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A LEVEL

Examiners' report

MATHEMATICS B (MEI)

H640

For first teaching in 2017

H640/03 Summer 2022 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

Paper 3 series overview

This was the first 'live' June series of this paper since 2019. Despite candidates' learning being disrupted, examiners were generally impressed by the competence of the maths shown by the majority of candidates.

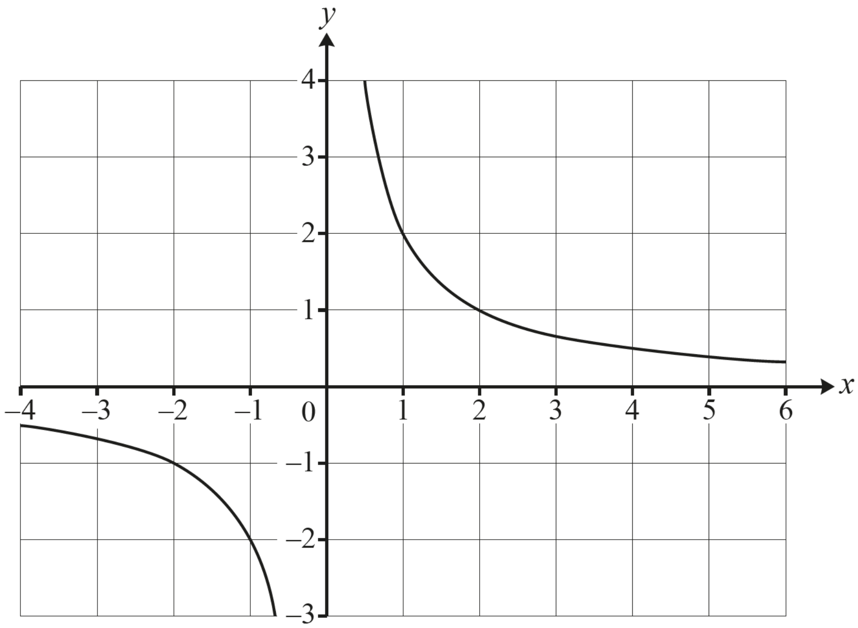
Candidates who did well on this paper generally did the following:	Candidates who did less well on this paper generally did the following:
• showed working clearly and methodically • took notice when '**detailed reasoning**' was required in a question • moved fluently between different forms of mathematical presentation, e.g. coordinate geometry, algebra, calculus, sketch graphs, etc.	• did not understand the meaning of 'prove' and regressed to giving many numerical examples • struggled with longer questions that required them to show multi-step chains of reasoning • demonstrated limited comprehension, with some questions receiving 'no response'.

Section A overview

Section A was the pure maths section. The questions ranged from straightforward low mark questions like Question 1 that most candidates scored well on to longer questions like Question 6 (b) and 8 (d) that required secure subject knowledge and the ability to maintain accuracy over an extended solution.

Question 1

1 A curve for which y is inversely proportional to x is shown below.



Find the equation of the curve. **[2]**

Most answered this correctly as y=2/*x*. A few answered y = 1/0.5*x* and although, on this occasion, this was not penalised, the specification does state that algebraic and numerical expressions should be simplified.

A few correctly said it is stretch scale factor 2 but gave the answer y = 1/2x.

The most common incomplete answer was y = 1/*x*.

A few tried, incorrectly, to use the equation of a straight line.

Question 2 (a), (b) (i) and (b) (ii)

2 The function $f(x) = \sqrt{x}$ is defined on the domain $x \geqslant 0$.

The function $g(x) = 25 - x^2$ is defined on the domain $\mathbb{R}$.

(a) Write down an expression for $fg(x)$. **[1]**

(b) **(i)** Find the domain of $fg(x)$. **[3]**

(ii) Find the range of $fg(x)$. **[2]**

Part (a) was generally well attempted. A significant number of candidates went on to give an incorrect final answer of $5 - x$.

In part (b) (i) of the question, many correctly stated that $x \leq 5$ but did not consider that $x \geq -5$

In part (b) (ii) of the question, some candidates correctly found one side of the inequality, but not the other. Some also got confused with notation, not using fg(x) correctly when stating the range.

