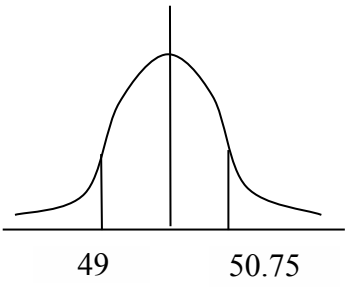


Paper 3: Statistics and Mechanics Mark Scheme

Question	Scheme	Marks	AOs
1(a)	Area = $8 \times 1.5 = 12 \text{ cm}^2$ Frequency = 8 so $1 \text{ cm}^2 = \frac{2}{3} \text{ hour (o.e.)}$	M1	3.1a
	Frequency of 12 corresponds to area of 18 so height = $18 \div 2.5 = 7.2 \text{ (cm)}$	A1	1.1b
	Width = $5 \times 0.5 = 2.5 \text{ (cm)}$	B1cao	1.1b
		(3)	
(b)	$[\bar{y} =] \frac{205.5}{31} = \text{awrt } 6.63$	B1cao	1.1b
	$[\sigma_y =] \sqrt{\frac{1785.25}{31} - \bar{y}^2} = \sqrt{13.644641} = \text{awrt } 3.69$	M1	1.1a
	allow $[s =] \sqrt{\frac{1785.25 - 31\bar{y}^2}{30}} = \text{awrt } 3.75$	A1	1.1b
		(3)	
(c)	Mean of Heathrow is higher than Hurn and standard deviation smaller suggesting Heathrow is more reliable	M1	2.4
	Hurn is South of Heathrow so does <u>not</u> support his belief	A1	2.2b
		(2)	
(d)	$\bar{x} + \sigma \approx 10.3$ so number of days is e.g. $\frac{(11 - "10.3")}{3} \times 8 (+5)$	M1	1.1b
	= 6.86 so 7 days	A1	1.1b
		(2)	
(e)	$[H = \text{no. of hours}] \quad P(H > 10.3) \text{ or } P(Z > 1) = [0.15865\dots]$	M1	3.4
	Predict $31 \times 0.15865\dots = \underline{\underline{4.9 \text{ or } 5 \text{ days}}}$	A1	1.1b
		(2)	
(f)	(5 or) 4.9 days < (7 or) 6.9 days so model may not be suitable	B1	3.5a
		(1)	
			(13 marks)

Question 1 continued**Notes:****(a)****M1:** for clear attempt to relate the area to frequency. Can also award if their height \times their width = 18**A1:** for height = 7.2 (cm)**(b)****M1:** for a correct expression for σ or s , can ft their value for mean**A1:** awrt 3.69 (allow $s = 3.75$)**(c)****M1:** for a suitable comparison of standard deviations to comment on reliability.**A1:** for stating Hurn is south of Heathrow and a correct conclusion**(d)****M1:** for a correct expression – ft their $\bar{x} + \sigma \approx 10.3$ **A1:** for 7 days but accept 6 (rounding down) following a correct expression**(e)****M1:** for a correct probability attempted**A1:** for a correct prediction**(f)****B1:** for a suitable comparison and a compatible conclusion

Question	Scheme	Marks	AOs
2(a)	e.g. It requires extrapolation so will be unreliable (o.e.)	B1	1.2
		(1)	
(b)	e.g. Linear association between w and t	B1	1.2
		(1)	
(c)	$H_0: \rho = 0$ $H_1: \rho > 0$	B1	2.5
	Critical value 0.5822	M1	1.1a
	Reject H_0		
	There is evidence that the product moment correlation coefficient is greater than 0	A1	2.2b
		(3)	
(d)	Higher \bar{t} suggests overseas and not Perth...lower wind speed so perhaps not close to the sea so suggest Beijing	B1	2.4
		(1)	
(6 marks)			
Notes:			
(a)	B1: for a correct statement (unreliable) with a suitable reason		
(b)	B1: for a correct statement		
(c)	B1: for both hypotheses in terms of ρ		
	M1: for selecting a suitable 5% critical value compatible with their H_1		
	A1: for a correct conclusion stated		
(d)	B1: for suggesting Beijing with some supporting reason based on t or w		
	Allow Jacksonville with a reason based just on higher \bar{t}		

