Section B: MECHANICS

Question	Scheme	Marks	AOs
6.	Integrate **v** w.r.t. time	M1	1.1a
	$\mathbf{r} = 2t^{\frac{1}{2}}\mathbf{i} - 2t^2\mathbf{j}\ \ (+\mathbf{C})$	A1	1.1b
	Substitute t = 4 and t = 1 into their **r**	M1	1.1b
	$t = 4, \mathbf{r} = 4\mathbf{i} - 32\mathbf{j}(+\mathbf{C});\ t = 1, \mathbf{r} = 2\mathbf{i} - 2\mathbf{j}(+\mathbf{C})$ or $(4, -32); (2, -2)$	A1	1.1b
	$\sqrt{2^2 + (-30)^2}$	M1	1.1b
	$\sqrt{904} = 2\sqrt{226}$	A1	1.1b
		(6)	
		(6 marks)	

Notes: Allow column vectors throughout

M1: At least one power increasing by 1.

A1: Any correct (unsimplified) expression

M1: Must have attempted to integrate **v**. Substitute t = 4 and t = 1 into their **r** to produce 2 vectors (or 2 points if just working with coordinates).

A1: $4\mathbf{i} - 32\mathbf{j}(+\mathbf{C})$ and $2\mathbf{i} - 2\mathbf{j}(+\mathbf{C})$ or $(4, -32)$ and $(2, -2)$. These can be seen or implied.

M1: Attempt at distance of form $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ for their points. Must have 2 non zero terms.

A1: $\sqrt{904} = 2\sqrt{226}$ or any equivalent surd (exact answer needed)

Question	Scheme	Marks	AOs
7(a)	Resolve vertically	M1	3.1b
	$R + 40\sin\alpha = 20g$	A1	1.1b
	Resolve horizontally	M1	3.1b
	$40\cos\alpha - F = 20a$	A1	1.1b
	$F = 0.14R$	B1	1.2
	$a = 0.396$ or 0.40 (m s^{-2})	A1	2.2a
		(6)	
(b)	Pushing will increase R which will increase available F	B1	2.4
	Increasing F will <u>decrease a</u> * GIVEN ANSWER	B1*	2.4
		(2)	
			(8 marks)

Notes:

(a)

M1: Resolve vertically with usual rules applying

A1: Correct equation**.** Neither g nor $\sin a$ need to be substituted

M1: Apply $F = ma$ horizontally, with usual rules

A1: Neither F nor $\cos a$ need to be substituted

B1: $F = 0.14R$ seen (e.g. on a diagram)

A1: Either answer

(b)

B1: Pushing increases R which produces an increase in available (limiting) friction

B1: F increase produces an <u>a decrease (need to see this)</u>

N.B. It is possible to score B0 B1 but for the B1, some "explanation" is needed to say why friction is increased e.g. by pushing into the ground.

Question	Scheme	Marks	AOs
8(a)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$: $(7\mathbf{i} - 10\mathbf{j}) = 2(2\mathbf{i} - 3\mathbf{j}) + \frac{1}{2}\mathbf{a}2^2$	M1	3.1b
	$\mathbf{a} = (1.5\mathbf{i} - 2\mathbf{j})$	A1	1.1b
	$\lvert\mathbf{a}\rvert = \sqrt{1.5^2 + (-2)^2}$	M1	1.1b
	$= 2.5$ m s^{-2} * GIVEN ANSWER	A1*	2.1
		(4)	
(b)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} - 3\mathbf{j}) + 2(1.5\mathbf{i} - 2\mathbf{j})$	M1	3.1b
	$= (5\mathbf{i} - 7\mathbf{j})$	A1	1.1b
	$\mathbf{v} = (5\mathbf{i} - 7\mathbf{j}) + t(4\mathbf{i} + 8.8\mathbf{j}) = (5 + 4t)\mathbf{i} + (8.8t - 7)\mathbf{j}$ and $(5 + 4t) = (8.8t - 7)$	M1	3.1b
	$t = 2.5$ (s)	A1	1.1b
		(4)	
		(8 marks)	

Notes: Allow column vectors throughout

(a)

No credit for individual component calculations

M1: Using a complete method to obtain the acceleration. **N.B.** Equation, in **a** only, could be obtained by two integrations

ALTERNATIVE

M1: Use velocity at half-time (t = 1) = Average velocity over time period

So at $t = 1$, $\mathbf{v} = \frac{1}{2}(7\mathbf{i} - 10\mathbf{j})$ so $\mathbf{a} = \frac{1}{2}(7\mathbf{i} - 10\mathbf{j}) - (2\mathbf{i} - 3\mathbf{j})$

N.B. could see $(7\mathbf{i} - 10\mathbf{j}) = (4\mathbf{i} - 6\mathbf{j}) + 2\mathbf{a}$ as first line of working

A1: Correct **a** vector

M1: Attempt to find magnitude of their **a** using form $\sqrt{a^2 + b^2}$

A1*: Correct GIVEN ANSWER obtained correctly

(b)