A few did not know the difference between domain and range and so always gave both or gave their answers as $\mathbb{R}$

Question 3 (a) and (b)

3 An infinite sequence $a_1, a_2, a_3, \ldots$ is defined by $a_n = \frac{n}{n+1}$, for all positive integers n.

(a) Find the limit of the sequence. **[1]**

(b) Prove that this is an increasing sequence. **[3]**

In part (a) examiners were looking for an unambiguous answer of 1 for the limit here. However many gave answers such as 'tends to 1', 'approaches 1', 'cannot get to 1' or '$\rightarrow$ 1' describing the behaviour of the sequence and not the limit.

There were also a few answers of -1.

3 (b) was probably the least successfully answered question on the paper. The most common attempts listed some numerical examples substituting consecutive values of *n* and often then commenting that the values were increasing. This approach showed no understanding of the concept of proof and therefore did not score. A few candidates scored a mark by writing expressions for the (*n*+1)th or (*n*-1)th term and the best candidates tended to find the difference [(*n*+1)th term – *n*th term] (or other similar approach) and then went on to explain why their answer was > 0.

Some candidates earned a special case (SC) mark for differentiating their term (wrt *n*) and explaining that as it was positive it showed the sequence was increasing. Most who attempted this method did not appreciate that the sequence applied to the discrete Natural Numbers ($\mathbb{N}^+$) rather than the continuous reals ($\mathbb{R}$). The very few who concluded that their argument was valid as $\mathbb{N}^+ \subset \mathbb{R}$ received a second mark.

It is worth noting that proof was not explicitly mentioned in the advance information.

Question 4

4 In this question you must show detailed reasoning.

Determine the exact solutions of the equation $2\cos^2 x = 3\sin x$ for $0 \leqslant x \leqslant 2\pi$. **[5]**

This was well done by most candidates. They took notice of the 'Detailed Reasoning' (**DR**) instruction and most gave a complete solution. Occasionally the incorrect $\cos^2 x = \sin^2 x - 1$ was used. Nearly all candidates gave answers in radians and scored 5/5. Some candidates did not explicitly show $2cos^2x$ as $2(1 - sin^2x)$.

Given that candidates should know the primary value of arccos (½) from GCSE this is a good example of a question that A Level candidates should be able to complete without their calculator and only using their calc to *check* their solution(s).

Detailed Reasoning

This was the first question in the paper requiring candidates to show Detailed Reasoning (**DR).**

Candidates knew that they needed to show their working but they also need to realise that they should 'do some maths' rather than rely on their calculators. Therefore if solving a quadratic they should use one of the methods they probably first used at GCSE such as factorising, the quadratic formula or completing the square. When needing to find gradients they should find $\frac{dy}{dx}$ and show a substitution, for areas they should show an algebraic integration and then show their substitutions. Abbreviating their working to just the final subtraction would not earn the marks.

A sensible use of their calculator would then be to use it to **check** solutions to equations or gradients or areas using their calculus features.

A sensible piece of advice for a DR question would be to put the graphical calculator to one side while they produce their answer and only pick it up when they are ready to check their answers.

OCR support

There is further advice on Detailed Reasoning and other command words in section 2d of the specification (page 10 onwards).

Question 5 (a)

5 A curve is defined implicitly by the equation $2x^2 + 3xy + y^2 + 2 = 0$.

(a) Show that $\frac{dy}{dx} = -\frac{4x+3y}{3x+2y}$. **[3]**

This part was generally done very well with nearly all candidates able to demonstrate both product rule and chain rule in this implicit context. Some struggled with the 'show that' part of the question and the minus was often 'fudged' in their solution.

Question 5 (b)

(b) **In this question you must show detailed reasoning.**

Find the coordinates of the stationary points of the curve. **[4]**

This part proved more challenging and a large number did not appreciate that if $\frac{a}{b} = 0$ then $a = 0$. This meant that many could not get as far as the first mark by expressing either *x* in terms of *y* or vice versa. Only then could they start to progress by substituting into the equation of the curve to get a quadratic. It is worth pointing out that as this was a **DR** question some working was expected to get the solutions which needed to be written as coordinates.

