AS

Further Mathematics

7366/2D – Discrete

Mark scheme

7366

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
dM	mark is dependent on one or more M marks and is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

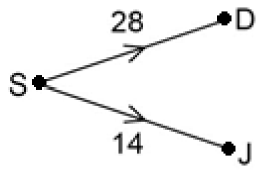
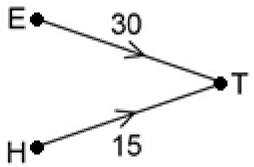
When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

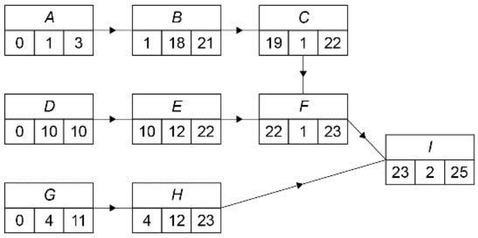
AS/A–Level Mathematics/Further Mathematics Assessment Objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Ticks correct box	AO1.1b	B1	Addition mod 4 and Multiplication mod 5
	Total		**1**	
2	Circles correct answer	AO1.1b	B1	0
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3(a)	Finds 4 or more correct row minima/col maxima	AO1.1a	M1	Row minima = $(2, 0, -2)$ Col maxima = $(2, 3, 5)$
	Correctly finds all row minima/col maxima	AO1.1b	A1	Max(row minima) = 2 Min(col maxima) = 2
	Explains correctly that as the max(row minima) is equal to the min(col maxima) then a stable solution exists and the value of the game is 2 Allow 'Maximin' and 'Minimax'	AO2.4	E1	As Max(row minima) = 2 = Min(col maxima), then a stable solution exists and the value of the game is 2
Q	**Marking Instructions**	**AO**	**Marks**	**Typical Solution**
3(a) ALT	Uses dominance to reduce the size of the pay-off matrix to 2×3 or 3×2	AO1.1a	M1	$\mathbf{A}_1$ (or $\mathbf{A}_2$) dominates $\mathbf{A}_3$, so remove $\mathbf{A}_3$ [from pay-off matrix]
	Uses dominance to reduce the pay-off matrix to 1×1 OR Correctly finds all remaining row minima/col maxima	AO1.1b	A1	$\mathbf{S}_1$ [now] dominates $\mathbf{S}_2$ and $\mathbf{S}_3$, so remove $\mathbf{S}_2$ and $\mathbf{S}_3$ $\mathbf{A}_1$ [now] dominates $\mathbf{A}_2$, so remove $\mathbf{A}_2$
	Explains that each player will play the same strategy each time which results in Alex gaining 2 each game OR Explains correctly that as the max(row minima) is equal to the min(col maxima) then a stable solution exists and the value of the game is 2 Allow 'Maximin' and 'Minimax'	AO2.4	E1	Alex will only ever play $\mathbf{A}_1$ and Sam will only ever play $\mathbf{S}_1$, which results in Alex gaining 2 each game. Therefore, the value of the game is 2.
3(b)	Identifies strategy for each player Condone stating '$\mathbf{A}_1$' and '$\mathbf{S}_1$'	AO1.1b	B1	Alex strategy $\mathbf{A}_1$ Sam strategy $\mathbf{S}_1$
	Total		**4**	

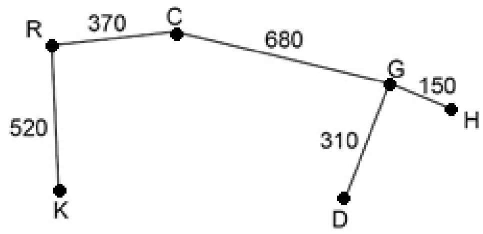
Q	Marking Instructions	AO	Marks	Typical Solution
4(a)(i)	Finds the value of the cut	AO1.1b	B1	37
4(a)(ii)	Deduces a weak inequality for the value of max flow based on their value of the cut Must see ≤ their value of the cut OE	AO2.2a	B1F	Max flow ≤ 37
4(b)(i)	Lists both sources CAO	AO1.1b	B1	D and J
4(b)(ii)	Adds supersource S with directed arcs to D and J labelled with weights at least 28 and at least 14 respectively	AO1.1b	B1	
4(c)(i)	Lists both sinks CAO	AO1.1b	B1	E and H
4(c)(ii)	Adds supersink T with directed arcs to E and H labelled with weights at least 30 and at least 15 respectively	AO1.1b	B1	
	Total		**6**	

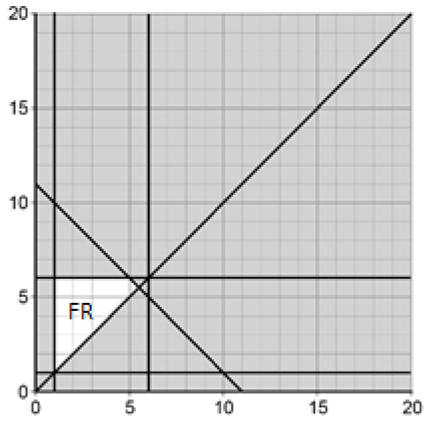
Q	Marking Instructions	AO	Marks	Typical Solution
5(a)(i)	Correctly completes precedence table	AO1.1b	B1	*D* *C*,*E* *G* *F*,*H*
5(a)(ii)	Correctly states earliest start times for all activities	AO1.1b	B1	
	Correctly states latest finish times for all activities	AO1.1b	B1	
5(b)(i)	States the correct activity (label or in words) from their completed activity network Only FT if their answer is *A*, *D* or *G*	AO1.1b	B1F	*D*, Chop vegetables
	Identifies their activity as critical OR States that the float of their activity is zero	AO2.5	E1F	*D* is the first activity on the critical path
5b(ii)	States their correct time using their completed activity network 6:30 pm + their minimum completion time Must see indication of pm OE	AO3.2a	B1F	6:55 pm