Question	Scheme	Marks	AOs
Q3(a)			
	$P(L > 50.98) = 0.025$	B1cao	3.4
	$\therefore \frac{50.98 - \mu}{0.5} = 1.96$	M1	1.1b
	$\therefore \mu = 50$	A1cao	1.1b
	$P(49 < L < 50.75)$	M1	3.4
	$= 0.9104\dots$ awrt 0.910	A1ft	1.1b
		(5)	
(b)	$S =$ number of strips that cannot be used so $S \sim B(10, 0.090)$	M1	3.3
	$= P(S \leq 3) = 0.991166\dots$ awrt 0.991	A1	1.1b
		(2)	
(c)	$H_0 : \mu = 50.1$ $H_1 : \mu > 50.1$	B1	2.5
	$\bar{X} \sim N\left(50.1, \frac{0.6^2}{15}\right)$ and $\bar{X} > 50.4$	M1	3.3
	$P(\bar{X} > 50.4) = 0.0264$	A1	3.4
	$p = 0.0264 > 0.01$ or $z = 1.936\dots < 2.3263$ and not significant	A1	1.1b
	There is insufficient evidence that the mean length of strips is greater than 50.1	A1	2.2b
		(5)	
(12 marks)			

Question 3 continued**Notes:****(a)****1st M1:** for standardizing with μ and 0.5 and setting equal to a z value ($|z| > 1$)**2nd M1:** for attempting the correct probability for strips that can be used**2nd A1ft:** awrt 0.910 (allow ft of their μ)**(b)****M1:** for identifying a suitable binomial distribution**A1:** awrt 0.991 (from calculator)**(c)****B1:** hypotheses stated correctly**M1:** for selecting a correct model (stated or implied)**1st A1:** for use of the correct model to find $p =$ awrt 0.0264 (allow $z =$ awrt 1.94)**2nd A1:** for a correct calculation, comparison and correct statement**3rd A1:** for a correct conclusion in context mentioning “mean length” and 50.1

Question	Scheme	Marks	AOs
4(a)	$P(A' B') = \frac{P(A' \cap B')}{P(B')} \text{ or } \frac{0.33}{0.55}$	M1	3.1a
	$= \frac{3}{5} \text{ or } 0.6$	A1	1.1b
		(2)	
(b)	e.g. $P(A) \times P(B) = \frac{7}{20} \times \frac{9}{20} = \frac{63}{400} \neq P(A \cap B) = 0.13 = \frac{52}{400}$ or $P(A' B') = 0.6 \neq P(A') = 0.65$	B1	2.4
		(1)	
(c)		B1	2.5
		M1	3.1a
		A1	1.1b
		M1	1.1b
		A1	1.1b
	(5)		
(d)	$P(B \cup C)' = 0.22 + 0.22 \text{ or } 1 - [0.56]$ or $1 - [0.13 + 0.23 + 0.09 + 0.11]$	M1	1.1b
	$= 0.44$	A1	1.1b
		(2)	
(10 marks)			
Notes:			
(a)			
M1: for a correct ratio of probabilities formula and at least one correct value.			
A1: a correct answer			
(b)			
for a fully correct explanation: correct probabilities and correct comparisons.			
(c)			
B1: for box with B intersecting A and C but C not intersecting A . (Or accept three intersecting circles, but with zeros entered for $A \cap C$ and $A \cap B \cap C$) No box is B0			
M1: for method for finding $P(B \cap C)$			
A1: for 0.09			
M1: for 0.13 and their 0.09 in correct places and method for their 0.23			
A1: fully correct			
(d)			
M1: for a correct expression – fit their probabilities from their Venn diagram.			
A1: cao			