M1: Using a complete method to obtain the velocity at A e.g.by use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with t = 2 and $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$ and their **a**

OR: by use of $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

OR: by integrating their **a,** with addition of $\mathbf{C} = 2\mathbf{i} - 3\mathbf{j}$, and putting t = 2

A1: correct vector

M1: Complete method to find equation in t only

e.g. by using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, with their **u** and equating **i** and **j** components
OR: by integrating (4**i** + 8.8**j**), with addition of a constant, and equating **i** and **j** components. **N.B.** Must be equating **i** and **j** components of <u>a velocity vector</u> and must be their velocity at *A*, to give an equation in *t* only for this M mark **A1:** 2.5 (s)

Question	Scheme	Marks	AOs
9(a)	Moments about A (or any other complete method)	M1	3.3
	$T2a\sin \alpha = Mga + 3Mgx$	A1	1.1b
	$T = \dfrac{Mg(a+3x)}{2a \times \frac{3}{5}} = \dfrac{5Mg(3x+a)}{6a}$ * GIVEN ANSWER	A1*	2.1
		(3)	
(b)	$\dfrac{5Mg(3x+a)}{6a}\cos \alpha = 2Mg$ **OR** $2Mg.2a\tan\alpha = Mga + 3Mgx$	M1	3.1b
	$x = \dfrac{2a}{3}$	A1	2.2a
		(2)	
(c)	Resolve vertically **OR** Moments about B	M1	3.1b
	$Y = 3Mg + Mg - \dfrac{5Mg(3.\frac{2a}{3}+a)}{6a}\sin \alpha$ $2aY = Mga + 3Mg(2a - \frac{2a}{3})$ **Or**: $Y = 3Mg + Mg - \left(\dfrac{2Mg}{\cos\alpha}\right)\sin\alpha$	A1**ft**	1.1b
	$Y = \dfrac{5Mg}{2}$ **N.B.** May use $R\sin\beta$ for Y and/or $R\cos\beta$ for X throughout	A1	1.1b
	$\tan\beta = \dfrac{Y}{X}$ or $\dfrac{R\sin\beta}{R\cos\beta}$ $= \dfrac{\frac{5Mg}{2}}{2Mg}$	M1	3.4
	$= \dfrac{5}{4}$	A1	2.2a
		(5)	
(d)	$\dfrac{5Mg(3x+a)}{6a} \leq 5Mg$ and solve for x	M1	2.4
	$x \leq \dfrac{5a}{3}$	A1	2.4
	For rope not to break, block can't be more than $\dfrac{5a}{3}$ from A oe Or just: $x \leq \dfrac{5a}{3}$, if no incorrect statement seen. **N.B. If the correct inequality is not found,** their comment must mention 'distance from A'.	B1 **A1**	2.4
		(3)	
			(13 marks)

Notes:
(a) **M1**: Using M(A), with usual rules, or any other complete method to obtain an equation in a, M, x and T only. **A1:** Correct equation **A1*:** Correct PRINTED ANSWER, correctly obtained, need to see $\sin\alpha = \frac{3}{5}$ used.
(b) **M1:** Using an appropriate strategy to find x. e.g. Resolve horizontally with usual rules applying OR Moments about C. Must use the <u>given</u> expression for T. **A1:** Accept $0.67a$ or better
(c) **M1:** Using a complete method to find Y (or $R\sin\beta$) e.g. resolve vertically or Moments about B, with usual rules **A1 ft:** Correct equation <u>with their x substituted in T expression</u> or using $T = \frac{2Mg}{\cos\alpha}$ **A1:** Y (or $R\sin\beta$) $= \frac{5Mg}{2}$ or $2.5Mg$ or $2.50Mg$ **M1:** For finding an equation **in tan β only** using $\tan\beta = \frac{Y}{X}$ or $\tan\beta = \frac{X}{Y}$ This is independent but must have found a Y. **A1:** Accept $\frac{-5}{4}$ if it follows from their working.
(d) **M1:** Allow $T = 5Mg$ or $T < 5Mg$ and solves for x, showing all necessary steps (M0 for $T > 5Mg$) **A1:** Allow $x = \frac{5a}{3}$ or $x < \frac{5a}{3}$. Accept $1.7a$ or better. **B1: Treat as A1.** For any appropriate equivalent fully correct comment or statement. E.g. maximum value of x is $\frac{5a}{3}$

