



GCE A LEVEL MARKING SCHEME

SUMMER 2023

**A LEVEL
MATHEMATICS
UNIT 3 PURE MATHEMATICS B
1300U30-1**

INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE A LEVEL MATHEMATICS
UNIT 3 PURE MATHEMATICS B
SUMMER 2023 MARK SCHEME

Q Solution	Mark Notes
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1 Let the first term = a

Let the common difference = d

$$a + (12 - 1)d = 41 \quad \text{B1 oe}$$

$$\frac{16}{2}(2a + (16 - 1)d) = 488 \quad \text{B1 oe}$$

$$2a + 15d = 61$$

$$2a + 22d = 82$$

$$7d = 21 \quad \text{M1 a valid attempt to eliminate one variable, FT their linear equations}$$

$$d = 3 \quad \text{A1 cao}$$

$$a (= 41 - 33) = 8 \quad \text{A1 cao}$$

Q Solution**Mark Notes**

$$\begin{aligned}2(a)(i) \quad & 5(\sin x + x^2)^4 f(x) \\& = 5(\sin x + x^2)^4 (\cos x + 2x)\end{aligned}$$

M1 $f(x) \neq 0, 1$, condone lack of brackets round $f(x)$
A1 brackets required in $(\cos x + 2x)$, ISW

$$\begin{aligned}2(a)(ii) \quad & x^3 f(x) + g(x) \cos x \\& = x^3(-\sin x) + (3x^2) \cos x\end{aligned}$$

M1 $f(x), g(x) \neq 0, 1$
A1 ISW

$$\begin{aligned}2(a)(iii) \quad & \frac{\sin 2x(f(x)) - e^{3x}(g(x))}{\sin^2 2x} \\& = \frac{\sin 2x(3e^{3x}) - e^{3x}(2\cos 2x)}{\sin^2 2x}\end{aligned}$$

M1 $f(x), g(x) \neq 0, 1$
A1 $f(x) = 3e^{3x}$
A1 $g(x) = 2\cos 2x$,
ISW

OR

$$y = e^{3x}(\sin 2x)^{-1}$$

$$y = (f(x))(\sin 2x)^{-1} + e^{3x}(g(x)) \quad (\text{M1}) \quad f(x), g(x) \neq 0, 1$$

$$\frac{dy}{dx} = 3e^{3x}(\sin 2x)^{-1} + e^{3x}(-1)(\sin 2x)^{-2}(2\cos 2x) \quad (\text{A1}) \quad f(x) = 3e^{3x}$$

$$(\text{A1}) \quad g(x) = (-1)(\sin 2x)^{-2}(2\cos 2x), \quad \text{ISW}$$

Q Solution**Mark Notes**

2(b) $8y \frac{dy}{dx} - (7x \frac{dy}{dx} + 7y) + 2x = 0$

B1 $8y \frac{dy}{dx}$

B1 $(7x \frac{dy}{dx} + 7y)$ or $7x \frac{dy}{dx} \pm 7y$

B1 $2x$ and 0

$$(8y - 7x) \frac{dy}{dx} = (7y - 2x)$$

$$\frac{dy}{dx} = \frac{7y - 2x}{8y - 7x}$$

At $(2, 4)$, $\frac{dy}{dx} = \frac{28 - 4}{32 - 14} \left(= \frac{24}{18} = \frac{4}{3} \right)$

M1 oe, correctly substitute values for x and y , FT their dy/dx for equivalent expression