Question	Scheme	Marks	AOs
5 (a)	The seeds would be destroyed in the process so they would have none to sell	B1	2.4
		(1)	
(b)	[$S = \text{no. of seeds out of 24 that germinate, } S \sim B(24, 0.55)$]		
	$T = \text{no. of trays with at least 15 germinating. } T \sim B(10, p)$	M1	3.3
	$p = P(S \geq 15) = 0.299126\dots$	A1	1.1b
	So $P(T \geq 5) = 0.1487\dots$ awrt <u>0.149</u>	A1	1.1b
		(3)	
(c)	n is large and p close to 0.5	B1	1.2
		(1)	
(d)	$X \sim N(132, 59.4)$	B1	3.4
	$P(X \geq 149.5) = P\left(Z \geq \frac{149.5 - 132}{\sqrt{59.4}}\right)$	M1	1.1b
	$= 0.01158\dots$ awrt <u>0.0116</u>	A1cso	1.1b
		(3)	
(e)	e.g The probability is very small therefore there is evidence that the company's claim is incorrect.	B1	2.2b
		(1)	
(9 marks)			
Notes:			
(a) B1: cao			
(b) M1: for selection of an appropriate model for T 1st A1: for a correct value of the parameter p (accept 0.3 or better) 2nd A1: for awrt 0.149			
(c) B1: both correct conditions			
(d) B1: for correct normal distribution M1: for correct use of continuity correction A1: cso			
(e) B1: correct statement			

Question	Scheme	Marks	AOs
6	Integrate \mathbf{a} w.r.t. time	M1	1.1a
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of \mathbf{C})	A1	1.1b
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s^{-1}	A1ft	1.1b
			(6 marks)
Notes:			
<p>1st M1: for integrating \mathbf{a} w.r.t. time (powers of t increasing by 1)</p> <p>1st A1: for a correct \mathbf{v} expression without \mathbf{C}</p> <p>2nd A1: for a correct \mathbf{v} expression including \mathbf{C}</p> <p>2nd M1: for putting $t = 4$ into their \mathbf{v} expression</p> <p>3rd M1: for finding magnitude of their \mathbf{v}</p> <p>3rd A1: ft for 100 m s^{-1}, follow through on an incorrect \mathbf{v}</p>			

Question	Scheme	Marks	AOs
7(a)	$R = mg\cos\alpha$	B1	3.1b
	Resolve parallel to the plane	M1	3.1b
	$-F - mg\sin\alpha = -0.8mg$	A1	1.1b
	$F = \mu R$	M1	1.2
	Produce an equation in μ only and solve for μ	M1	2.2a
	$\mu = \frac{1}{4}$	A1	1.1b
		(6)	
(b)	Compare $\mu mg\cos\alpha$ with $mg\sin\alpha$	M1	3.1b
	Deduce an appropriate conclusion	A1 ft	2.2a
		(2)	
			(8 marks)
Notes:			
(a)			
B1: for $R = mg\cos\alpha$			
1st M1: for resolving parallel to the plane			
1st A1: for a correct equation			
2nd M1: for use of $F = \mu R$			
3rd M1: for eliminating F and R to give a value for μ			
2nd A1: for $\mu = \frac{1}{4}$			
(b)			
M1: comparing size of limiting friction with weight component down the plane			
A1ft: for an appropriate conclusion from their values			

Question	Scheme	Marks	AOs
8(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{at}$: $(10.5\mathbf{i} - 0.9\mathbf{j}) = 0.6\mathbf{j} + 15\mathbf{a}$	M1	3.1b
	$\mathbf{a} = (0.7\mathbf{i} - 0.1\mathbf{j}) \text{ m s}^{-2}$ Given answer	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r} = \mathbf{ut} + \frac{1}{2} \mathbf{at}^2$	M1	3.1b
	$\mathbf{r} = 0.6\mathbf{j} t + \frac{1}{2}(0.7\mathbf{i} - 0.1\mathbf{j}) t^2$	A1	1.1b
		(2)	
(c)	Equating the i and j components of r	M1	3.1b
	$\frac{1}{2} \leftarrow 0.7 t^2 = 0.6 t - \frac{1}{2} \leftarrow 0.1 t^2$	A1ft	1.1b
	$t = 1.5$	A1	1.1b
		(3)	
(d)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{at}$: $\mathbf{v} = 0.6\mathbf{j} + (0.7\mathbf{i} - 0.1\mathbf{j}) t$	M1	3.1b
	Equating the i and j components of v	M1	3.1b
	$t = 0.75$	A1 ft	1.1b
		(3)	
			(10 marks)
Notes:			
(a)			
M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{at}$			
A1: for given answer correctly obtained			
(b)			
M1: for use of $\mathbf{r} = \mathbf{ut} + \frac{1}{2} \mathbf{at}^2$			
A1: for a correct expression for r in terms of <i>t</i>			
(c)			
M1: for equating the i and j components of their r			
A1ft: for a correct equation following their r			
A1: for $t = 1.5$			
(d)			
M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{at}$ for a general <i>t</i>			
M1: for equating the i and j components of their v			
A1ft: for $t = 0.75$, or a correct follow through answer from an incorrect equation			

