GCE

MATHEMATICS

UNIT 4: APPLIED MATHEMATICS B

SAMPLE ASSESSMENT MATERIALS

(1 hour 45 minutes)

SECTION A – Statistics

SECTION B – Differential Equations and Mechanics

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

• a 12 page answer book;
• a Formula Booklet;
• a calculator;
• statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Take g as 9.8 ms^{-2}.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

SECTION A – Statistics

1.   It is known that 4% of a population suffer from a certain disease. When a diagnostic test is applied to a person with the disease, it gives a positive response with probability 0.98. When the test is applied to a person who does not have the disease, it gives a positive response with probability 0.01.

   (a)   Using a tree diagram, or otherwise, show that the probability of a person who does not have the disease giving a negative response is 0.9504.   [2]

   The test is applied to a randomly selected member of the population.

   (b)   Find the probability that a positive response is obtained.   [2]

   (c)   Given that a positive response is obtained, find the probability that the person has the disease.   [2]

2.   Mary and Jeff are archers and one morning they play the following game. They shoot an arrow at a target alternately, starting with Mary. The winner is the first to hit the target. You may assume that, with each shot, Mary has a probability 0.25 of hitting the target and Jeff has a probability p of hitting the target. Successive shots are independent.

   (a)   Determine the probability that Jeff wins the game

   i) with his first shot,

   ii) with his second shot.   [4]

   (b)   Show that the probability that Jeff wins the game is   [3]

$$\frac{3p}{1+3p}$$

   (c)   Find the range of values of p for which Mary is more likely to win the game than Jeff.   [2]

3.   A string of length 60 cm is cut a random point.

   (a)   Name a distribution, including parameters, that can be used to model the length of the longer piece of string and find its mean and variance.   [3]

   (b)   The longer string is shaped to form the perimeter of a circle. Find the probability that the area of the circle is greater than 100 cm^2.   [4]

4. Automatic coin counting machines sort, count and batch coins. A particular brand of these machines rejects 2p coins that are less than 6.12 grams or greater than 8.12 grams.

    (a) The histogram represents the distribution of the weight of UK 2p coins supplied by the Royal Mint. This distribution has mean 7.12 grams and standard deviation 0.357 grams.

Weight of UK two pence coins
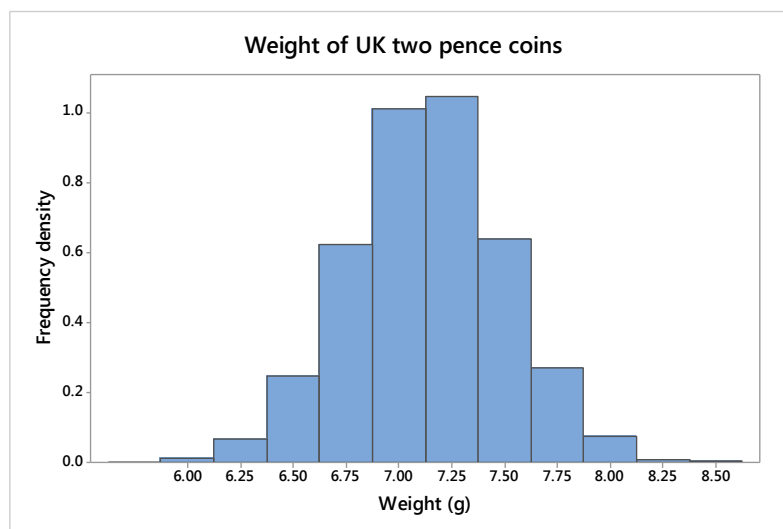
Explain why the weight of 2p coins can be modelled using a normal distribution.   [1]

    (b) Assume the distribution of the weight of 2p coins is normally distributed. Calculate the proportion of 2p coins that are rejected by this brand of coin counting machine.   [2]

    (c) A manager suspects that a large batch of 2p coins is counterfeit. A random sample of 30 of the suspect coins is selected. Each of the coins in the sample is weighed. The results are shown in the summary statistics table.

Summary statistics Weights (in grams) for a random sample of 30 UK 2p coins						
Mean	Standard deviation	Minimum	Lower quartile	Median	Upper quartile	Maximum
6.89	0.296	6.45	6.63	6.88	7.08	7.48

    i) What assumption must be made about the weights of coins in this batch in order to conduct a test of significance on the sample mean? State, with a reason, whether you think this assumption is reasonable.   [2]

    ii) Assuming the population standard deviation is 0.357 grams, test at the 1% significance level whether the mean weight of the 2p coins in this batch is less than 7.12 grams.   [6]

5. A hotel owner in Cardiff is interested in what factors hotel guests think are important when staying at a hotel. From a hotel booking website he collects the ratings for 'Cleanliness', 'Location', 'Comfort' and 'Value for money' for a random sample of 17 Cardiff hotels.

    (Each rating is the average of all scores awarded by guests who have contributed reviews using a scale from 1 to 10, where 10 is 'Excellent'.)

