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A LEVEL

Examiners' report

MATHEMATICS B (MEI)

H640
For first teaching in 2017

H640/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper H640/01 series overview

This was the first paper for this new A Level and all the candidates had prepared for this examination in one year. The marks were generally very good as many candidates are also Further Mathematics candidates. This paper contributes 36.4% of the total A Level and assesses content from Pure Mathematics and Mechanics.

Candidates showed a good understanding on Mechanics as well as Pure Mathematics. Candidates were well able to cope with extended answers (for example question 10). Questions requiring proof or an explanation were less well answered (for example question 12(iii) 13(v) and 14(ii)).

To do well in this component, candidates need to be able to apply their knowledge of the syllabus content in a variety of modelling contexts and to make efficient use of calculator technology.

Section A overview

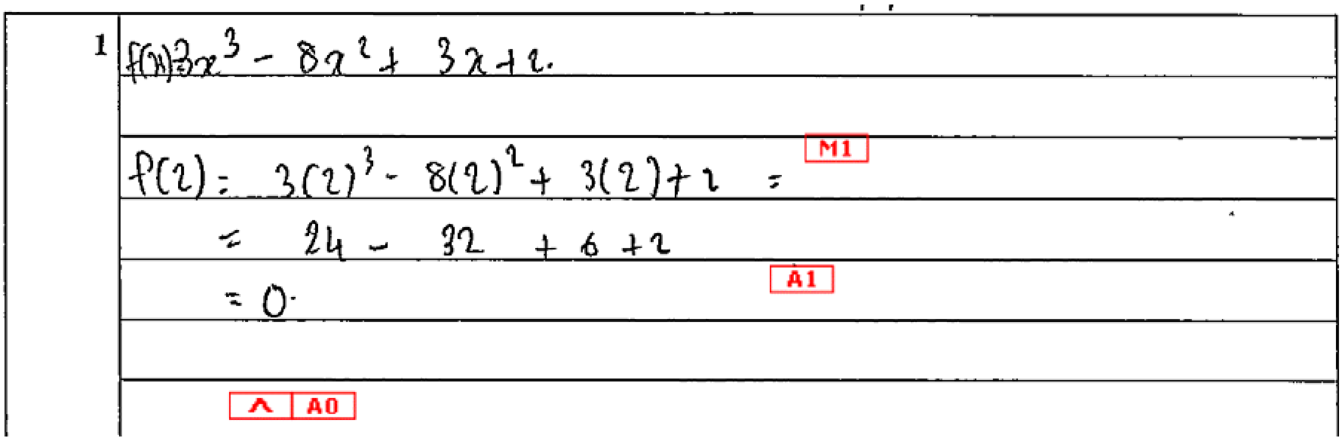
Section A questions are designed to give all candidates an opportunity to do some of the questions on the paper as they require little reading or interpretation. Most candidates did very well in Section A.

Question 1

1 Show that $(x-2)$ is a factor of $3x^3-8x^2+3x+2$. **[3]**

There were many good answers and candidates could choose whether to use the factor theorem or to divide. Some candidates lost the mark as they presented the evidence but did not write that $f(2)=0$ implied that $(x-2)$ is a factor, or that no remainder implied that $(x-2)$ is a factor.

Exemplar 1


1 $f(x) 3x^3 - 8x^2 + 3x + 2.$

$f(2) = 3(2)^3 - 8(2)^2 + 3(2) + 2 =$ M1

$= 24 - 32 + 6 + 2$

$= 0.$ A1

^ A0

This exemplar shows why some candidates were only credited 2 of 3 marks.

Question 2

2 By considering a change of sign, show that the equation $e^x - 5x^3 = 0$ has a root between 0 and 1. **[2]**

This question was generally well answered with only a few candidates making an arithmetical error that cost a mark.

Question 3

3 **In this question you must show detailed reasoning.**

Solve the equation $\sec^2\theta + 2\tan\theta = 4$ for $0° \leqslant \theta < 360°$. **[4]**

Candidates who used the identity $\sec^2\theta = 1+\tan^2\theta$ generally went on to obtain most of the marks. Only a few candidates tried to rewrite the equation in terms of $\cos\theta$ as this is a much more difficult method requiring both sides to be squared and spurious solutions eliminated. Candidates did not get far enough into this method to obtain the method mark.

Question 4 (i)

4 Rory pushes a box of mass 2.8 kg across a rough horizontal floor against a resistance of 19 N. Rory applies a constant horizontal force. The box accelerates from rest to $1.2\,\mathrm{m\,s^{-1}}$ as it travels 1.8 m.