Gradient of tangent at $(2, 4) = \frac{4}{3}$

m1 si

Equation of tangent is

$$y - 4 = \frac{4}{3}(x - 2)$$

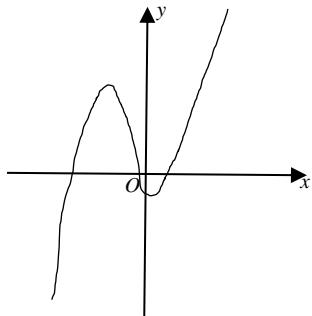
A1 oe, cao, ISW

$$3y = 4x + 4$$

Q	Solution	Mark Notes
3(a)	$\frac{9}{(1-x)(1+2x)^2} \equiv \frac{A}{(1-x)} + \frac{B}{(1+2x)} + \frac{C}{(1+2x)^2}$ M1	
	$9 = A(1+2x)^2 + B(1-x)(1+2x) + C(1-x)$ m1 oe	
	Put $x = 1$	
	$9 = 9A, A = 1$	A1 one correct coefficient
	Put $x = -\frac{1}{2}$,	
	$9 = \frac{3}{2}C, C = 6$	
	Coefficient $x^2, 0 = 4A - 2B, B = 2$	A1 all 3 correct
	$\frac{9}{(1-x)(1+2x)^2} \equiv \frac{1}{(1-x)} + \frac{2}{(1+2x)} + \frac{6}{(1+2x)^2}$	
3(b)	$(1-x)^{-1} = 1 + x + \frac{(-1)(-2)}{2}(-x)^2 + \dots$	B1 si
	$= 1 + x + x^2 + \dots$	
	$(1+2x)^{-1} = 1 - 2x + \frac{(-1)(-2)}{2}(2x)^2 + \dots$	B1 si
	$= 1 - 2x + 4x^2 + \dots$	
	$(1+2x)^{-2} = 1 - 4x + \frac{(-2)(-3)}{2}(2x)^2 + \dots$	B1 si
	$= 1 - 4x + 12x^2 + \dots$	
	$(1-x)^{-1} + 2(1+2x)^{-1} + 6(1+2x)^{-2}$	
	$= (1+x+x^2) + 2(1-2x+4x^2)$	
	$+ 6(1-4x+12x^2) + \dots$	M1 Adding candidate's 2 or 3 series, with their A, B, C
	$= 9 - 27x + 81x^2 + \dots$	A1 2 correct terms cao
		A1 all 3 correct cao, ISW
	Expansion is valid when $ x < \frac{1}{2}$	B1 $-\frac{1}{2} < x < \frac{1}{2}$ Mark final answer.

Q Solution**Mark Notes**

4(a)



$$f(-1) = 30, f(0) = -6, f(1) = 28$$

There are two roots in $[-1, 1]$.

M1 looking at signs of $f(x)$ in $[-1, 1]$ for at least 2 values, or attempt to find roots.
(Correct roots are $-6, -\frac{1}{3}, \frac{1}{2}$).

Or sketch of graph between $x = -1$ and $x = 1$

A1 2 changes of signs or roots $-\frac{1}{3}, \frac{1}{2}$

Award M1A1 for stating '2 roots', provided not based on incorrect mathematics,
e.g. $30 - 28 = 2$ roots M1A0

Q Solution**Mark Notes**

4(b)(i) $f'(x) = 18x^2 + 70x - 7$

B1 seen anywhere

$$x_{n+1} = x_n - \frac{6x_n^3 + 35x_n^2 - 7x_n - 6}{18x_n^2 + 70x_n - 7}$$

M1 si

$$x_0 = 1$$

$$x_1 = \frac{53}{81} = 0.6543209877$$

A1

4(b)(ii) ($x_2 = 0.5234785163$)

$$(x_3 = 0.5007059775)$$

$$(x_4 = 0.5000006736)$$

$$\text{root} = 0.5$$

A1

4(c) e.g. $x_1 = 2.049390153 \left(= \frac{\sqrt{105}}{5} \right)$

B1 si

$$7x_1 + 6 - 6x_1^3 = -31.29890079 < 0,$$

E1 $7x_1 + 6 - 6x_1^3 < 0$

or $\frac{7x_1 + 6 - 6x_1^3}{35} = -0.89\dots < 0$

or $\frac{7x_1 + 6 - 6x_1^3}{35} < 0$

so the square root will not be real.

Hence the method cannot be used
to find a root of $f(x) = 0$.If no values seen, award B1E1 provided
reference has been made to x_2 , explicitly or
implicitly