    The scatter graph shows the relationship between 'Value for money' and 'Cleanliness' for the sample of Cardiff hotels.


    Value for money versus Cleanliness for a random sample of 17 Cardiff hotels
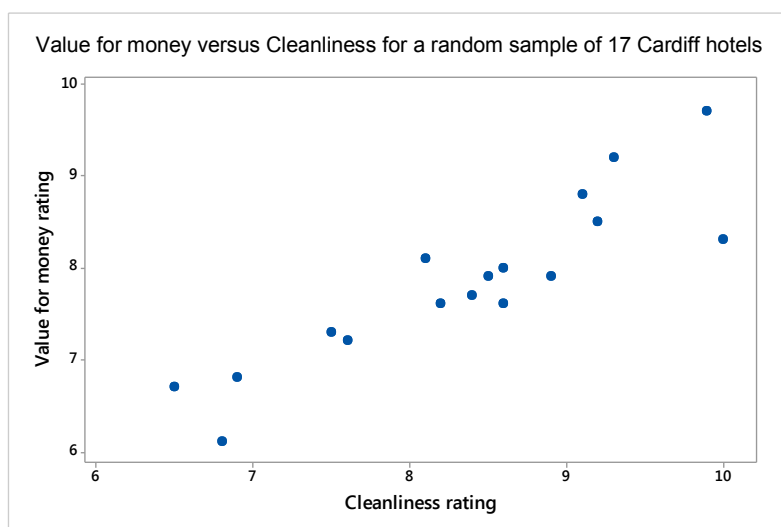
    (a) The product moment correlation coefficient for 'Value for money' and 'Cleanliness' for the sample of 17 Cardiff hotels is 0.895.
        Stating your hypotheses clearly, test, at the 5% level of significance, whether this correlation is significant. State your conclusion in context.   [5]

    (b) The hotel owner also wishes to investigate whether 'Value for money' has a significant correlation with 'Cost per night'. He used a statistical analysis package which provided the following output which includes the Pearson correlation coefficient of interest and the corresponding *p*-value.

	Value for money	Cost per night
Value for money	1	
Cost per night	0.047 (0.859)	1

    Comment on the correlation between 'Value for money' and 'Cost per night'.
                                                                        [2]

SECTION B – Differential Equations and Mechanics

6.   An object of mass 4 kg is moving on a horizontal plane under the action of a constant force $4\mathbf{i} - 12\mathbf{j}$ N. At time $t = 0$ s, its position vector is $7\mathbf{i} - 26\mathbf{j}$ with respect to the origin O and its velocity vector is $-\mathbf{i} + 4\mathbf{j}$.

(a)   Determine the velocity vector of the object at time $t = 5$ s.   [3]

(b)   Calculate the distance of the object from the origin when $t = 2$ s.   [5]

7.   The diagram below shows an object of weight 160 N at a point C, supported by two cables AC and BC inclined at angles of 23° and 40° to the horizontal respectively.
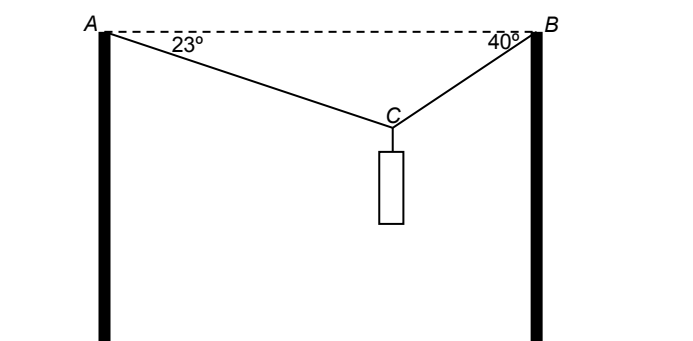
(a)   Find the tension in AC and the tension in BC.   [6]

(b)   State two modelling assumptions you have made in your solution.   [2]

8.   The rate of change of a population of a colony of bacteria is proportional to the size of the population P, with constant of proportionality k. At time $t = 0$ (hours), the size of the population is 10.

(a)   Find an expression, in terms of k, for P at time t.   [6]

(b)   Given that the population doubles after 1 hour, find the time required for the population to reach 1 million.   [3]

9.   A particle of mass 12 kg lies on a rough horizontal surface. The coefficient of friction between the particle and the surface is 0.8. The particle is at rest.
It is then subjected to a horizontal tractive force of magnitude 75 N.
Determine the magnitude of the frictional force acting on the particle, giving a reason for your answer.   [5]

10.   A body is projected at time $t = 0$ s from a point O with speed V ms^{-1} in a direction inclined at an angle of θ to the horizontal.

(a)   Write down expressions for the horizontal and vertical components x m and y m of its displacement from O at time t s.   [2]

(b)   Show that the range R m on a horizontal plane through the point of projection is given by

$$R = \frac{V^2}{g} \sin 2\theta$$

[3]

(c)   Given that the maximum range is 392 m, find, correct to one decimal place,

i)   the speed of projection,

ii)   the time of flight,

iii)   the maximum height attained.   [5]