(i) Calculate the acceleration of the box. **[2]**

Most candidates successfully used the *suvat* equations to reach the correct answer.

Question 4 (ii)

(ii) Find the magnitude of the force that Rory applies. **[2]**

Newton's second law was well understood and most candidates were successful here.

Question 5 (i)

5 The position vector **r** metres of a particle at time t seconds is given by

$$\mathbf{r} = (1 + 12t - 2t^2)\mathbf{i} + (t^2 - 6t)\mathbf{j}.$$

(i) Find an expression for the velocity of the particle at time t. **[2]**

Most candidates used vector notation accurately and successfully differentiated to obtain a correct expression for velocity.

Question 5 (ii)

(ii) Determine whether the particle is ever stationary. **[2]**

This was typically well answered with most candidates realising the requirement for both components to be zero at the same time.

Misconception It is not sufficient to equate the components and solve to find t = 3. From this starting point, candidates would need to check that the components were zero to achieve the method mark.

Question 6 (i)

6 Aleela and Baraka are saving to buy a car. Aleela saves £50 in the first month. She increases the amount she saves by £20 each month.

(i) Calculate how much she saves in two years. **[2]**

Some candidates used a brute force method, writing out the complete list of monthly payments and adding them. Most successfully identified this as an arithmetic series and used the correct formula to find the sum of 24 terms.

Partial credit was not awarded where a candidate found the 24th term but did not then attempt to find the sum total.

Question 6 (ii)

Baraka also saves £50 in the first month. The amount he saves each month is 12% more than the amount he saved in the previous month.

(ii) Explain why the amounts Baraka saves each month form a geometric sequence. **[1]**

Many candidates realised that the 12% increase could be achieved by multiplying by 1.12 which leads to a geometric series. The value 1.12 had to be seen in part (ii) for the mark to be credited.

AfL

Even if 1.12 was used in part (iii) this mark could not be credited if 1.12 was not seen in part (ii).

Exemplar 2

6(ii)	Because the amount saved is going up in a common ratio, ~~and the 150~~ it is not the same difference each time. ^ B0

Some candidates were not credited the mark as their answer was too vague, as in the exemplar above.

Question 6 (iii)

(iii) Determine whether Baraka saves more in two years than Aleela. **[3]**

Again the formula for the total of terms was needed to earn the method mark. The final mark was credited to candidates who compared their totals and follow-through was available to those who had made an arithmetic mistake but not to those who had not earned the method marks in both part (i) and part (iii).

Section B overview

Section B contains longer questions and more problem solving than Section A. The questions are graded in difficulty and this was reflected in the marks credited, although some candidates who were well prepared in Mechanics found question 14 straightforward.

Question 7 (i)

7 A rod of length 2 m hangs vertically in equilibrium. Parallel horizontal forces of 30 N and 50 N are applied to the top and bottom and the rod is held in place by a horizontal force F N applied x m below the top of the rod as shown in Fig. 7.

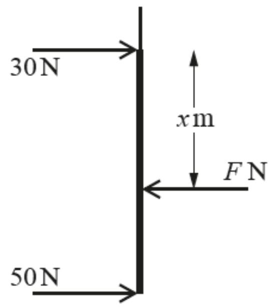


Fig. 7

(i) Find the value of F. **[1]**

This was a routine mark for which almost all candidates were credited, the only mistake seen was to find the difference between 50 N and 30 N,

Question 7 (ii)

(ii) Find the value of x. **[2]**

Most candidates successfully took moments about the top of the rod to easily obtain the correct answer. Candidates that attempted to take moments about the centre of the rod often encountered problems with their positioning of F with respect to the centre.

Question 8 (i)

8 **(i)** Show that $8\sin^2 x\cos^2 x$ can be written as $1-\cos 4x$. **[3]**

This question posed considerable difficulty to many candidates. Some had success by writing both $\sin^2 x$ and $\cos^2 x$ in terms of $\cos 2x$ and multiplying out. Many then did not go on to complete the proof. The first 2 marks were credited for any two applications of the double angle formulae and the final mark only when there was a convincing proof so many candidates were credited 2 out of 3 marks.

Question 8 (ii)

(ii) Hence find $\int \sin^2 x\cos^2 x\,dx$. **[3]**

Most candidates realised they needed to use the identity from part (i) although many omitted the factor of $\frac{1}{8}$. Some candidates lost the final mark for omitting the $+c$.

