

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Sample Question Paper

Date – Morning/Afternoon

Version 2.1

Time allowed: 2 hours

You must have:

- Printed Answer Booklet

You may use:

- a scientific or graphical calculator



* o o o o o o *

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{1}{2}\theta^2, \tan \theta \approx \theta \text{ where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2} h \{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B) \quad \text{or} \quad P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Section A: Pure MathematicsAnswer **all** the questions

- 1 (a) If $|x| = 3$, find the possible values of $|2x - 1|$. [3]
- (b) Find the set of values of x for which $|2x - 1| > x + 1$.
Give your answer in set notation. [4]
- 2 (a) Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for $\int_0^1 \frac{1}{\sqrt{1+x^2}} dx$. [3]
- (b) Explain how the trapezium rule might be used to give a better approximation to the integral given in part (a). [1]
- 3 **In this question you must show detailed reasoning.**
- Given that $5 \sin 2x = 3 \cos x$, where $0^\circ < x < 90^\circ$, find the exact value of $\sin x$. [4]
- 4 For a small angle θ , where θ is in radians, show that $1 + \cos \theta - 3 \cos^2 \theta \approx -1 + \frac{5}{2} \theta^2$. [4]

5 (a) Find the first three terms in the expansion of $(1+px)^{\frac{1}{3}}$ in ascending powers of x . [3]

(b) The expansion of $(1+qx)(1+px)^{\frac{1}{3}}$ is $1+x-\frac{2}{9}x^2+\dots$.

Find the possible values of the constants p and q . [5]

6 A curve has equation $y = x^2 + kx - 4x^{-1}$ where k is a constant.

Given that the curve has a minimum point when $x = -2$

- find the value of k
- show that the curve has a point of inflection which is not a stationary point. [7]

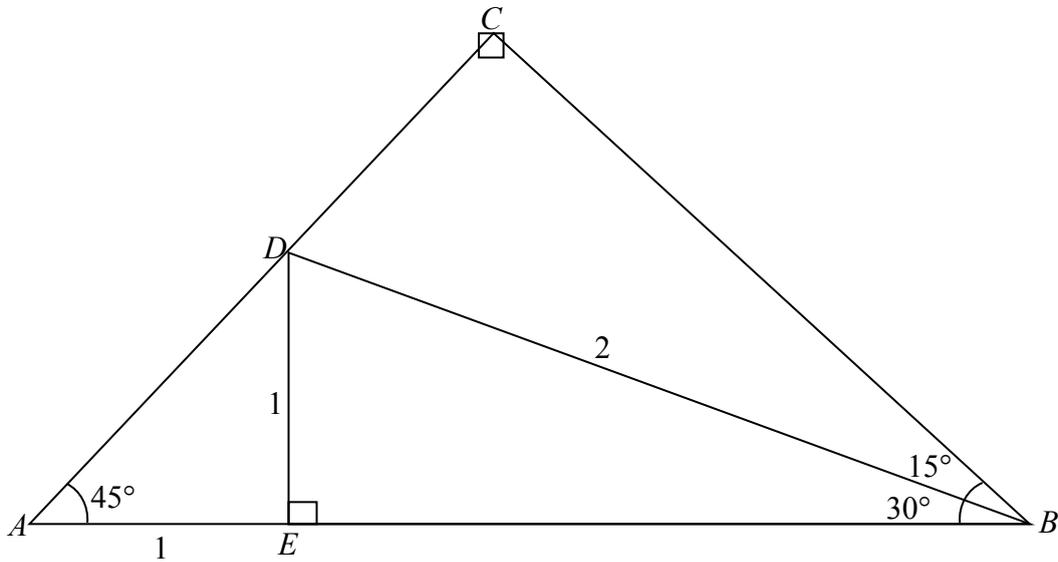
7 (a) Find $\int 5x^3 \sqrt{x^2+1} \, dx$. [5]

(b) Find $\int \theta \tan^2 \theta \, d\theta$.

You may use the result $\int \tan \theta \, d\theta = \ln|\sec \theta| + c$. [5]

8 In this question you must show detailed reasoning.

The diagram shows triangle ABC .



The angles CAB and ABC are each 45° , and angle $ACB = 90^\circ$.

The points D and E lie on AC and AB respectively. $AE = DE = 1$, $DB = 2$.

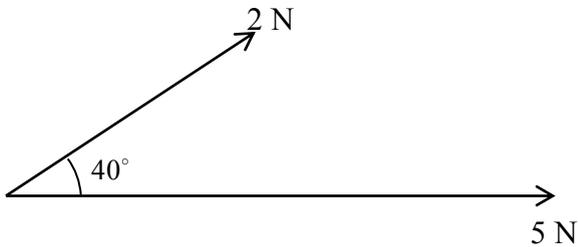
Angle $BED = 90^\circ$, angle $EBD = 30^\circ$ and angle $DBC = 15^\circ$.

(a) Show that $BC = \frac{\sqrt{2} + \sqrt{6}}{2}$. [3]

(b) By considering triangle BCD , show that $\sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$. [3]

Section B: Mechanics
Answer **all** the questions

- 9 Two forces, of magnitudes 2 N and 5 N, act on a particle in the directions shown in the diagram below.