Exemplar 1

5(b)	$-\frac{4x+3y}{3x+2y} = 0 \quad \therefore -(4x+3y)=0$
	$\therefore 4x+3y=0 \quad \therefore$ at stationary point, $y = \frac{-4x}{3}$
	Substituting into first equation:
	$2x^2+3x\left(\frac{-4x}{3}\right)+\left(\frac{-4x}{3}\right)^2+2=0$
	$2x^2 - \cancel{6} - \frac{12x^2}{3} + \frac{16x^2}{9}+2=0$
	$2x^2-4x^2+\frac{16x^2}{9}+2=0$
	$\frac{2}{9}x^2=2 \quad x^2=9 \quad x=\pm 3$
	$y=\frac{-4\times 3}{3} = -4 \quad (3,-4)$
	$y=\frac{-4\times -3}{3}=4 \quad (-3,4)$

A concise but clear example of a correct solution with sufficient Detailed Reasoning to score full marks.

The candidate puts the expression for $\frac{dy}{dx} = 0$. They put the numerator of the fraction = 0 and rearrange it to get *y* in terms of *x*. This earns M1. They substitute this into the equation of the curve earning the second M1. Their clear working leads to the correct *x* values earning the first accuracy mark, A1. They then show their working to get the *y*-values and show them as coordinates as requested by the question.

Question 6 (a)

6 A hot drink is cooling. The temperature of the drink at time t minutes is T°C.

The rate of decrease in temperature of the drink is proportional to $(T-20)$.

(a) Write down a differential equation to describe the temperature of the drink as a function of time. **[2]**

Probably the standard way to answer this question would be by writing $\frac{dT}{dt} = -k(T-20)$ and hence getting k = 0.07. However examiners accepted both with or without the – provided it was dealt with eventually. Only a few candidates lost a mark by not including the k here.

Question 6 (b)

(b) When $t = 0$, the temperature of the drink is 90 °C and the temperature is decreasing at a rate of 4.9 °C per minute.

Determine how long it takes for the drink to cool from 90 °C to 40 °C. **[6]**

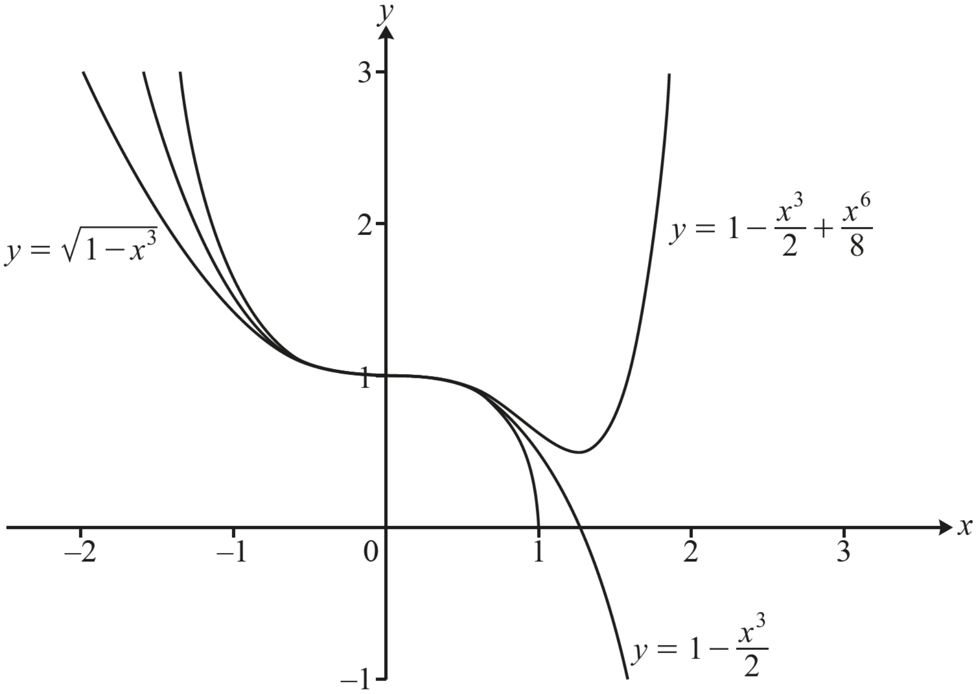
Few candidates gained full marks. The majority would correctly separate the variables and integrate, but use of ln and e thereafter led to issues with accuracy. Most candidates were able to get at least 2 of the method marks. Many left 'k' in their working and so didn't get the B1. The main issue was resolving the sign for the final answer. Quite often this was fudged and the sign quietly changed hoping examiners would not notice. Others did not hide their answer, giving t = -17.9 and then saying it had to be positive.