Question 9 (i)

9 A pebble is thrown horizontally at $14\,\text{m s}^{-1}$ from a window which is $5\,\text{m}$ above horizontal ground. The pebble goes over a fence $2\,\text{m}$ high $d\,\text{m}$ away from the window as shown in Fig. 9. The origin is on the ground directly below the window with the x-axis horizontal in the direction in which the pebble is thrown and the y-axis vertically upwards.

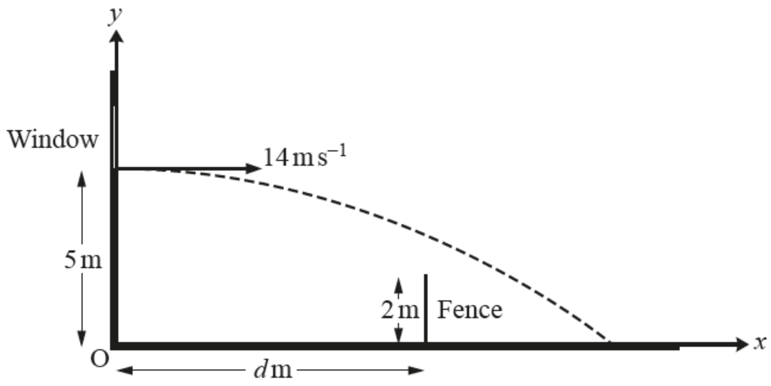


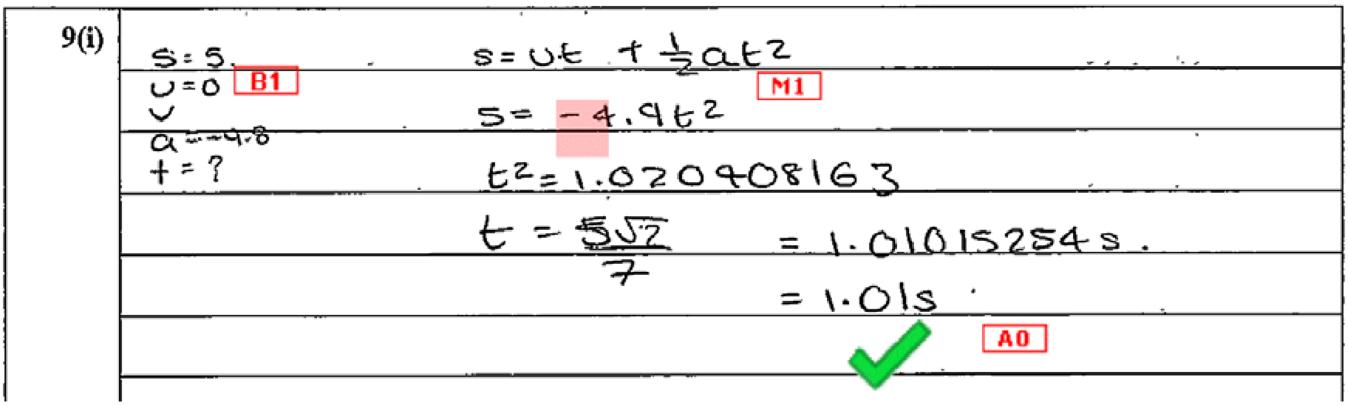
Fig. 9

(i) Find the time the pebble takes to reach the ground. **[3]**

Most candidates realised that the initial velocity in the vertical direction was zero and successfully completed this question.

AfL It is important to make use of a consistent sign convention, with acceleration and displacement either both positive or both negative. A clear statement of the positive direction at the start of the answer helps avoid problems.

Exemplar 3



9(i)

$s: 5$ B1

$u = 0$

v

$a = -9.8$

$t = ?$

$s = ut + \frac{1}{2}at^2$ M1

$s = -4.9t^2$

$t^2 = 1.020408163$

$t = \frac{5\sqrt{2}}{7} = 1.01015254\text{ s}.$

$= 1.01\text{s}$

A0

This candidate was credited B1 using u = 0 and M1 for the equation with a sign error. Although the correct answer is seen, it comes from incorrect working and was therefore not credited the final mark. This candidate obviously recognised that there was an issue with the working and should have gone back to identify and correct their mistake ($s = -5$ or $a = +9.8$).

Question 9 (ii)

(ii) Find the cartesian equation of the trajectory of the pebble. **[4]**

Many correct answers were seen. The most common error was to omit the initial height of the pebble. The origin is given in the question, so the correct equation is $y = 5 - 4.9t^2$.