Question	Scheme	Marks	AOs
9(a)	Take moments about A (or any other complete method to produce an equation in S , W and α only)	M1	3.3
	$W \cos \alpha + 7W \cos \alpha = S \sin \alpha$	A1 A1	1.1b 1.1b
	Use of $\tan \alpha = \frac{5}{2}$ to obtain S	M1	2.1
	$S = 3W$ *	A1*	2.2a
		(5)	
(b)	$R = 8W$	B1	3.4
	$F = \frac{1}{4} R (= 2W)$	M1	3.4
	$P_{\text{MAX}} = 3W + F$ or $P_{\text{MIN}} = 3W - F$	M1	3.4
	$P_{\text{MAX}} = 5W$ or $P_{\text{MIN}} = W$	A1	1.1b
	$W \leq P \leq 5W$	A1	2.5
		(5)	
(c)	M(A) shows that the reaction on the ladder at B is unchanged	M1	2.4
	also R increases (resolving vertically)	M1	2.4
	which increases max F available	M1	2.4
		(3)	
			(13 marks)

Question 9 continued**Notes:****(a)****1st M1:** for producing an equation in S , W and α only**1st A1:** for an equation that is correct, or which has one error or omission**2nd A1:** for a fully correct equation**2nd M1:** for use of $\tan \alpha = \frac{5}{2}$ to obtain S in terms of W only**3rd A1*:** for given answer $S = 3W$ correctly obtained**(b)****B1:** for $R = 8W$ **1st M1:** for use of $F = \frac{1}{4} R$ **2nd M1:** for either $P = (3W + \text{their } F)$ or $P = (3W - \text{their } F)$ **1st A1:** for a correct max or min value for a correct range for P **2nd A1:** for a correct range for P **(c)****1st M1:** for showing, by taking moments about A , that the reaction at B is unchanged by the builder's assistant standing on the bottom of the ladder**2nd M1:** for showing, by resolving vertically, that R increases as a result of the builder's assistant standing on the bottom of the ladder**3rd M1:** for concluding that this increases the limiting friction at A

Question	Scheme	Marks	AOs
10(a)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$36 = U t \cos \alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-18 = U t \sin \alpha - \frac{1}{2}gt^2$	A1	1.1b
	Correct strategy for solving the problem by setting up two equations in t and U and solving for U	M1	3.1b
	$U = 15$	A1	1.1b
		(6)	
(b)	Using the model and horizontal motion: $U \cos \alpha$ (12)	B1	3.4
	Using the model and vertical motion: $v^2 = (U \sin \alpha)^2 + 2(-10)(-7.2)$	M1	3.4
	$v = 15$	A1	1.1b
	Correct strategy for solving the problem by finding the horizontal and vertical components of velocity and combining using Pythagoras: Speed = $\sqrt{(12^2 + 15^2)}$	M1	3.1b
	$\sqrt{369} = 19 \text{ m s}^{-1}$ (2sf)	A1 ft	1.1b
		(5)	
(c)	Possible improvement (see below in notes)	B1	3.5c
	Possible improvement (see below in notes)	B1	3.5c
		(2)	
			(13 marks)

Question 10 continued**Notes:****(a)****1st M1:** for use of $s = ut$ horizontally**1st A1:** for a correct equation**2nd M1:** for use of $s = ut + \frac{1}{2}at^2$ vertically**2nd A1:** for a correct equation**3rd M1:** for correct strategy (need both equations)**2nd A1:** for $U = 15$ **(b)****B1:** for $U\cos\alpha$ used as horizontal velocity component**1st M1:** for attempt to find vertical component**1st A1:** for 15**2nd M1:** for correct strategy (need both components)**2nd A1ft:** for 19 m s^{-1} (2sf) following through on incorrect component(s)**(c)****B1, B1:** for any two of

e.g. Include air resistance in the model of the motion

e.g. Use a more accurate value for g in the model of the motion

e.g. Include wind effects in the model of the motion

e.g. Include the dimensions of the stone in the model of the motion