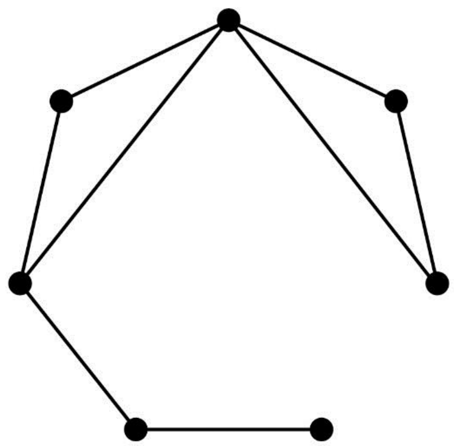
<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td>5(c)(i)</td><td>Completes precedence table 2 with correct durations

Condone correct durations in seconds</td><td>AO3.1b</td><td>B1</td><td><table><tr><th>Duration</th><th>Immediate Predecessors</th></tr><tr><td>10.25</td><td>–</td></tr><tr><td>8.25</td><td>J</td></tr><tr><td>0.5</td><td>K</td></tr></table></td></tr>
<tr><td rowspan="2">5(c)(ii)</td><td>Uses their minimum completion time and 19 in consistent units to find the float in the second project</td><td>AO1.1a</td><td>M1</td><td>25 – 19 = 6 minutes</td></tr>
<tr><td>States the correct time CAO

Condone omission of pm</td><td>AO1.1b</td><td>A1</td><td>6:36 pm</td></tr>
<tr><td></td><td>Total</td><td></td><td>9</td><td></td></tr>
</table>

Q	Marking Instructions	AO	Marks	Typical Solution
6	Sets up model and identifies one correct weight	AO3.3	M1	 Total distance = 2030 metres 2030×0.60 = £1218
	Uses model to identify at least 4 correct weights Must identify no more than 5 weights	AO3.4	M1	
	Identifies the correct minimum spanning tree for the network, stating clearly that it is a minimum spanning tree. Could be implied by clear evidence that Kruskal's algorithm or Prim's algorithm has been used.	AO1.1b	A1	
	Finds the correct total weight of the minimum spanning tree CAO PI	AO1.1b	A1	
	Uses their total weight to find their correct minimum cost with consistent unit	AO3.2a	A1F	
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)(i)	Draws $y = x$	AO1.1a	M1	
	Clearly labels (or shows by shading) the correct feasible region	AO1.1b	A1	
7(a)(ii)	Calculates the correct value of $5x + 4y$ at $(5, 6)$ or $(5.5, 5.5)$	AO1.1a	M1	Value at $(5, 6) = 49$ Value at $(5.5, 5.5) = 49.5$
	Finds the correct maximum value of $5x + 4y$	AO1.1b	A1	Maximum value is 49.5
7(b)(i)	Explains the implication of the graph being connected Must refer explicitly to 'connected'	AO1.2	B1	The graph is connected, so no vertex can have degree 0.
7(b)(ii)	Explains the implication of the graph being simple Must refer explicitly to 'simple'	AO1.2	B1	The graph is simple, so each vertex can only connect at most once to each of the 6 other vertices.
7(b)(iii)	Explains why x and y cannot both be 6	AO2.4	E1	If x and y are both 6 then there would be two vertices that are both adjacent to the other five vertices. However, this would mean that there would be no vertex of degree 1 so one of x or y has to be at most 5. Hence $x + y \leq 5 + 6$
	Completes rigorous argument for $x + y \leq 11$	AO2.1	R1	
7(b)(iv)	Identifies the need for integer solutions	AO3.5b	B1	x and y must both be integers

Q	Marking Instructions	AO	Marks	Typical Solution
7(c)(i)	Uses degree sum = 2×8 to deduce $x + y$	AO2.2a	M1	$1 + 2 + 3 + v + w + x + y = 2 \times 8$ $16 - (1 + 2 + 3 + 4) = x + y$ $x + y = 6$ $x = 1, \quad y = 5;$ $x = 2, \quad y = 4;$ $x = 3, \quad y = 3.$
	Infers there are multiple solutions and finds two correct feasible pairs of x and y values	AO2.2b	M1	
	Finds all three pairs and no others Condone pairs written as coordinates	AO1.1b	A1	
7(c)(ii)	Starts to show configuration of G by drawing a connected graph with exactly 8 edges OR by listing/labelling the vertex degrees: 1, 2, 2, 2, 2, 3, 4	AO3.1a	M1	
	Draws a correct graph with degrees: 1, 2, 2, 2, 2, 3, 4 NB Multiple correct solutions, check by counting degrees	AO3.2a	A1	
	Total		**14**	
	TOTAL		**40**	