AS
FURTHER MATHEMATICS
7366/2D

Paper 2 Discrete

Mark scheme

June 2021

Version: 1.1 Final Mark Scheme

216A7366/2D/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1(a)	Circles correct answer	1.1b	B1	7
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
1(b)	Circles correct answer	1.1b	B1	7
	Total		**1**	

	Question total		**2**	

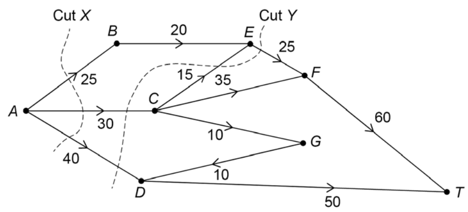
Q	Marking instructions	AO	Marks	Typical solution
2(a)	Uses correct values to label row and column headings of the Cayley table	2.5	B1	<table><tr><td>$+_8$</td><td>0</td><td>2</td><td>4</td><td>6</td></tr><tr><td>0</td><td>0</td><td>2</td><td>4</td><td>6</td></tr><tr><td>2</td><td>2</td><td>4</td><td>6</td><td>0</td></tr><tr><td>4</td><td>4</td><td>6</td><td>0</td><td>2</td></tr><tr><td>6</td><td>6</td><td>0</td><td>2</td><td>4</td></tr></table>
	Finds two correct rows or two correct columns	1.1a	M1	
	Completes the table correctly Condone $+_8$ missing from table	1.1b	A1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
2(b)	States the correct identity element	1.2	B1	0
	Total		**1**	

	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
3(a)	Finds the correct value of Cut X Condone missing/incorrect units	1.1b	B1	$25 + 30 + 40$ $= 95\ \text{m}^3\,\text{s}^{-1}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(b)	Finds the correct value of Cut Y Condone missing/incorrect units	1.1b	B1	$25 + 30 + 40$ $= 95\ \text{m}^3\,\text{s}^{-1}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(c)	Identifies at least one sink of the network PI	1.1a	M1	Nodes F and D are sinks 
	Draws correct arcs with arrows and appropriate weights	1.1b	A1	
	Total		**2**	

