy mark.

Section B overview

Two general points with regards to the answering of certain mechanics questions should be made in this overview. The first is that unless told otherwise, the value that candidates should use for the acceleration due to gravity, *g*, is 9.8 and not 10 or 9.81 (and this value is stated explicitly on the front cover of the examination paper). Secondly, when applying Newton's second law in the context of connected particles, centres (when teaching) and candidates (when answering examination questions) are strongly encouraged to apply F = ma to each particle separately rather than attempting to apply this equation to the whole system. These attempts generally result in either the incorrect number of forces on the left-hand side of the equation or errors with the mass/acceleration of the combined system on the right-hand side. Often these attempts score no marks (as was commonly seen in this paper in Question 10 (b)).

Question 8

8

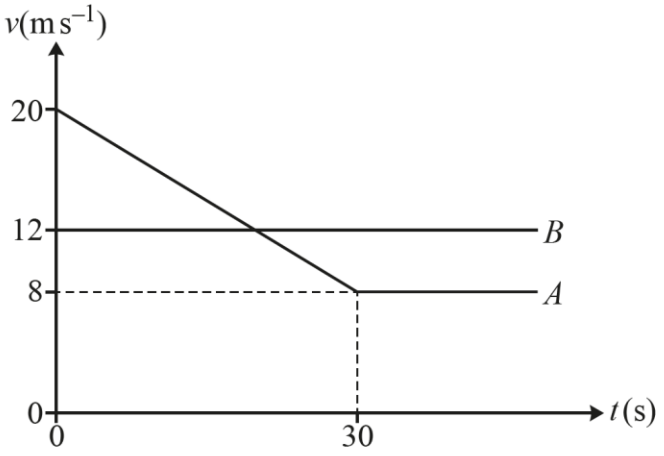
A child attempts to drag a sledge along horizontal ground by means of a rope attached to the sledge. The rope makes an angle of 15° with the horizontal (see diagram).

Given that the sledge remains at rest and that the frictional force acting on the sledge is 60 N, find the tension in the rope. **[2]**

This was answered extremely well with nearly all candidates correctly resolving horizontally and obtaining the correct value for tension in the rope from the equation $T\cos 15 = 60$. The most common errors were to either use sine rather than cosine or to use an inequality rather than equals when finding the required tension.

Question 9 (a)

9



The diagram shows a velocity-time graph representing the motion of two cars A and B which are both travelling along a horizontal straight road. At time $t = 0$, car B, which is travelling with constant speed 12 m s^{-1}, is overtaken by car A which has initial speed 20 m s^{-1}.

From $t = 0$ car A travels with constant deceleration for 30 seconds. When $t = 30$ the speed of car A is 8 m s^{-1} and the car maintains this speed in its subsequent motion.

(a) Calculate the deceleration of car A. **[2]**

This was probably the best answered question of the whole paper. Strictly speaking the deceleration of car *A* was 0.4, however, examiners accepted either this or the corresponding negative value.

Question 9 (b)

(b) Determine the value of t when B overtakes A. **[4]**

In comparison, candidates found the second part of this question much more demanding than expected. Many candidates attempted to apply an equation of constant acceleration for *A*'s entire journey even though the acceleration of *A* changed after 30 seconds. Although there were many correct ways of answering this part the most common way was to first work out the distance that each car travelled in the first 30 seconds (for *B* this was 360 m and for *A* this was 420 m) and then solved $12T + 360 = 8T + 420$ and then add the 30 to *T*, therefore giving the correct value of *t* as 45.

Question 10 (a)

10

A rectangular block B is at rest on a horizontal surface. A particle P of mass 2.5 kg is placed on the upper surface of B. The particle P is attached to one end of a light inextensible string which passes over a smooth fixed pulley. A particle Q of mass 3 kg is attached to the other end of the string and hangs freely below the pulley. The part of the string between P and the pulley is horizontal (see diagram).

The particles are released from rest with the string taut. It is given that B remains in equilibrium while P moves on the upper surface of B. The tension in the string while P moves on B is 16.8 N.

(a) Find the acceleration of Q while P and B are in contact. **[2]**

This question was answered very well with most candidates correctly applying Newton's second law vertically for particle *Q*. Occasionally, candidates attempted to apply Newton's second law to either the entire system or to *P* (but without any success).

Question 10 (b)

(b) Determine the coefficient of friction between P and B. **[3]**

Like part (a) this question was answered extremely well with most candidates scoring the method marks for applying Newton's second law horizontally for *P* and using $F = \mu R$. Occasionally, candidates incorrectly used the mass of *Q* in their equation or attempted to apply Newton's second law for the entire system, but these were almost always unsuccessful.