AfL Take careful note of the origin and remember to include the initial position in the equations.

Question 9 (iii)

(iii) Find the range of possible values for d. **[3]**

The question was designed so that the simplest way to answer this was to substitute $y = 2$ in the equation of the trajectory leading to $x = 10.95$. Common sense was enough to use this as a boundary value for the inequality – the pebble would go over the wall if it were nearer the window than that value.

Many candidates went back to the original model, found the time to drop to the height of the wall, used that to work out the boundary value for d, and received full credit.

Question 10

10 Fig. 10 shows the graph of $y = (k - x)\ln x$ where k is a constant ($k > 1$).

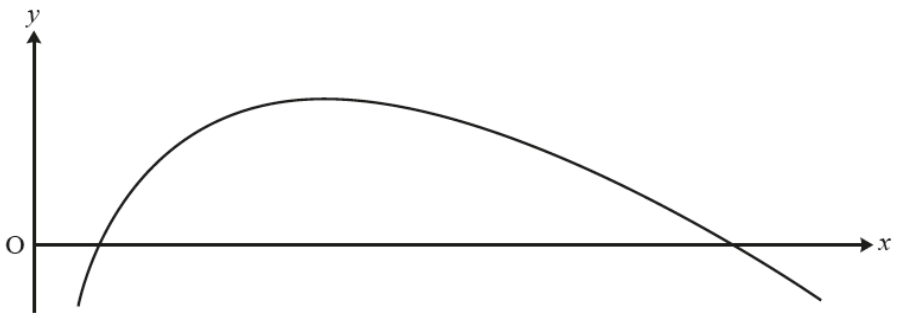


Fig. 10

Find, in terms of k, the area of the finite region between the curve and the x-axis. **[8]**

This is an exemplar of a question requiring an extended answer – there are 4 method marks in the scheme. Candidates had to structure their answer. Had a value for *k* been given, the definite integral would have been possible on many calculators and the question may have become a "detailed reasoning" question.

Successful candidates generally used integration by parts with $\frac{dv}{dx} = k - x$. It needed much more work to expand the bracket and split the integral in two. Some candidates lost the final mark, as they did not tidy up their answer so had too many terms.

AfL

In an unstructured question like this one, do not give up or leave it blank because you do not know how to calculate the limits. Find the indefinite integral to make sure of 4 out of 8 marks. Using incorrect limits could also have been credited a method mark.

Question 11 (i)

11 Fig. 11 shows two blocks at rest, connected by a light inextensible string which passes over a smooth pulley. Block A of mass 4.7 kg rests on a smooth plane inclined at 60° to the horizontal. Block B of mass 4 kg rests on a rough plane inclined at 25° to the horizontal. On either side of the pulley, the string is parallel to a line of greatest slope of the plane. Block B is on the point of sliding up the plane.

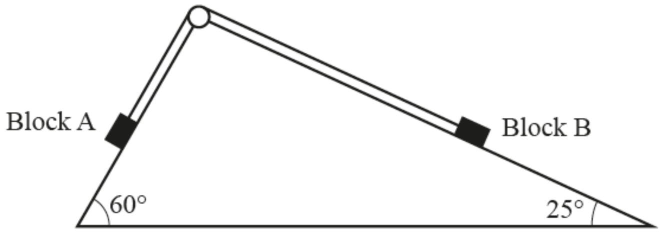


Fig. 11

(i) Show that the tension in the string is 39.9 N correct to 3 significant figures. **[2]**

This was generally well answered but some candidates lost a mark as all they had written was a component of weight and it was not clear that they had equated that to the tension. Since the answer was given in the question, the response needed to be a full mathematical justification to show the given answer.

AfL Make sure that you set up an equilibrium equation even if there are only two terms.

Question 11 (ii)

(ii) Find the coefficient of friction between the rough plane and Block B. **[5]**

The problem solving element of this question stems from the lack of help that candidates were given in structuring their answer. They had to realise that they had to calculate the normal reaction and the frictional force before they could calculate the coefficient of friction. Some answers were very fragmented with very little help given by candidates to the examiner who were not always able to tell whether finding the normal reaction and the frictional forces had been attempted.

Question 12 (i)

12 Fig. 12 shows the circle $(x-1)^2+(y+1)^2=25$, the line $4y=3x-32$ and the tangent to the circle at the point A (5, 2). D is the point of intersection of the line $4y=3x-32$ and the tangent at A.

