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Examiners' report

FURTHER MATHEMATICS A

H235
For first teaching in 2017

Y535/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y535/01 series overview

For both candidates and centres, this was the very first "live" example of a question paper for the additional pure further maths module and there was a considerable variety of responses to the paper. Many candidates seemed to be very carefully prepared for the full range of ideas, skills and techniques that arose. Others showed a lack of certainty when dealing with one or two of the topics and they often chose lengthy or clumsy approaches in their solution attempts. Overall, each of the seven questions delivered the appropriate kinds of outcomes, in terms of candidates' responses. Some candidates did not attempt question 7, or produced only partial solutions. This is most likely due to their having spent more time than necessary on earlier questions.

One aspect of the examination that differentiated between the good, the better and the best performances lay in the fact that many of the questions allowed for a choice on the part of the solver as to how best to proceed; or even, to begin with, which path should be followed in order to make progress. For instance, in question 6 an inductive proof is required, but while the structure makes this straightforward to spot, candidates were not explicitly told to use this form of proof. This "decision-making" factor also arose (to a lesser or greater extent) in each of questions.1, 3, 4, 5 and 7.

Another, very important, feature of this additional pure maths module is that the sensible use of calculators is very much encouraged. However, candidates do need to keep in mind the instructions in the specification for questions which include the phrase '***In this question you must show detailed reasoning*'** and to show clearly what they have entered into their calculators. This point needs to be stressed further: even when calculators are being deployed routinely (for instance, in the multiplication of matrices in this paper's question 4), there still needs to be clear indications from candidates as to what it is that has been calculated, even if this involves little more than noting that the matrix just written down is **ABA**, say. I would direct teachers and candidates to the Practice Papers available from OCR: these give a lot of carefully constructed guidance as to what should be written by examination candidates, as well as what need not.

Finally, I would like to stress the very important point made earlier that candidates need to be very keenly aware that the most valuable asset at their disposal during the sitting of these papers is time. Unnecessarily lengthy and repeated work can be a major drain on this resource and some candidates this year did themselves a great disservice by spending it unwisely.

Question 1(i)

1 The points A, B and C have position vectors $6\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$, $13\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$ and $16\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ respectively.

(i) Using the vector product, calculate the area of triangle ABC. [5]

For the most part, this question was handled both confidently and accurately. However, a small proportion of the candidates worked with the position vectors **a**, **b** and **c** instead of the vectors that represented the sides of the triangles, <u>AB</u>, <u>BC</u> and <u>CA</u>. The calculation of a vector product was usually good and almost everyone was able to apply the appropriate vector formula for the area of a triangle.

Question 1(ii)

(ii) Hence find, in simplest surd form, the perpendicular distance from C to the line through A and B. [3]

Stronger candidates had little trouble with this but many others did not grasp how to proceed. A clear, simple diagram usually pointed the way ahead as it essentially boiled down to a backwards treatment of the formula "area = half base × height" for a triangle, with a bit of surds work thrown in for good measure.

Question 2(i)

2 The surface with equation $z = 6x^3 + \frac{1}{9}y^2 + x^2y$ has two stationary points.

(i) Verify that one of these stationary points is at the origin. [4]

Although the majority of candidates were able to do the work for both parts of Q2 with little difficulty, most of them spent far too much time on working that was largely unnecessary … or, in many cases, repetitive. As a result of a lack of careful planning, the working for parts (i) and (ii) were often mixed together or, as mentioned previously, repeated in (ii) having already appeared in (i). The shrewder candidates noted that part (i) only required them to find the two first partial derivatives of *z*, and then to show that they were zero when both *x* and *y* were zero (and to verify that *z* was also zero at this point) in order to answer the question.

Question 2(ii)

(ii) Find the coordinates of the second stationary point. [5]

Most candidates appeared content to repeat much of the work they had already produced in part (i), by setting the two first partial derivatives equal to zero and solving. The biggest obstacle to completely correct solutions appeared to lie in the poor handling of minus signs in the ensuing algebra.

Question 3

3 Given that n is a positive integer, show that the numbers $(4n + 1)$ and $(6n + 1)$ are co-prime. **[3]**

There were 3 marks available for this question but most attempts scored only 2 of them. There were two principal approaches deployed, one of which is not actually on the syllabus. The expected approach was to use the result that any common factor of X and Y must also be a common factor of $aX + bY$. Here, X is $4n + 1$ and Y is $6n + 1$, and most candidates noted that $a = 3$, $b = -2$ gives $aX + bY = 1$. In itself, however, this does not automatically establish that h, the hcf of X and Y, is 1. A bit of explanation is needed to note that since $h \mid 1$ it **must** be equal to 1. Consider the similar argument in which $a = 6$, $b = -4$ gives $aX + bY = 2$; the conclusion here is that $h \mid 2$ and could then be either 1 or 2. Indeed, a few candidates did reason this way and they then completed the deal by observing that, since both X and Y are clearly odd, $h \neq 2$ and so $h = 1$.