Question 10 (c)

(c) Given that the coefficient of friction between B and the horizontal surface is $\frac{5}{49}$, determine the least possible value for the mass of B. **[3]**

Not very many candidates made any real progress with this question even though it was a relatively straightforward application of Newton's third law. Very few candidates realised that when resolving vertically for *B* that $R_B = 2.5g + Mg$ (where *M* was the mass of *B*), and that horizontally for *B* the frictional forces acted in opposite directions but were of equal magnitude. It was very rare for examiners to see the correct response of 3.8 (with the most common incorrect response being 6.3 coming from $R_B = Mg$, so ignoring the normal contact force of *P* on *B*).

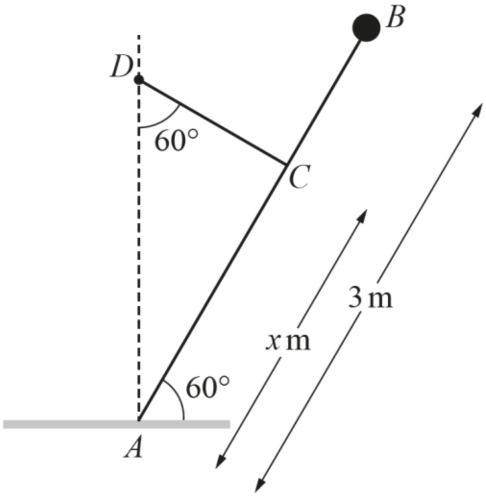
Exemplar 3

10(c)	$\mu = \frac{5}{49}$
	$R = Mg$
	$6\cdot3 = Mg \times \frac{5}{49}$
	$\frac{6\cdot3}{\frac{5}{49} \times 9\cdot8} = M$
	$M = 6\cdot3$ kg

This response scored zero. Although the candidate has used the correct value of 6.3 for the friction between *P* and *B*, they have not considered the force of *P* acting on *B* vertically. This is indicated by the candidate suggesting that the magnitude of the contact force between the horizontal surface and *B* is only equal to weight of *B* which therefore ignores the fact that *P* exists and lies on the upper surface of *B*.

Question 11 (a)

11



A uniform rod AB of mass 4 kg and length 3 m rests in a vertical plane with A on rough horizontal ground.

A particle of mass 1 kg is attached to the rod at B. The rod makes an angle of 60° with the horizontal and is held in limiting equilibrium by a light inextensible string CD. D is a fixed point vertically above A and CD makes an angle of 60° with the vertical. The distance AC is x m (see diagram).

(a) Find, in terms of g and x, the tension in the string. **[3]**

Although most candidates realised that the best way to find the tension in the string was to take moments about point *A* for the rod, many struggled with the corresponding geometry of the situation. Although many did obtain the correct equation $xT = 4g\left(\frac{3}{2}\cos 60\right) + g\left(3\cos 60\right)$ a number of candidates left their response either in terms of cos(60) or gave their response as $\dfrac{44.1}{x}$ even though the question specifically asked for a response in terms of *g* and *x*.

Assessment for learning

Candidates are strongly advised when taking moments to make it clear to the examiner which point (in this case on the rod) they are taking moments about. A number of candidates attempted to take moments about another point other than *A* together with resolve forces vertically and/or horizontally; these attempts were usually unsuccessful.

Question 11 (b)

The coefficient of friction between the rod and the ground is $\frac{9\sqrt{3}}{35}$.

(b) Determine the value of x. **[4]**

Many struggled with this part and even those candidates who realised that the most efficient way of determining the value of *x* was to resolve vertically and horizontally for the rod (and then apply $F = \mu R$) many could not cope with the mechanics of the situation or the corresponding algebraic demands.

Question 12 (a)

12 In this question the unit vectors **i** and **j** are in the directions east and north respectively.

A particle P is moving on a smooth horizontal surface under the action of a single force $\mathbf{F}$ N. At time t seconds, where $t \geqslant 0$, the velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ of P, relative to a fixed origin O, is given by

$$\mathbf{v} = (1-2t)\mathbf{i} + (2t^2 + t - 13)\mathbf{j}.$$

(a) Show that P is never stationary. **[2]**

The most common way of showing that *P* was never stationary was to solve either $1-2t=0$ or $2t^2+t-13=0$ and show that the value of *t* from one equation was not consistent with the value(s) of *t* from the other equation. Those candidates who instead decided to find an expression for the speed of *P* at time *t* (which in most cases was a 5-term quartic equation) were rarely successful in either deriving this equation correctly or articulating their findings when they attempted to solve it.