Q	Solution	Mark Notes
5(a)	Amount of growth after 10 years $= 32 + 32(0.9) + 32(0.9)^2 + \dots + 32(0.9)^9$	B1 si, GP $a = 32$, $r = 0.9$, or at least 3 terms
	$= 32(1 + 0.9 + 0.9^2 + \dots + 0.9^9)$	
	$= 32\left(\frac{1(1 - 0.9^{10})}{1 - 0.9}\right)$	M1 correct use of correct formula, for their GP provided $r = 0.9$. Award M1 for $(A)\left(\frac{1(1 - 0.9^9)}{1 - 0.9}\right)$ or $(A)\left(\frac{1(1 - 0.9^{11})}{1 - 0.9}\right)$
	$= 32 \times 6.5132 = 208.42\dots$	A1 si, Award A1 for 521.06 ($A = 80$, $n = 10$) or 729.48 ($A = 112$, $n = 10$)
	Height after 10 years = $80 + 208.42$	
	Height after 10 years = $288(.42)$ (cm)	A1 oe, cao
5(b)	Maximum growth of tree = $32\left(\frac{1}{1 - 0.9}\right)$	M1 correct use of correct formula $(A)\left(\frac{1}{1 - 0.9}\right)$ gets M1
	(Maximum growth of tree = 320)	
	Maximum height of tree = 400 (cm)	A1 oe, cao

Q Solution**Mark Notes**

6(a) $\cos 75^\circ = \cos(30^\circ + 45^\circ)$ M1

$$= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}} \quad \text{A1 oe}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4} \quad \text{A1 convincing}$$

Q Solution**Mark Notes**

6(b) $2\cot^2x + \operatorname{cosec}x = 4$

$2(\operatorname{cosec}^2x - 1) + \operatorname{cosec}x = 4$

M1 $\cot^2x + 1 = \operatorname{cosec}^2x$

$2 \operatorname{cosec}^2x + \operatorname{cosec}x - 6 = 0$

$(2\operatorname{cosec}x - 3)(\operatorname{cosec}x + 2) = 0$

$\operatorname{cosec}x = \frac{3}{2}, -2$

A1

$\sin x = \frac{2}{3}$

B1 or $\sin x = -\frac{1}{2}$

$x = 41.81^\circ, 138.19^\circ$

B1 $0.7297^\circ, 2.4119^\circ$

$\sin x = -\frac{1}{2},$

$x = 210^\circ$

B1 $\frac{7\pi}{6}$

$x = 330^\circ$

B1 $\frac{11\pi}{6}$

OR $2\frac{\cos^2x}{\sin^2x} + \frac{1}{\sin x} = 4$

(B1)

$2(1 - \sin^2x) + \sin x = 4\sin^2x$

(M1) use of $\sin^2x + \cos^2x = 1$

$6\sin^2x - \sin x - 2 = 0$

$(3\sin x - 2)(2\sin x + 1) = 0$

$\sin x = \frac{2}{3}, -\frac{1}{2}$

(A1)

$x = 41.81^\circ, 138.19^\circ$

(B1) $0.7297^\circ, 2.4119^\circ$

$x = 210^\circ$

(B1) $\frac{7\pi}{6}$

$x = 330^\circ$

(B1) $\frac{11\pi}{6}$

NOTES

Mark each branch separately.

FT 2 branches only if different signs.

Do not FT for other trig functions.

For each branch, -1 for a 3_{rd} root in the range $0^\circ < \theta < 360^\circ$,
 -1 for a 4_{th} root in the range $0^\circ < \theta < 360^\circ$.

Ignore roots outside the range $0^\circ < \theta < 360^\circ$.

Q Solution**Mark Notes**

6(c)(i) $R\cos(\theta + \alpha) = 7\cos\theta - 24\sin\theta$

$$R\cos\theta\cos\alpha - R\sin\theta\sin\alpha \equiv 7\cos\theta - 24\sin\theta$$

$$R\cos\alpha = 7, R\sin\alpha = 24$$

M1 si, must be from correct identity
M0 if $\cos\alpha = 7, \sin\alpha = 24$

$$R = \sqrt{7^2 + 24^2} = 25$$

B1

$$\alpha = \tan^{-1}\left(\frac{24}{7}\right) = 73.74^\circ$$

A1

$$7\cos\theta - 24\sin\theta \equiv 25\cos(\theta + 73.74^\circ)$$

6(c)(ii) $25\cos(\theta + 73.74) = 5$

$$\cos(\theta + 73.74) = \frac{5}{25}$$

M1 ft similar expressions if possible.

$$\theta + 73.74 = \cos^{-1}(0.2)$$

$$\theta + 73.74 = 78.46, 281.54$$

$$\theta = 4.72^\circ,$$

A1 FT for one angle only

$$\theta = 207.80^\circ$$

A1

NOTES

Do not FT for other trig functions.

-1 for a 3_{rd} root in the range $0^\circ < \theta < 360^\circ$,

-1 for a 4_{th} root in the range $0^\circ < \theta < 360^\circ$.