Question 7 (a)

7 A student is trying to find the binomial expansion of $\sqrt{1-x^3}$.

She gets the first three terms as $1-\frac{x^3}{2}+\frac{x^6}{8}$.

She draws the graphs of the curves $y=\sqrt{1-x^3}$, $y=1-\frac{x^3}{2}$ and $y=1-\frac{x^3}{2}+\frac{x^6}{8}$ using software.



(a) Explain why $1-\frac{x^3}{2}+\frac{x^6}{8} \geqslant 1-\frac{x^3}{2}$ for all values of x. **[1]**

The most common error on this question was to conclude that $\frac{x^6}{8}>0$ and not that it could also be equal to zero.

Question 7 (b)

(b) Explain why the graphs suggest that the student has made a mistake in the binomial expansion. **[1]**

Many students concluded with a correct statement, and these included that the shape of the graph did not follow the original graph; that the approximation with more terms should be better, but it is not.

Question 7 (c)

(c) Find the first **four** terms in the binomial expansion of $\sqrt{1-x^3}$. **[3]**

This question provided a good source of marks for candidates and many provided a clear and detailed worked method which was pleasing to see. The most common errors were: some did not use $(-x^3)$ in the expansion but just (x) or (x^3) ; some were unable to properly evaluate a term involving a negative term being taken to an even power.

Question 7 (d)

(d) State the set of values of x for which the binomial expansion in part **(c)** is valid. **[1]**

Examiners saw many correct answers but examples of incorrect answers include x ≤1; x ϵ R; x ≥ 1; $-1<x^3<1$; |*x*| > 1 |*x*| < -1

Question 7 (e)

(e) Sketch the curve $y = 2.5\sqrt{1-x^3}$ on the grid in the Printed Answer Booklet. **[2]**

This part on the whole scored 2 marks for nearly all candidates. The most common error was not realising the point (1,0) was invariant, and instead putting the transformed graph with a starting position of (2.5,0).

Question 7 (f)

(f) **In this question you must show detailed reasoning.**

The end of a bus shelter is modelled by the area between the curve $y = 2.5\sqrt{1-x^3}$, the lines $x = -0.75$, $x = 0.75$ and the x-axis. Lengths are in metres.

Calculate, using your answer to part **(c)**, an approximation for the area of the end of the bus shelter as given by this model. **[4]**

In this **DR** question it was pleasing to see that many candidates showed all of their working, and achieved the correct answer to the integration. The most common errors included: the student not multiplying their integral by 2.5; not integrating their expression correctly; and missing the units off their final answer. As it is a **DR** question examiners were expecting to see the substitution of the limits to get the third method mark and hence the accuracy mark (**A1**).

Exemplar 2

7(f) **(continued)**

$$\left(1 - \frac{1}{2}x^3 - \frac{1}{8}x^6 - \frac{1}{16}x^9\right) 2.5$$

$$\int_{-0.75}^{0.75} 2.5 - \frac{5}{4}x^3 - \frac{5}{16}x^6 - \frac{5}{32}x^9 .$$

$$\left[2.5x - \frac{5}{16}x^4 - \frac{5}{112}x^7 - \frac{1}{64}x^{10}\right]_{-0.75}^{0.75}$$

$$\cancel{\frac{15}{8}} \quad \cancel{\frac{405}{4096}} \qquad 1.769284 - -1.96879775$$

$$= 3.738\ \text{m}^2 .$$

✓

It is interesting to note that although the candidate gets the correct answer, as this is a DR question they only earn 2 out of a maximum 4 marks. They write their (correct) expansion from 7 (c) as an integral and multiply it by 2.5 so earning the first M1. They then integrate each term earning the second M1. The third M1 is for a clear substitution of the limits into each term of the integrand and the subtraction shown. Here the substitution is not shown so they do not earn this M1 and as they have not earned all of the method marks they can not score the final accuracy mark (even though their answer is correct).