Question	Scheme	Marks	AOs
10(a)	Using the model and vertical motion: $0^2 = (U\sin\alpha)^2 - 2g \times (3 - 2)$	M1	3.3
	$U^2 = \dfrac{2g}{\sin^2\alpha}$ * GIVEN ANSWER	A1*	2.2a
		(2)	
(b)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-\frac{5}{4} = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	sub for t: $-\frac{5}{4} = U\sin\alpha\left(\frac{20}{U\cos\alpha}\right) - \frac{1}{2}g\left(\frac{20}{U\cos\alpha}\right)^2$	M1 (I)	3.1b
	sub for U^2	M1(II)	3.1b
	$-\frac{5}{4} = 20\tan\alpha - 100\tan^2\alpha$	A1(I)	1.1b
	$(4\tan\alpha - 1)(100\tan\alpha + 5) = 0$	M1(III)	1.1b
	$\tan\alpha = \frac{1}{4} \Rightarrow \alpha = 14^0$ or better	A1(II)	2.2a
		(9)	
	N.B. For the last 5 marks, they may set up a quadratic in t, by substituting for $U\sin\alpha$ first, then solve the quadratic to find the value of t, then use $20 = Ut\cos\alpha$ to find α. The marks are the same but earned in a different order. Enter on ePen in the corresponding M and A boxes above, as indicated below.		
	Sub for $U\sin\alpha$ to give equation in t only	M1(II)	
	$-\frac{5}{4} = \sqrt{2g}t - \frac{1}{2}gt^2$	A1(I)	
	Solve for t	M1(III)	
	$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13 and use $20 = Ut\cos\alpha$	M1(I)	
	$\alpha = 14^0$ or better	A1(II)	
(b)	**ALTERNATIVE**		

	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos a$	A1	1.1b
	A to top: $s = vt - \frac{1}{2}at^2$ <u>and</u> top to T: $s = ut + \frac{1}{2}at^2$		
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ <u>and</u> $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	M1	3.4
	$= \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}} \quad (= \frac{5}{\sqrt{2g}})$	A1	1.1b
	$20 = U\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for t)	M1	3.1b
	$20 = \sqrt{\frac{2g}{\sin^2\alpha}}\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for U)	M1	3.1b
	$\tan a = \frac{1}{4}$	A1	1.1b
	Solve for α	M1	1.1b
	Þ $a = 14^0$ or better	A1	2.2a
		(9)	
(c)	The target will have dimensions so in practice there would be a range of possible values of α **Or** There will be air resistance **Or** The ball will have dimensions **Or** Wind effects **Or** Spin of the ball	B1	3.5b
		(1)	
(d)	Find U using their α e.g. $U = \sqrt{\frac{2g}{\sin^2\alpha}}$	M1	3.1b
	Use $20 = Ut\cos a$ (or use vertical motion equation)	A1 **M1**	1.1b
	$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13	B1 **A1**	1.1b
		(3)	
(d)	**ALTERNATIVE**		

	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$	M1	3.1b
	$1 = \frac{1}{2}g{t_1}^2$ => $t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}g{t_2}^2$ => $t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	A1 **M1**	1.1b
	$= = \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}}$ $(= \frac{5}{\sqrt{2g}})$ = 1.1 or 1.13 (s)	B1 **A1**	1.1b
		(3)	
(15 marks)			

Notes:

(a)
M1: Or any other complete method to obtain an equation in U, g and a **only**
A1*: Correct GIVEN ANSWER

(b)
M1: Using horizontal motion
A1: Correct equation
M1: Using vertical motion . N.B. M0 if they use $s = \pm 2$ or ± 3, but allow $s = \pm 1.25$ or ± 0.75 or ± 2.25 or ± 2.75
A1: Correct equation
M1: Using $20 = Ut\cos a$ to sub. for t
M1: Substituting for U^2 using (a)
A1: Correct quadratic equation (in tan a **or cot a)**
M1: Solve a 3 term quadratic, either by factorisation or formula (or by calculator (implied) if answer is correct) **and find a**
A1: $a = 14^{\circ}$ or better (No restriction on accuracy since g's cancel)
N.B. If answer is correct, previous M mark can be implied, but if answer is incorrect, an explicit attempt to solve must be seen to earn the previous M mark.

(b) ALTERNATIVE
M1: Using the model with the usual rules applying to the equation
A1: Correct equation
M1: Using the model to obtain the **total** time from A to T
A1: Correct **total** time t
M1: Substitute for t in $20 = Ut\cos a$
M1: Substitute for U in $20 = Ut\cos a$, using part (a)
A1: Correct equation in tan a **only**
M1: Solve equation for a
A1: $a = 14^{\circ}$ or better (No restriction on accuracy since g's cancel)

N.B. If they quote the equation of the trajectory $y = x\tan\alpha - \dfrac{gx^2}{2U^2\cos^2\alpha}$ oe **AND** put in values for *x* and *y*, could score first 5 marks, M1A1M1A1M1 (nothing for the equation only); wrong *x* value loses first A mark and wrong *y* value loses second A mark

(c)

B1: Give one limitation of the model e.g. the ball will have dimensions, or there will be air resistance or wind effects or spin

N.B. B0 if any incorrect extra(s) but ignore extra consequences.

(d)

M1: Using their α to find a value for *U*

A1: Treat as M1: Using their *U* to find a value for *t*

B1: Treat as A1 : $t = 1.1$ or 1.10 (since depends on $g = 9.8$)

(d) ALTERNATIVE

M1: Using their α to find a value for *U*

A1: Treat as M1: Using their *U* to find a value for *t*

B1: Treat as A1 : $t = 1.1$ or 1.10 (since depends on $g = 9.8$)