Ignore roots outside the range $0^\circ < \theta < 360^\circ$.

Q Solution**Mark Notes**

7(a) $5x - 3 = 2x + 3$

M1

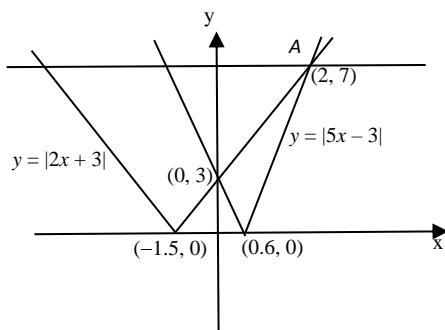
$3x = 6$

$x = 2, y = 7$

A1 AG

allow verification (both lines for M1)

7(b)

B1 correct sketch $y = |2x + 3|$ B1 correct sketch $y = |5x - 3|$ B1 $(-1.5, 0), (0.6, 0),$ B1 $(0, 3), (2, 7)$

7(c) Area = triangle in first quadrant

M1 identify correct area, allow shown on diagram,
Only FT if graphs are in 1st and 2nd quadrants $= \text{trapezium} - 2 \text{ triangles}$

m1 method for area, oe

$= \frac{1}{2}(3 + 7) \times 2 - \frac{1}{2} \times 0.6 \times 3 - \frac{1}{2} \times (2 - 0.6) \times 7$

A1 correct expression, si

$= 10 - 0.9 - 4.9$

$= 4.2$

A1 cao

Q Solution**Mark Notes**

$$8(a) \quad f(x) = \frac{(4x^2 + 12x + 9)}{(2x^2 + x - 3)}$$

$$= \frac{(2x+3)(2x+3)}{(2x+3)(x-1)}$$

$$= \frac{2x+3}{x-1} \left(= \frac{2(x-1)+2+3}{x-1} \right)$$

$$= 2 + \frac{5}{x-1}$$

B1 correct factorisation of one quadratic

B1 common factor cancelled

B1 convincing

OR

$$f(x) = \frac{4x^2 + 12x + 9}{2x^2 + x - 3} = \frac{2(2x^2 + x - 3) + 10x + 15}{2x^2 + x - 3}$$

$$= 2 + \frac{10x + 15}{2x^2 + x - 3} \quad (\text{B1})$$

$$= 2 + \frac{5(2x+3)}{(2x+3)(x-1)} \quad (\text{B1}) \quad \text{correct factorisation of denominator}$$

$$= 2 + \frac{5}{x-1} \quad (\text{B1}) \quad \text{common factor cancelled}$$

$$8(b) \quad \int_3^7 f(x) dx = \int_3^7 \left(2 + \frac{5}{x-1} \right) dx$$

M1

$$= [2x + 5\ln|x-1|]_3^7$$

B1 correct integration, condone omission of modulus signs

$$= (14 + 5\ln 6) - (6 + 5\ln 2)$$

m1 correct use of limits

$$= 8 + 5\ln 3$$

A1 cao. Note $5\ln 3 = \ln 243$
M0 for 13.493 not supported by workings.

Q Solution**Mark Notes**

9(a) $y = \frac{1}{4}\sqrt{144 - 9x^2}$

M1 Allow $y = \sqrt{\frac{144-9x^2}{16}}$,
 $4y = \sqrt{144 - 9x^2}$

Area = $\int_0^4 \frac{1}{4}\sqrt{144 - 9x^2} dx$

m1 correct integral, si

Volume = $V = 0.06 \int_0^4 \frac{1}{4}\sqrt{144 - 9x^2} dx$

Volume = $V = 0.015 \int_0^4 \sqrt{144 - 9x^2} dx$

A1 AG, convincing

9(b) x $f(x) = \sqrt{144 - 9x^2}$

$(f(x) = 0.015\sqrt{144 - 9x^2})$

0 12 (0.18)

0.8 $11.757 = \left(\frac{24\sqrt{6}}{5}\right)$ (0.1763632615)

1.6 $10.998 = \left(\frac{12\sqrt{21}}{5}\right)$ (0.164972725)

2.4 $9.6 = \left(\frac{48}{5}\right)$ (0.144)

3.2 $7.2 = \left(\frac{36}{5}\right)$ (0.108)

4 0 (0)