The second approach commonly used was the application of the Euclidean algorithm, which was credited full marks if presented properly and with an effective explanatory note at the end to justify the conclusion.

Question 4(i)

4 The group G consists of a set of six matrices under matrix multiplication. Two of the elements of G are

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}.$$

(i) Determine each of the following:

- $\mathbf{A}^2$
- $\mathbf{B}^2$ **[2]**

The use of the command word "Determine" does indicate that some written justification is needed, even if the actual answer is found using the matrix functionality of a calculator.

Question 4(ii)

(ii) Determine all the elements of G. **[4]**

Most candidates were sensible enough to list the elements of G both as matrices and by labels. Repeats appeared seldom, there were some who did not identify all six elements, while others wasted valuable time producing a Cayley table.

Question 4(iii)

(iii) State the order of each non-identity element of G. **[3]**

Candidates who had produced clear working in part (ii) generally fared better than those whose working was cluttered and sporadic, as it made this part of the question more of a "write down" than anything else.

Question 4(iv)

(iv) State, with justification, whether G is

- abelian
- cyclic. **[2]**

There were many interesting – and occasionally quite lengthy – answers to this part of the question and candidates must learn to be guided by the number of marks as well as to the allocated space in the Printed Answer Booklet. To earn the mark for noting that *G* is not abelian, it was sufficient to observe that **AB** ≠ **BA**; it was not good enough to reason that matrix multiplication is not generally commutative. To support the claim that *G* is not cyclic, the anticipated response was that *G* contained no element of order equal to the order of the group. Most candidates realised that "orders" was what the question was all about and responded accordingly. However, it was not sufficient to make a more general claim that *G* had no generator without explaining why this was so.

Question 5(i)

5 For integers a and b, with $a \geqslant 0$ and $0 \leqslant b \leqslant 99$, the numbers M and N are such that

$$M = 100a + b \quad \text{and} \quad N = a - 9b.$$

(i) By considering the number $M + 2N$, show that $17 \mid M$ if and only if $17 \mid N$. **[4]**

In this question, candidates were supposed to note that the number $M + 2N = 17(6a - b)$ and is thus a multiple of 17. From this, it is easy to deduce that if M is a multiple of 17 then so is $2N$ and hence N, and if N is a multiple of 17 then so is M, these results following with very little further algebra or explanation. However, many candidates made a lot of hard work out of this, offering up to half a page of working for each part of an *if and only if* proof. The principal reason for this was that very few indeed realised that the form of $M + 2N$ was intended to be a helpful start along the way. Although alternative methods were equally acceptable, there was a mark allocated for this introductory observation and rather too many candidates simply ignored it and thus did far more working than was necessary *and* lost a mark. A significant proportion of candidates did not realise that an *if and only if* proof requires either two parts to the proof or a careful justification that a single argument is reversible.

Question 5(ii)

(ii) Demonstrate step-by-step how an algorithm based on the result of part **(i)** can be used to show that $2\,058\,376\,813\,901$ is a multiple of 17. **[4]**

This was one of those areas that caused some confusion amongst candidates, possibly arising from a lack of prior experimentation with such ideas. The key to such recursive processes is that the scale of the problem is reduced at each step of the algorithm, in this case by two digits at a time. A number of candidates had clearly been taught an alternative method for deciding divisibility and then produced substantial amounts of work in applying it only to gain a maximum of 1 mark as the wording of this question actually required them to demonstrate a process based on the results given in part (i).

Question 6(i)

6 The Fibonacci sequence $\{F_n\}$ is defined by $F_0 = 0, F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all $n \geqslant 2$.

(i) Show that $F_{n+5} = 5F_{n+1} + 3F_n$ **[3]**

This question was often poorly handled. Some candidates thought it should be approached using induction, but then did not see how; others merely demonstrated that it worked in a couple of specific cases. In fact, it was simply a matter of applying the Fibonacci definition several times over, starting with F_{n+5} in terms of F_{n+4} and F_{n+3} and then working down to F_{n+1} and F_n (or, for that matter, working upwards). This should take half-a-dozen lines at most and not the majority of a whole page. All that was required of candidates here was that they should be willing to play around with the subscripts of a sequence defined recurrently.