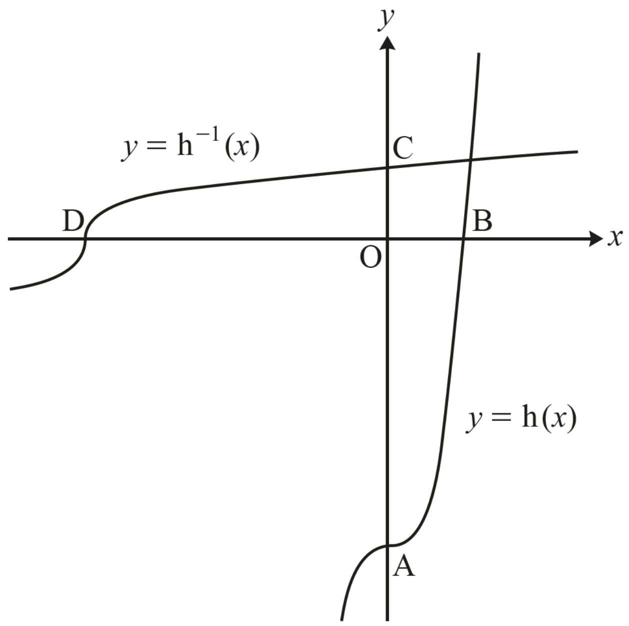
Good advice would be to do the question in the traditional way and only use the calculus function on their calculator to *check* their answer.

Question 8 (a)

8 The curves $y = \mathrm{h}(x)$ and $y = \mathrm{h}^{-1}(x)$, where $\mathrm{h}(x) = x^3 - 8$, are shown below.

The curve $y = \mathrm{h}(x)$ crosses the x-axis at B and the y-axis at A.

The curve $y = \mathrm{h}^{-1}(x)$ crosses the x-axis at D and the y-axis at C.



(a) Find an expression for $\mathrm{h}^{-1}(x)$. **[2]**

This part was answered well by most candidates. However a large number went on to incorrectly simplify the inverse expression, with the most common incorrect simplification being $\sqrt[3]{x} + 2$. A few interpreted $h^{-1}(x)$ as being the reciprocal of $h(x)$.

Question 8 (b)

(b) Determine the coordinates of A, B, C and D. **[5]**

This part was answered correctly by nearly all candidates. The most common error was a missing negative sign on the y-ordinate in the point A or the x-ordinate in point D. A few just gave the *x* or *y* values of the points rather than the coordinates.

Question 8 (c)

(c) Determine the equation of the perpendicular bisector of AB. Give your answer in the form $y = mx + c$, where m and c are constants to be determined. **[4]**

This part was well answered by most candidates. The most common error was not obtaining the correct gradient, or using a point A, B, C, or D instead of the midpoint in the equation. A small number of candidates reached the required answer but then proceeded to put it in a form that was not as requested in the question.

Question 8 (d)

(d) Points A, B, C and D lie on a circle.

Determine the equation of the circle. Give your answer in the form $(x-a)^2 + (y-b)^2 = r^2$, where a, b and r^2 are constants to be determined. **[5]**

Many found the most straight forward way to find the centre using the midpoint of the chords AC and BD.

Most substituted some or all of A, B, C or D into the equation of the circle and solved with greater or lesser ease depending on whether they used two equations that enabled them to eliminate a^2 and r^2 (or b^2 and r^2). Otherwise they ended up with having to solve simultaneous equations in a and b. Those that did this were in general less successful due to mistakes in the algebraic manipulation.

The next most used method was to recognise that the centre lies on y = x and find the intersection of y=x with the perp bisector they found in part c.

Also several found perp bisector of CD and intersection with perp bisector of AB.

With all of the above the most common reason for not solving correctly was poor algebraic manipulation.

A significant number did not recognise the need to find the intersection of perp bisectors of chords. Quite a few thought that AD was the diameter, and so the centre would be at (-4, -4).

Section B overview

This comprehension proved more accessible than most with most candidates being able to make a start on it and fewer than usual feeling unable to attempt the later questions.

Question 9

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

9 Show that $y = x$ has the same gradient as $y = \sin x$ when $x = 0$, as stated in line 5. **[2]**

This was well done. There were some unconventional notations/presentations that we condoned, e.g. $\frac{\mathrm{d}y}{\mathrm{dx}} = 1, \frac{\mathrm{d}y}{\mathrm{d}x} = \cos x = \cos 0 = 1$, where there was no sense of what 'y' the derivative was of. A small number attempted to use small angle approximations which did not earn any marks.

Question 10

10 **In this question you must show detailed reasoning.**

Fig. C2.2 indicates that the curve $y = \dfrac{4x(\pi - x)}{\pi^2} - \sin x$ has a stationary point near $x = 3$.