Question 12 (b)

(b) Find, in terms of **i** and **j**, the acceleration of P at time t. **[1]**

This question was almost always answered correctly by differentiating the expression given in the stem to part (a). Only occasionally did examiners report cases of candidates using either constant acceleration formulae or integration.

Question 12 (c)

The mass of P is 0.5 kg.

(c) Determine the magnitude of **F** when P is moving in the direction of the vector $-2\mathbf{i} + \mathbf{j}$. Give your answer correct to **3** significant figures. **[5]**

How to deal with '…moving in the direction of the vector $-2\mathbf{i}+\mathbf{j}$' was not well known by most candidates (and it would seem that many candidates did not realise that the direction of motion of *P* is governed by the corresponding velocity vector and not the displacement vector). For those candidates who did realise that $\begin{pmatrix}-2\\1\end{pmatrix}=k\begin{pmatrix}1-2t\\2t^2+t-13\end{pmatrix}$ for some constant *k* most usually went on to find the correct value of *t* as 2.5.

In comparison, the vector expression for **F** (and corresponding magnitude) were much better known by candidates and even if they did not obtain the correct value of *t* they could still access some marks for their knowledge of these results.

Question 12 (d)

When $t = 1$, P is at the point with position vector $\frac{1}{6}\mathbf{j}$.

(d) Determine the bearing of P from O at time $t = 1.5$. **[5]**

This question was much better answered than part (c) with many candidates correctly integrating the velocity vector to obtain the corresponding displacement vector. Some candidates did not include a vector constant of integration (and some just automatically assumed that it would be a zero vector). However, many correctly found the displacement at time 1.5 as $-\frac{3}{4}\mathbf{i}-\frac{33}{8}\mathbf{j}$ and although many used the tan ratio to work out a suitable angle only some correctly found the required bearing.

Question 13 (a)

13 A small ball B moves in the plane of a fixed horizontal axis Ox, which lies on horizontal ground, and a fixed vertically upwards axis Oy. B is projected from O with a velocity whose components along Ox and Oy are $U\,\mathrm{m\,s^{-1}}$ and $V\,\mathrm{m\,s^{-1}}$, respectively. The units of x and y are metres.

B is modelled as a particle moving freely under gravity.

(a) Show that the path of B has equation $2U^2y = 2UVx - gx^2$. **[3]**

Overall, the responses to this final question were mixed with a number of candidates leaving many of the parts completely blank. However, this first part was answered well and for those that began by applying $s = ut + 0.5at^2$ both horizontally (with *s* = *x* and *a* = 0) and vertically (with *s* = *y* and *a* = -*g*) the required show that usually followed.

Question 13 (b)

During its motion, B just clears a vertical wall of height $\frac{1}{2}a$ m at a horizontal distance a m from O. B strikes the ground at a horizontal distance $3a$ m beyond the wall.

(b) Determine the angle of projection of B. Give your answer in degrees correct to **3** significant figures. **[5]**

Many candidates did not realise the significance of the given response in part (a) of the question in answering part (b) of the question. It was also clear that most candidates did not read the question carefully. Most candidates took the point of the ball hitting the ground as at a distance of 3a from O rather than 4a. Of those that did realise the significance of the equation given in part (a) and that the ball passed through the points $(a, 0.5a)$ and $(4a, 0)$ only the most able could find suitable expressions for *U* and/or *V* and use $\tan\theta = \frac{V}{U}$ to find the correct angle of projection as 33.7 degrees.

Question 13 (c)

(c) Given that the speed of projection of B is $54.6\,\mathrm{m\,s^{-1}}$, determine the value of a. **[2]**

Only some candidates attempted (and correctly answered) this question with the most common method being to use the result that $a = \frac{UV}{2g}$ even though the result that $U^2 + V^2 = 54.6^2$ was slightly more efficient.

Question 13 (d)

(d) Hence find the maximum height of B above the ground during its motion. **[3]**

Of those that attempted this question the most common method was to use $v^2 = u^2 + 2as$ with $v = 0$ and $a = -g$. From here candidates were evenly split between those that used an expression for V^2 from (b) (which if correct was $V^2 = \frac{4ga}{3}$) or those that used $V = 54.6\sin\theta$ with θ being the angle found in (b).

Question 13 (e)

(e) State **one** refinement of the model, other than including air resistance, that would make it more realistic. **[1]**

Many candidates who had left the previous parts of the question unanswered did attempt this final part. Although there were many suitable correct responses seen by examiners (for example, using a more accurate value of *g*, considering the dimensions or spin of *B*, etc.) the most common incorrect responses were those that referred to the mass or weight or shape of *B*.

Assessment for learning

When candidates are asked to give **one** written response (in this case a refinement to a model) they should do just that. If multiple responses are given, then all must be valid to score the single mark available.

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