	Question total		**4**	

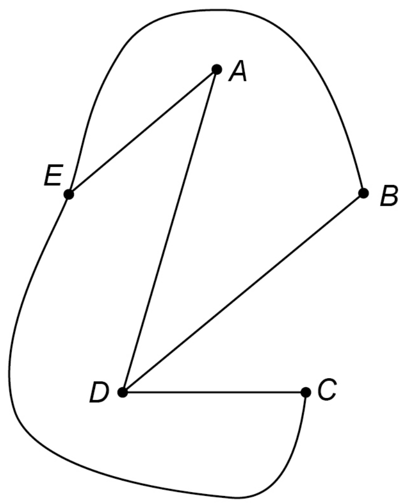
Q	Marking instructions	AO	Marks	Typical solution
4(a)	Sets up a test for commutativity using 2 distinct elements by considering $b * a$	1.1a	M1	$a * b = ab + 1$ $b * a = ba + 1$ As $ab + 1 = ba + 1$ then $a * b = b * a$ Therefore $*$ is commutative
	Constructs a rigorous mathematical argument to prove that $*$ is commutative	2.1	R1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Sets up a test for associativity using 3 elements	1.1a	M1	$(1 * 2) * 3 = (1 \times 2 + 1) * 3$ $= 3 * 3$ $= 10$ $1 * (2 * 3) = 1 * (2 \times 3 + 1)$ $= 1 * 7$ $= 8$ As $(1 * 2) * 3 \neq 1 * (2 * 3)$ then $*$ is not associative
	Finds two correct values for a proof by counter example or Finds two correct simplified algebraic expressions	1.1b	A1	
	Constructs a rigorous mathematical argument to prove that $*$ is not associative	2.1	R1	
	Total		**3**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Identifies a missing edge in the adjacency matrix	1.1a	M1	There is no edge *AD*, meaning that each vertex is not adjacent to every other vertex so *G* is not complete.
	Explains why *G* is not complete	2.4	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Explains correctly that *G* is not Eulerian by noting that *G* is not connected or *G* contains two vertices of odd degree	2.4	M1	*D* and *E* are only connected to each other, meaning that *G* is not connected. Hence *G* is neither Eulerian nor semi-Eulerian.
	Deduces correctly that *G* is neither Eulerian nor semi-Eulerian	2.2a	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(c)	Draws a graph with 5 vertices which are labelled *A*, *B*, *C*, *D* and *E*	1.1a	M1	
	Draws a simple-connected graph with 5 vertices with degrees of 2, 2, 2, 3 and 3	1.1a	M1	
	Draws a fully correct graph	1.1b	A1	
	Total		**3**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Finds row minima	1.1a	M1	Row minima = (–250, –100, –200) max(row minima) = –100
	Correctly finds play-safe strategy for Vaya	1.1b	A1	Play-safe strategy for Vaya is $\mathbf{V}_2$
	Finds column maxima	1.1a	M1	Column maxima = (75, 300, 75) min(column maxima) = 75
	Correctly finds both play-safe strategies for Wynne	1.1b	A1	Play-safe strategies for Wynne are $\mathbf{W}_1$ and $\mathbf{W}_3$
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Translates the problem to a mathematical process by identifying possible outcomes or strategies FT the play-safe strategies from **part (a)**	3.1b	M1	Wynne always plays $\mathbf{W}_2$ Vaya either plays $\mathbf{V}_1$ or $\mathbf{V}_3$ Best outcome for Wynne is to gain 200 each time the game is played.
	Deduces correct best outcome for Wynne	2.2a	A1F	
	Total		**2**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Sets up a model by identifying the problem as a route inspection problem and identifying the four odd nodes	3.3	M1	Odd nodes are *A*, *C*, *G* and *I* Shortest lengths between odd nodes *A*–*C*: 30 *G*–*I*: 26 *A*–*G*: 22 *C*–*I*: 22 *A*–*I*: 43 *C*–*G*: 43 Pairings (*A*–*C*)(*G*–*I*) = 30 + 26 = 56 (*A*–*G*)(*C*–*I*) = 22 + 22 = 44* (*A*–*I*)(*C*–*G*) = 43 + 43 = 86 44 mm is the shortest length to be repeated Total length = 240 mm + 44 mm = 284 mm
	Uses the model to find at least four correct shortest distances between odd nodes	3.4	M1	
	Finds the correct minimum pair of shortest distances from the three pairs	1.1b	A1	
	Determines correctly the minimum length CSO Condone missing/incorrect units	1.1b	A1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Refines the model to semi-Eulerian by identifying suitable start and end nodes PI	3.5c	M1	Start at *A* and end at *G* 240mm + 22 mm = 262 mm
	Finds the correct minimum length Condone missing/incorrect units	3.2a	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)	States a plausible reason for starting from *B*	3.5b	B1	The pendant would be symmetrical if starting from *B*
	Total		**1**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
8	Finds gradient of one constraint line or Finds expressions for P at two points of intersection	3.1a	M1	gradient $= \frac{11-6}{5-1} = \frac{5}{4}$ gradient $= \frac{11-9}{5-13} = -\frac{1}{4}$ $P = ax + y \quad \Rightarrow \quad y = -ax + P$ gradient $= -a$
	Finds gradient of objective line or Finds correct expressions for P at all points of intersection	1.1b	A1	$-\frac{5}{4} \le a \le \frac{1}{4}$ $-\frac{25}{4} + 11 \le P \le \frac{5}{4} + 11$
	Finds the correct absolute value of at least one critical value for a or for P	3.1a	M1	$4.75 \le P \le 12.25$
	Finds both correct critical values for a or for P	1.1b	A1	
	Obtains correct range for P Condone strict inequalities	1.1b	A1	
	Total		**5**	

	Paper total		**40**	