Question 6(ii)

(ii) Prove that F_n is a multiple of 5 when n is a multiple of 5. **[5]**

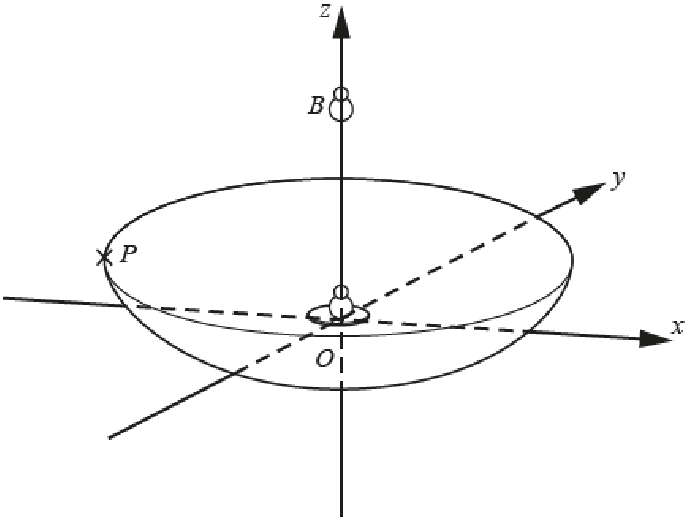
Although there is a really nice alternative solution (which is also covered by the specification – see the marking scheme) most candidates rightly saw this as an induction question. On the other hand, very few of them realised that part (i) had given them all the necessary help towards establishing the inductive step: that is, IF F_n is a multiple of 5 THEN F_{n+5} is a multiple of 5 plus three times a multiple of 5 etc. etc. All that essentially remains is to establish the baseline case and round off the explanation of the inductive logic. Nevertheless, even those who did spot the connection felt obliged to produce a sizeable amount of working in order to complete the matter. Also, rather a lot of candidates seemed to think that they were trying to prove the defining Fibonacci relationship $F_{n+2} = F_{n+1} + F_n$, while many other attempts seemed to mix up the idea of n (or k) with 5n (or 5k) caused by having to describe multiples of 5 as part of their working and this, again, highlighted a common lack of confidence in dealing with 'dummy variables' such as the position indicators in the subscripts.

Question 7(i)(a)

7 The 'parabolic' TV satellite dish in the diagram can be modelled by the surface generated by the rotation of part of a parabola around a vertical z-axis. The model is represented by part of the surface with equation $z = \mathrm{f}(x, y)$ and O is on the surface.

The point P is on the rim of the dish and directly above the x-axis.

The object, B, modelled as a point on the z-axis is the receiving box which collects the TV signals reflected by the dish.



(i) The horizontal plane Π_1, containing the point P, intersects the surface of the model in a contour of the surface.

(a) Sketch this contour in the Printed Answer Booklet. **[1]**

The whole of this question is based on an application of a mathematical model to a well-known real world setting using the work on Surfaces, which will be a new A-level topic to most. Although each part is relatively simple, it still requires a small amount of careful thought. One of the most significant features of the question as a whole is that a certain amount of generalisation was called for. Many candidates were clearly uncomfortable with this degree of choice and preferred to work with specific values (which were not penalised directly). However, this then made it difficult for them to make a realistic response to the calculational work that comprised part (iv).

For part (i) (a), most candidates realised that the contour required was a circle.

Question 7(i)(b)

(b) State a suitable equation for this contour. **[1]**

Any one of several correct answers could have been offered here and all suitable possibilities were accepted.

Question 7(ii)(a)

(ii) A second plane, Π_2, containing both P and the z-axis, intersects the surface of the model in a section of the surface.

(a) Sketch this section in the Printed Answer Booklet. **[1]**

Most attempts had a parabola drawn, although not all of them passed through the origin (as described in the question).

Question 7(ii)(b)

(b) State a suitable equation for this section. **[1]**

As with (i) (b), many candidates picked up the mark but did so by choosing numerical, rather than general, values for any proposed constants.

Question 7(iii)

(iii) A proposed equation for the surface is $z = ax^2 + by^2$. What can you say about the constants a and b within this equation? Justify your answers. **[3]**

There were many sensible answers given here, although very few candidates ultimately offered more than the one suggestion. Thus, the two possibilities – "they must be positive" and "they must be equal" – appeared equally often, but seldom appeared together.

Question 7(iv)

(iv) The real TV satellite dish has the following measurements (in metres): the height of P above O is 0.065 and the perimeter of the rim is 2.652. Using this information, calculate correct to three decimal places the values of

- a and b,
- any other constants stated within the answers to parts **(i)(b)** and **(ii)(b)**. **[4]**

This was the only part of the question in which direct calculations needed to be made. However, a deficiency in any of parts (i) (b), (ii) (b) and (iii) – such as not observing that *a* and *b* in (iii) had to be equal, for instance – meant that not all the marks could be gained by candidates who had already assigned a specific value to a term which they now needed to be working out.

Question 7(v)

(v) Incoming satellite signals arrive at the dish in linear "beams" travelling parallel to the z-axis. They are then 'bounced' off the dish to the receiving box at B.

- On the diagram for part **(ii)(a)** in the Printed Answer Booklet draw some of these beams and mark B.
- If the values of a and b were changed, what would happen? **[2]**

Most candidates who attempted this last part of the paper were able to realise that a change in the parameters would mean that *B* would have to be moved or, equivalently, that the beams would no longer arrive at the receiving box. Even more were happy to draw some incoming beams being reflected to *B* on their parabola drawn in (ii) (a).

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