

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Sample Question Paper

Version 2.1

Date – Morning/Afternoon

Time allowed: 2 hours

You must have:

- Printed Answer Booklet
- the Insert

You may use:

- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient Rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small Angle Approximations

$$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{1}{2}\theta^2, \tan \theta \approx \theta \text{ where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2} h \{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B) \quad \text{or} \quad P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Sample Variance

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The Binomial Distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^n C_r p^r q^{n-r}$ where $q = 1 - p$

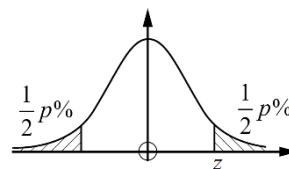
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2$$

$$s = \frac{1}{2} (u + v) t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2} at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2} \mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2} (\mathbf{u} + \mathbf{v}) t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2} \mathbf{a}t^2$$

Answer **all** the questions

Section A (60 marks)

1 Express $\frac{2}{x-1} + \frac{5}{2x+1}$ as a single fraction. [2]

2 Find the first four terms of the binomial expansion of $(1-2x)^{\frac{1}{2}}$.

State the set of values of x for which the expansion is valid. [4]

3 Show that points A (1, 4, 9), B (0, 11, 17) and C (3, -10, -7) are collinear. [4]

4 Show that $\sum_{r=1}^4 \ln \frac{r}{r+1} = -\ln 5$. [3]

5 In this question you must show detailed reasoning.

Fig. 5 shows the circle with equation $(x-4)^2 + (y-1)^2 = 10$.

The points $(1, 0)$ and $(7, 0)$ lie on the circle. The point C is the centre of the circle.

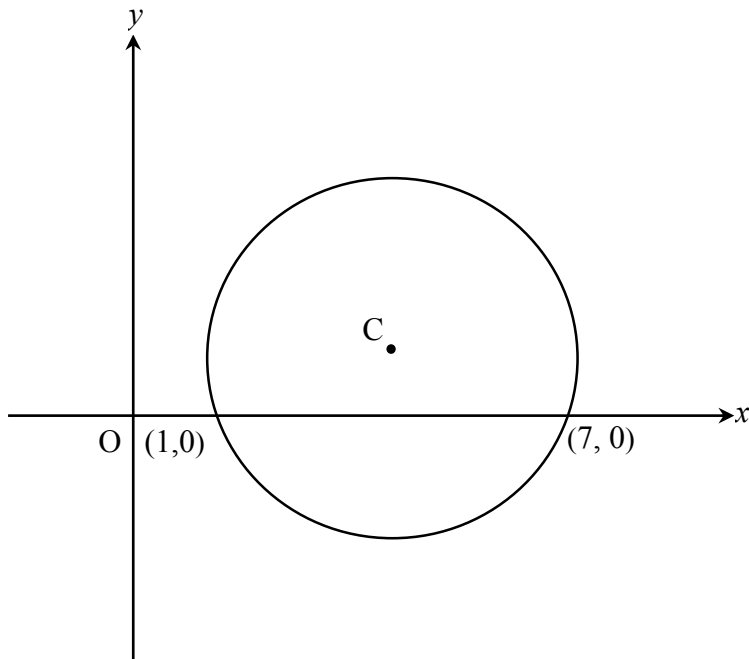


Fig. 5

Find the area of the part of the circle below the x -axis.

[5]

- 6 Fig. 6 shows the curve with equation $y = x^4 - 6x^2 + 4x + 5$.

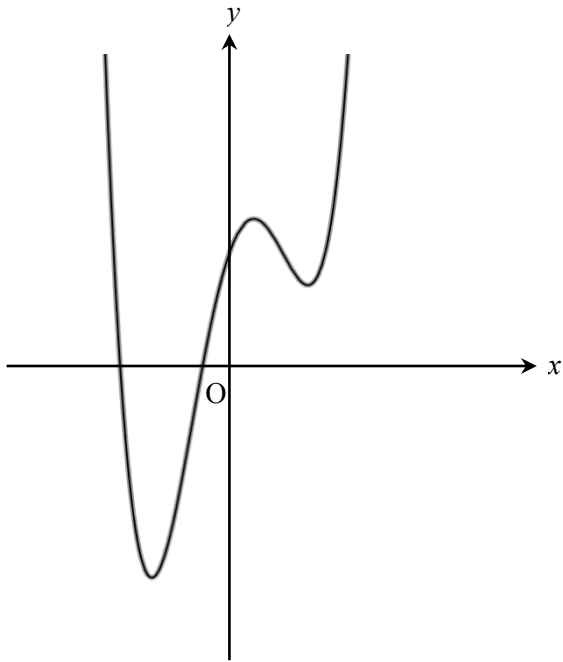


Fig. 6

Find the coordinates of the points of inflection.

[5]

- 7 By finding a counter example, disprove the following statement.

If p and q are non-zero real numbers with $p < q$, then $\frac{1}{p} > \frac{1}{q}$.

[2]

- 8 In **Fig. 8**, OAB is a thin bent rod, with $OA = 1$ m, $AB = 2$ m and angle $OAB = 120^\circ$. Angles θ , ϕ and h are as shown in **Fig. 8**.