- Verify that the x-coordinate of this stationary point is between 2.6 and 2.7.
- Show that this stationary point is a maximum turning point. **[5]**

Many candidates recognised that they had to differentiate the function, but sometimes went about it in a rather long way, for example by using the quotient rule. Candidates should be reminded that the instruction 'verify that' means that they should use the given values, they do not have to find them. Most understood this and successfully substituted these values into their gradient function, interpreting the results correctly. While they were able to confirm that the stationary point was a maximum from these values, some went on to find the second derivative, substituting the given values into this. It was evident that some used the graphing facility on their graphics calculator to locate the stationary point and a **DR** solution should not use the graphical features for anything other than checking solutions.

Exemplar 3

10	$y = \frac{4}{\pi^2}x(\pi - x) - \sin(x)$
	$= \frac{4}{\pi}x - \frac{4}{\pi^2}x^2 - \sin(x)$
	$\frac{dy}{dx} = \frac{4}{\pi} - \frac{8}{\pi^2}x - \cos(x)$
	~~let $\frac{dy}{dx}$~~
	~~let~~ Let $x = 2.6$
	$\frac{4}{\pi} - \frac{8}{\pi^2}(2.6) - \cos(2.6)$
	$= 0.0276$
	Let $x = 2.7$
	$\frac{4}{\pi} - \frac{8}{\pi^2}(2.7) - \cos(2.7)$
	$= -0.01122$
	hence there is a sign change
	so $\frac{dy}{dx} = 0$ in the interval ~~$2.6 \le x \le 2.7$~~ $2.6 < x < 2.7$
	therefor a stationary point must lie inbetween.
	Finished in additional space!

10) (continued) $-x^2$

$$\frac{d^2y}{dx^2} = \sin(x) - \frac{8}{\pi^2}$$

$$\sin(2.65) - \frac{8}{\pi^2} = -0.3385$$

~~the~~ negative therefor the stationary point is a local maximum.

Although this is a question in the comprehension section it is really a quite traditional question on calculus and the theory around iteration.

This candidate knows that they need to find $\frac{dy}{dx}$ if they are going to find a stationary point and they do that correctly labelling it clearly earning the first M1. They understand that 'verify' means they should use the given values of 2.6 and 2.7 to correctly find the gradients of 0.0226 and -0.0112 and this gets them a further M1 and A1. They get the E1 (Explanation mark) for saying that the 'sign change' shows the 'stationary point lies in between'.

They do not score the final B1 as finding the second derivative half way between the two values of 2.6 and 2.7 is not a lot of help as at this point they do not know where the stationary point is. Examiners saw a number of methods to justify that it is a maximum. Some found the second derivative at both 2.6 and 2.7 and stated that both were negative implying a maximum, some used their calculator to` find the stationary point more accurately (2.67) and then found the second derivative (-0.356). However the easiest way was to comment how the gradient went from positive to zero to negative implying the maximum. Some even supplied a useful sketch as well.

Question 11

11 Show that, for the angle 45°, the formula $\sin\theta \approx \frac{4\theta(180-\theta)}{40500-\theta(180-\theta)}$ given in line 28 gives the same approximation for the sine of the angle as the formula $\sin x \approx \frac{16x(\pi-x)}{5\pi^2-4x(\pi-x)}$ given in line 23. **[3]**

There were many correct responses to this question. Candidates had to show that the two expressions led to the same fraction to show that they were equivalent; showing that they were equivalent to a number of significant figures is insufficient to show that they are exactly the same. Most knew that 45° = $\frac{\pi}{4}$ radians.

Question 12 (a)

12 **(a)** Show that $\cos x = \sin\left(x + \frac{\pi}{2}\right)$. **[2]**

The more successful approach was to expand sin(x + π/2) using the compound angle formula and then substitute cos π/2 = 0 and sin π/2 = 1 to score both marks. A few tried graphs or a transformation approach but it was disappointing to see 'shifted by' or 'moved' rather than 'translation'.

Question 12 (b)

(b) Hence show that $\sin x \approx \frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}$ gives the approximation $\cos x \approx \frac{\pi^2 - 4x^2}{\pi^2 + x^2}$, as stated in line 31. **[3]**

Provided candidates knew how to start this question, they generally went on to give a full and correct response. Those who gained partial marks, tripped over sign issues in the expansion of the brackets.

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