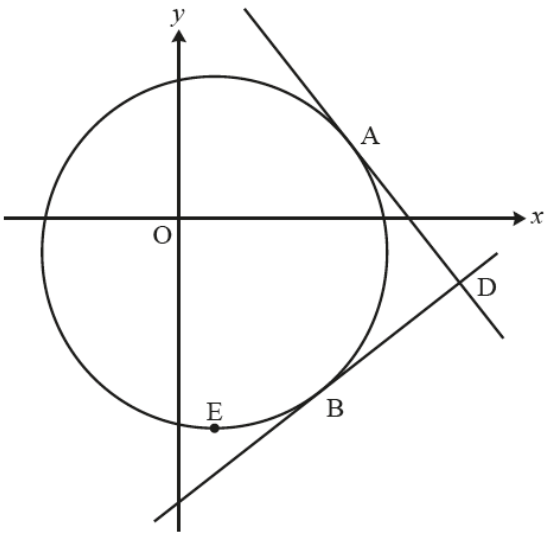


Fig. 12

(i) Write down the coordinates of C, the centre of the circle. **[1]**

Most candidates were correct here.

Question 12 (ii) (A)

(ii) (*A*) Show that the line $4y=3x-32$ is a tangent to the circle. **[4]**

There was quite a lot of confusion in this question between the tangent to the circle at A (whose equation is not given nor required) and the line $4y=3x-32$ which can be proved to be the tangent to the circle at B. Some candidates set out to find the equation of the tangent at A hoping to obtain the given equation.

Successful candidates solved the equation of the line and the circle simultaneously although many struggled with the fractions that resulted.

Question 12 (ii) (B)

(*B*) Find the coordinates of B, the point where the line $4y=3x-32$ touches the circle. **[1]**

There was only one mark here as the *x*-coordinate had normally been found in part A, so there was only the *y*-coordinate to find.

Question 12 (iii)

(iii) Prove that ADBC is a square. **[3]**

This proved quite challenging for many candidates. The first 2 marks were given for two facts that form part of a complete proof, and the third only when a complete proof was seen. Some candidates assumed that AD and BD were perpendicular in order to find the equation of AD and subsequently used their equation to complete their answer – but this is not a proper proof. Candidates needed to show they had used the gradient of AC or the derivative of the circle to find the gradient of the tangent at A.

Question 12 (iv)

(iv) The point E is the lowest point on the circle. Find the area of the sector ECB. **[5]**

The problem solving aspect of this question involved the need to find the angle ECB before the formula for the area of the sector could be used. Many candidates managed to write down the coordinates of E for the first mark. There were several valid methods of finding the required angle but it was often difficult to follow candidates thinking unless the correct answer was obtained. The second method mark was dependent on the first so that it was not enough just to guess an angle and use that.

AfL Write a few words of explanation to the examiner, or clear headings, indicating your method so that you can be given a method mark even if your answer is not quite right. Two method marks and the follow-through answer mark may depend on it.

Question 13 (i)

13 The function $f(x)$ is defined by $f(x) = \sqrt[3]{27 - 8x^3}$. Jenny uses her scientific calculator to create a table of values for $f(x)$ and $f'(x)$.

x	$f(x)$	$f'(x)$
0	3	0
0.25	2.9954	–0.056
0.5	2.9625	–0.228
0.75	2.8694	–0.547
1	2.6684	–1.124
1.25	2.2490	–1.977
1.5	0	ERROR

(i) Use calculus to find an expression for $f'(x)$ and hence explain why the calculator gives an error for $f'(1.5)$. **[3]**

This was well answered as most candidates used the chain rule successfully and realised that substituting x = 1.5 gives a zero in the denominator

Question 13 (ii)

(ii) Find the first three terms of the binomial expansion of $\mathrm{f}(x)$. **[3]**

Many candidates dealt successfully with the 27, but when that was done without clear working shown, could cost 2 marks here. Some candidates simplified their coefficients early and incorrectly, so it was not always clear that the binomial expansion had been used, costing the method mark.

AfL Make your method clear by writing down the factors of each term before simplifying.

Question 13 (iii)

(iii) Jenny integrates the first three terms of the binomial expansion of $\mathrm{f}(x)$ to estimate the value of $\int_0^1 \sqrt[3]{27-8x^3}\,\mathrm{d}x$. Explain why Jenny's method is valid in this case. (You do not need to evaluate Jenny's approximation.) **[2]**

One of the assessment objectives in the specification is to test the ability of a candidate to assess the validity of an argument as in this question. Not many candidates realised that the key to this explanation was to find the range of values for which the binomial expansion is valid. The limits lie well within the valid range so the method is valid.