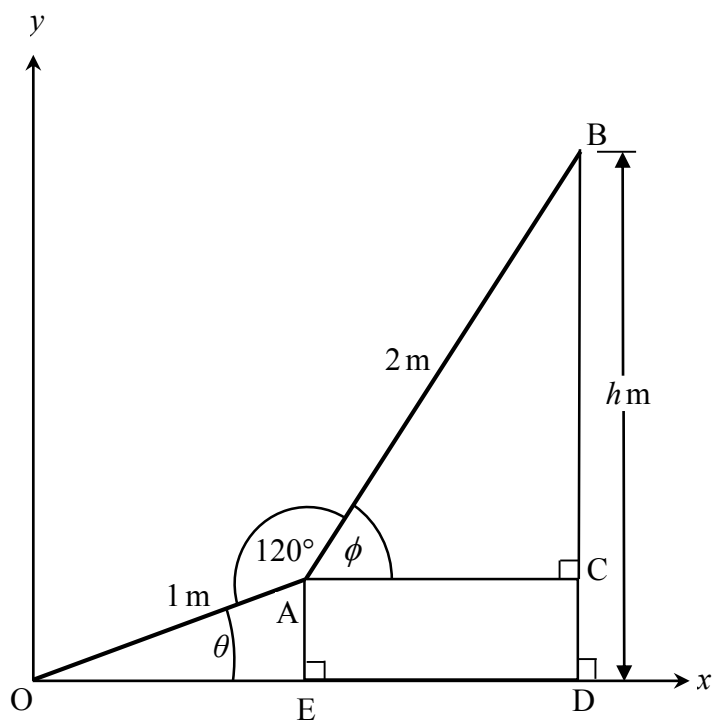


Fig. 8

- (a) Show that $h = \sin \theta + 2 \sin(\theta + 60^\circ)$. [3]

The rod is free to rotate about the origin so that θ and ϕ vary. You may assume that the result for h in part (a) holds for all values of θ .

- (b) Find an angle θ for which $h = 0$. [5]

- 9 (a) Express $\cos \theta + 2 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $0 < \alpha < \frac{1}{2}\pi$ and R is positive and given in exact form. [4]

The function $f(\theta)$ is defined by $f(\theta) = \frac{1}{(k + \cos \theta + 2 \sin \theta)}$, $0 \leq \theta \leq 2\pi$, k is a constant.

- (b) The maximum value of $f(\theta)$ is $\frac{(3 + \sqrt{5})}{4}$.
Find the value of k . [3]

10 The function $f(x)$ is defined by $f(x) = x^4 + x^3 - 2x^2 - 4x - 2$.

- (a) Show that $x = -1$ is a root of $f(x) = 0$. [1]
- (b) Show that another root of $f(x) = 0$ lies between $x = 1$ and $x = 2$. [2]
- (c) Show that $f(x) = (x + 1)g(x)$, where $g(x) = x^3 + ax + b$ and a and b are integers to be determined. [3]
- (d) Without further calculation, explain why $g(x) = 0$ has a root between $x = 1$ and $x = 2$. [1]
- (e) Use the Newton-Raphson formula to show that an iteration formula for finding roots of $g(x) = 0$ may be written

$$x_{n+1} = \frac{2x_n^3 + 2}{3x_n^2 - 2}.$$

Determine the root of $g(x) = 0$ which lies between $x = 1$ and $x = 2$ correct to 4 significant figures. [3]

11 The curve $y = f(x)$ is defined by the function $f(x) = e^{-x} \sin x$ with domain $0 \leq x \leq 4\pi$.

(a) (i) Show that the x -coordinates of the stationary points of the curve $y = f(x)$, when arranged in increasing order, form an arithmetic sequence.

(ii) Show that the corresponding y -coordinates form a geometric sequence. **[9]**

(b) Would the result still hold with a larger domain? Give reasons for your answer. **[1]**

Answer **all** the questions

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

- 12 Explain why the smaller regular hexagon in **Fig. C1** has perimeter 6. [1]
- 13 Show that the larger regular hexagon in **Fig. C1** has perimeter $4\sqrt{3}$. [3]
- 14 Show that the two values of b given on line 36 are equivalent. [3]

- 15 Fig. 15 shows a unit circle and the escribed regular polygon with 12 edges.

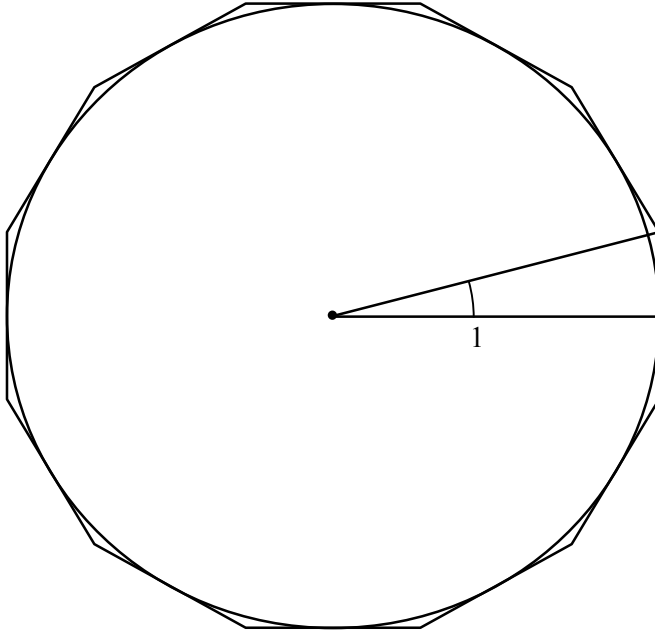


Fig. 15

- (a) Show that the perimeter of the polygon is $24 \tan 15^\circ$. [2]
- (b) Using the formula for $\tan(\theta - \phi)$ show that the perimeter of the polygon is $48 - 24\sqrt{3}$. [3]
- 16 On a unit circle, the inscribed regular polygon with 12 edges gives a lower bound for π , and the escribed regular polygon with 12 edges gives an upper bound for π .

Calculate the values of these bounds for π , giving your answers:

- (i) in surd form
- (ii) correct to 2 decimal places. [3]

END OF QUESTION PAPER

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