B1 if any 2 terms correct

B1 all terms correct, si

$I = \frac{0.8}{2}[12 + 2(11.757 + 10.998 + 9.600$

+ 7.200) + 0]

M1 correct formula used

$I = 0.4 \times 91.111 = 36.444$

$V = 0.015 \times 36.444 = 0.5467 \text{ (m}^3\text{)}$

A1 Accept 0.547, 0.55, but not 0.5

9(c) The answer is an underestimate AND reason,

e.g. the trapeziums are all under the curve. B1 curve concave

Q Solution**Mark Notes**

10(a)(i)
$$fg(x) = f((x - 2)^2)$$

M1 si

$$= \frac{8}{(x - 2)^2 - 4}$$

A1 Mark final answer

10(a)(ii) $fg(x)$ does not exist when $g(x) = 4$ M1 si, FT their $fg(x)$

$$(x - 2)^2 = 4$$

$$x - 2 = (\pm) 2$$

$$x = 4$$

A1

$$x = 0$$

A1

SC1 for unsupported answer of $x = 4$ only

10(b) $y = \frac{8}{x - 4}$

M1

$$xy - 4y = 8, \quad x = \frac{8 + 4y}{y}$$

A1 put x as subject, or y as subject if x and y interchanged earlier. Allow one slip

$$f^{-1}(x) = \frac{8 + 4x}{x} \text{ or } f^{-1}(x) = \frac{8}{x} + 4$$

A1 cao, ' $f^{-1}(x)$ ' required

Q Solution**Mark Notes**

11(a) $f'(x) = 15x^2 + 4x - 3 (= (5x + 3)(3x - 1))$ B1

$f''(x) = 30x + 4$ B1 FT their $f'(x)$

When $f''(x) = 0$ M1 used

$30x + 4 = 0$

$x = -\frac{2}{15}$ A1 cao, condone $-\frac{4}{30}$

[$f''(x) < 0$ if $x < -\frac{2}{15}$, $f''(x) > 0$ if $x > -\frac{2}{15}$, so $x = -\frac{2}{15}$ is the x -coordinate of a point of inflection.]

Valid reason

Eg. $f'\left(-\frac{2}{15}\right) \left(= -\frac{49}{15}\right) \neq 0$ or stationary points only at $x = -\frac{3}{5}, \frac{1}{3}$

AND Non-stationary point of inflection. B1 FT their x value

11(b) If C is concave, $f''(x) < 0$. M1 oe si allow \leq

$30x + 4 < 0$

$x < -\frac{2}{15}$ A1 condone $x < -\frac{4}{30}$, allow \leq

FT linear $f''(x)$

Q Solution

12(a) $\frac{dy}{dx} = ky$

Mark Notes

B1 allow $\frac{dy}{dx} = -ky$, allow $\frac{dy}{dx} = 4y$

Award if seen in (b).

12(b) $x = 1, y = 0.5, \frac{dy}{dx} = 2$

$2 = 0.5k, k = 4$

B1

$\frac{dy}{dx} = 4y$

$\int \frac{dy}{y} = \int 4 dx$

M1 Separation of variables

$\ln|y| = 4x (+ C)$

A1

$\ln|0.5| = 4 + C$

m1 Use of conditions

$C = \ln|0.5| - 4 (= -4.693\dots)$

When $x = 3$,

$\ln|y| = 4 \times 3 + \ln(0.5) - 4 (= 7.3068\dots)$

m1 $\ln|y| - \ln(0.5) = 8, 2y = e^8$

$y = 0.5e^8 = 1490(.478\dots)$

A1 Condone values of y that round to 1491

OR

$\int \frac{dy}{y} = \int k dx$

(M1) Separation of variables

$\ln|y| = kx + C$

(A1)

$y = Ae^{kx}$

$x = 1, y = 0.5, \frac{dy}{dx} = 2; 2 = k \times 0.5; k = 4$

(B1)

$0.5 = Ae^4$

(m1) Use of conditions

$A = 0.5e^{-4}$

When $x = 3, y = 0.5e^{-4} \times e^{12}$

(m1)