Question 13 (iv)

(iv) Use the trapezium rule with 4 strips to obtain an estimate for $\int_0^1 \sqrt[3]{27-8x^3}\,\mathrm{d}x$. **[3]**

This was generally done very well.

Question 13 (v)

The calculator gives 2.921 174 38 for $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. The graph of $y = f(x)$ is shown in Fig. 13.

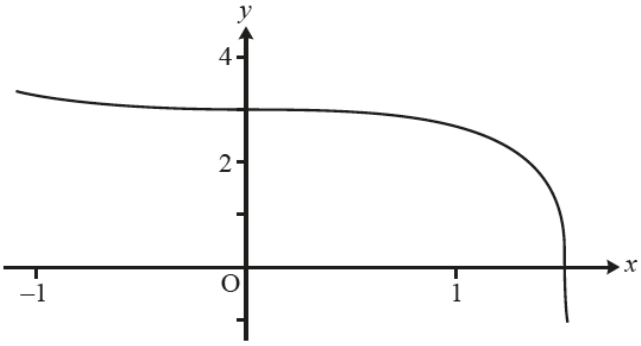


Fig. 13

(v) Explain why the trapezium rule gives an underestimate. **[1]**

Most candidates were able to explain this clearly. Some had learned that a curve being concave downwards would give an underestimate but gave no indication as to why that would be, so lost the mark.

AfL Clear annotated sketches can support a mathematical explanation better than an extended written response.

Question 14 (i)

14 The velocity of a car, $v\,\mathrm{m\,s^{-1}}$ at time t seconds, is being modelled. Initially the car has velocity $5\,\mathrm{m\,s^{-1}}$ and it accelerates to $11.4\,\mathrm{m\,s^{-1}}$ in 4 seconds.

In model A, the acceleration is assumed to be uniform.

(i) Find an expression for the velocity of the car at time t using this model. **[3]**

The key to this question was to calculate the acceleration of the car. The required expression is then found by substituting the values for *u* and *a* into the equation $v = u + at$. Many fully correct answers were seen.

Question 14 (ii)

(ii) Explain why this model is not appropriate in the long term. **[1]**

Candidates were required to recognise the limitations of this model. Most successful answers stated that the velocity would eventually get much too big or that the car would have to slow down or stop at some point.

Question 14 (iii)

Model A is refined so that the velocity remains constant once the car reaches $17.8\,\text{m}\,\text{s}^{-1}$.

(iii) Sketch a velocity-time graph for the motion of the car, making clear the time at which the acceleration changes. **[3]**

Most candidates correctly had a graph consisting of two line segments but a common error was to begin the graph at the origin when the initial velocity was $5\,\text{ms}^{-1}$. Some did not fully label their graph so lost a mark.

AfL Make sure the axes are labelled and that all key points are clearly indicated on the graph.

Question 14 (iv)

(iv) Calculate the displacement of the car in the first 20 seconds according to this refined model. **[3]**

Many candidates were successful in finding the area under their graph with only a few arithmetical errors.

Question 14 (v)

In model B, the velocity of the car is given by

$$v = \begin{cases} 5 + 0.6t^2 - 0.05t^3 & \text{for } 0 \leqslant t \leqslant 8, \\ 17.8 & \text{for } 8 < t \leqslant 20. \end{cases}$$

(v) Show that this model gives an appropriate value for v when $t = 4$. **[1]**

This mark was credited for seeing the substitution of $t = 4$ into the equation. It would have been good to see this followed by a comment that the value was close to the given value.

Question 14 (vi)

(vi) Explain why the value of the acceleration immediately before the velocity becomes constant is likely to mean that model B is a better model than model A. **[3]**

Some candidates were very vague in their answer here and were not credited many marks. There is a very clear instruction that it is the value of acceleration that is needed, so 2 of the 3 marks were given for finding this. Many candidates were able to comment that model B gives a gradual change in acceleration as it approaches the maximum speed avoiding the very sudden change seen in model A.

Question 14 (vii)

(vii) Show that model B gives the same value as model A for the displacement at time 20 s. **[3]**

Many candidates realised that the distance was the definite integral that gave the distance travelled in the first 8s. It would have been sufficient to clearly write the integral with its limits and use a calculator to evaluate it. Most candidates decided to give a full solution with the substitution of limits made clear. Only a few candidates omitted the part of the journey beyond 8s.

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