- (a) Calculate the magnitude of the resultant force on the particle. [3]
- (b) Calculate the angle between this resultant force and the force of magnitude 5 N. [1]
- 10 A body of mass 20 kg is on a rough plane inclined at angle α to the horizontal. The body is held at rest on the plane by the action of a force of magnitude P N. The force is acting up the plane in a direction parallel to a line of greatest slope of the plane. The coefficient of friction between the body and the plane is μ .
- (a) When $P = 100$, the body is on the point of sliding down the plane.
- Show that $g \sin \alpha = g\mu \cos \alpha + 5$. [4]
- (b) When P is increased to 150, the body is on the point of sliding up the plane.
- Use this, and your answer to part (a), to find an expression for α in terms of g . [3]

11 In this question the unit vectors \mathbf{i} and \mathbf{j} are in the directions east and north respectively.

A particle of mass 0.12 kg is moving so that its position vector \mathbf{r} metres at time t seconds is given by

$$\mathbf{r} = 2t^3\mathbf{i} + (5t^2 - 4t)\mathbf{j}.$$

- (a) Show that when $t = 0.7$ the bearing on which the particle is moving is approximately 044° . **[3]**
- (b) Find the magnitude of the resultant force acting on the particle at the instant when $t = 0.7$. **[4]**
- (c) Determine the times at which the particle is moving on a bearing of 045° . **[2]**

12 A girl is practising netball.

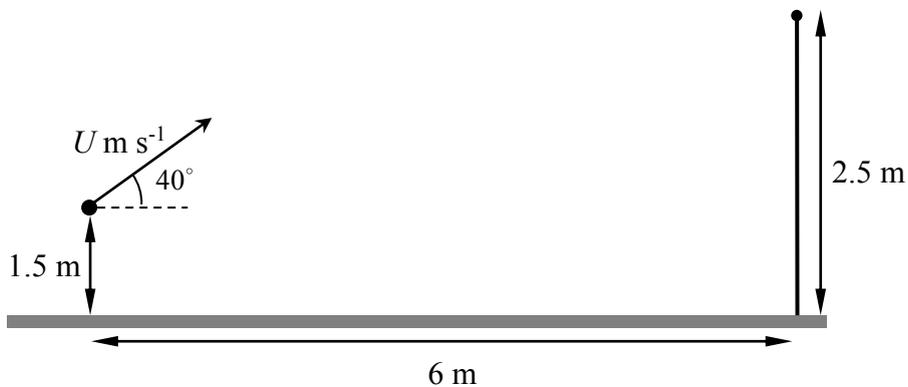
She throws the ball from a height of 1.5 m above horizontal ground and aims to get the ball through a hoop.

The hoop is 2.5 m vertically above the ground and is 6 m horizontally from the point of projection.

The situation is modelled as follows.

- The initial velocity of the ball has magnitude $U \text{ m s}^{-1}$.
- The angle of projection is 40° .
- The ball is modelled as a particle.
- The hoop is modelled as a point.

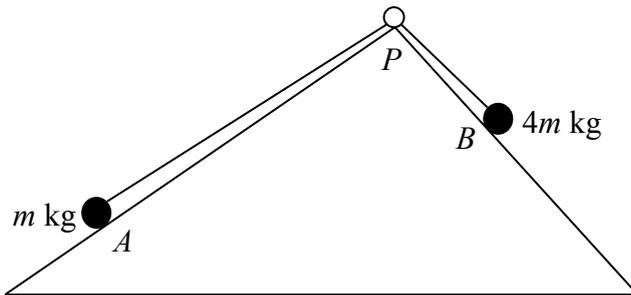
This is shown on the diagram below.



- (a) For $U = 10$, find
- the greatest height above the ground reached by the ball [5]
 - the distance between the ball and the hoop when the ball is vertically above the hoop. [4]
- (b) Calculate the value of U which allows her to hit the hoop. [3]
- (c) How appropriate is this model for predicting the path of the ball when it is thrown by the girl? [1]
- (d) Suggest one improvement that might be made to this model. [1]

- 13** Particle A , of mass m kg, lies on the plane Π_1 inclined at an angle of $\tan^{-1} \frac{3}{4}$ to the horizontal. Particle B , of $4m$ kg, lies on the plane Π_2 inclined at an angle of $\tan^{-1} \frac{4}{3}$ to the horizontal. The particles are attached to the ends of a light inextensible string which passes over a smooth pulley at P . The coefficient of friction between particle A and Π_1 is $\frac{1}{3}$ and plane Π_2 is smooth. Particle A is initially held at rest such that the string is taut and lies in a line of greatest slope of each plane.

This is shown on the diagram below.



- (a) Show that when A is released it accelerates towards the pulley at $\frac{7g}{15} \text{ m s}^{-2}$. [6]
- (b) Assuming that A does not reach the pulley, show that it has moved a distance of $\frac{1}{4}$ m when its speed is $\sqrt{\frac{7g}{30}} \text{ m s}^{-1}$. [2]
- 14** A uniform ladder AB of mass 35 kg and length 7 m rests with its end A on rough horizontal ground and its end B against a rough vertical wall. The ladder is inclined at an angle of 45° to the horizontal. A man of mass 70 kg is standing on the ladder at a point C , which is x metres from A . The coefficient of friction between the ladder and the wall is $\frac{1}{3}$ and the coefficient of friction between the ladder and the ground is $\frac{1}{2}$. The system is in limiting equilibrium.
- Find x . [8]

END OF QUESTION PAPER

BLANK PAGE

Copyright Information:

OCR is committed to seeking permission to reproduce all third-party content that it uses in the assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.