$y = 0.5e^8 = 1490(.478\dots)$

(A1) Condone values of y that round to 1491

Q	Solution	Mark Notes
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13	Attempt to eliminate p or q for C_1 or C_2	M1 si
	$C_1: \quad y = (x - 1)^2$	A1 any correct form ISW
	$C_2: \quad 2y = x$	A1 any correct form ISW
Graphs meet when		
	$2(x - 1)^2 = x \quad \text{or} \quad y = (2y - 1)^2$	m1
	$2x^2 - 5x + 2 = 0 \quad \text{or} \quad 4y^2 - 5y + 1 = 0$	
	$(2x - 1)(x - 2) = 0 \quad \text{or} \quad (4y - 1)(y - 1) = 0$	m1 si $ax^2 + bx + c = (dx + e)(fx + g)$ $df = a$ and $eg = c$
	$x = \frac{1}{2}, 2 \quad \text{or} \quad y = \frac{1}{4}, 1$	A1 cao, One correct pair
	$y = \frac{1}{4}, 1 \quad \text{or} \quad x = \frac{1}{2}, 2$	A1 cao, All correct

Points of intersection are $\left(\frac{1}{2}, \frac{1}{4}\right), (2, 1)$

Q	Solution	Mark Notes
13.	OR	
	Graphs meet when x and y coordinates for the two curves are equal	(M1) si
	$3p + 1 = 4q$	
	$9p^2 = 2q$	(A1) at least one correct equation
	Solving simultaneously	(m1) one variable eliminated
	$3p + 1 = 2 \times 9p^2$ or $(4q - 1)^2 = 2q$	
	$18p^2 - 3p - 1 = 0$ or $16q^2 - 10q + 1 = 0$	
	$(6p + 1)(3p - 1) = 0$ or $(8q - 1)(2q - 1) = 0$	(m1) attempt to solve quadratic equation
		$ax^2 + bx + c = (dx + e)(fx + g)$ $df = a$ and $eg = c$

$$p = -\frac{1}{6}, \frac{1}{3} \quad \text{or} \quad q = \frac{1}{8}, \frac{1}{2} \quad (\text{A1}) \quad \text{cao}$$

When $p = -\frac{1}{6}$ or $q = \frac{1}{8}$, point is $\left(\frac{1}{2}, \frac{1}{4}\right)$ (A1) cao, $x = \frac{1}{2}$, $x = 2$ or $y = \frac{1}{4}$, $y = 1$

When $p = \frac{1}{3}$ or $q = \frac{1}{2}$, point is $(2, 1)$ (A1) cao, All correct

Q Solution**Mark Notes**

$$\begin{aligned}
 14(a) \quad & \int_0^1 ((3x - 1)e^{2x}) dx \\
 &= [(3x - 1)Ae^{2x}]_0^1 - \int_0^1 Ae^{2x} \times 3 dx \\
 &= \left[(3x - 1) \frac{e^{2x}}{2} \right]_0^1 - \int_0^1 \frac{e^{2x}}{2} \times 3 dx \\
 &= \left[(3x - 1) \frac{e^{2x}}{2} \right]_0^1 - \left[\frac{3}{4} e^{2x} \right]_0^1 \\
 &= \left(e^2 + \frac{1}{2} \right) - \left(\frac{3e^2}{4} - \frac{3}{4} \right) \\
 &= \frac{1}{4}e^2 + \frac{5}{4} = 3.097(264025)
 \end{aligned}$$

M1 attempt at integration by parts, at least one term correct. Limits not required.
 Only allow $A = 2$ or $\frac{1}{2}$
 A1 all correct
 m1 correct use of limits, evidence required
 A1 cao, at least 1dp

If M0, SC1 for

$$\left[\left(\frac{3x^2}{2} - x \right) e^{2x} \right]_0^1 - \int_0^1 \left(\frac{3x^2}{2} - x \right) 2e^{2x} dx$$

at least 1 correct term.
 No more marks available.

Note

No marks for answer unsupported by workings.

$$14(b) \quad u = 1 - 2\cos x$$

$$\begin{aligned}
 du = 2\sin x dx & \quad B1 \quad \frac{du}{dx} = 2\sin x \\
 \int \frac{\sin x}{1 - 2\cos x} dx &= \frac{1}{2} \int \frac{1}{u} du \quad M1 \\
 &= \frac{1}{2} \ln|u| (+ C) \quad A1 \quad \text{oe, allow if modulus sign missing} \\
 &= \frac{1}{2} \ln|1 - 2\cos x| + C \quad A1 \quad \text{oe, modulus sign and } + C \text{ required.}
 \end{aligned}$$