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AS LEVEL

*Examiners' report*

# FURTHER MATHEMATICS A

**H235**
For first teaching in 2017

## Y532/01 Summer 2019 series

Version 1

# Contents



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# Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the exam paper can be downloaded from OCR.


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# Paper Y532 series overview

This was the Statistics paper of AS Further Mathematics. In order to do well on this paper candidates not only had to be able to carry out routine calculations and processes but to show understanding of the statistical ideas and be able to draw inferences from the results of calculations. In particular, there was a premium on understanding the issues for each individual question, as opposed to trying to fit a standard answer that had been seen before, or even rote-learnt.

Candidates were good at answering questions that required routine calculations; most were confident about using new calculator technology.

There was a wide range of quality in responses to this paper. Some excellent scripts were seen, with careful and detailed answers to most of the questions. Inevitably there were some under-prepared candidates, and this year it seemed that there was a significant number of candidates who needed more practice on distinguishing between related parts of the syllabus. For instance, formulae for the binomial and geometric distributions were confused, or the statement of hypotheses in tests involving the goodness-of-fit test was confused with that for correlation, or were given the wrong way round. Linear examinations make more demands on the skills of recognition and selection of correct methods and formulae than do modular examinations.

Many Centres had noted the sort of answers that were required from the Specimen Materials and practice papers. The mark schemes of these papers give a clear indication of what is expected, for example with respect to statements of hypotheses. Understandably, topics new to the specification were less confidently handled than those retained from the modular papers. For example, conclusions to hypothesis tests were generally well stated (even in the new contexts), but many struggled with problems involving probabilities, such as Questions 4(b) and 8.

The emphasis on problem-solving in the new specification (as required by OfQual) means that, even more than in the past, candidates must expect to be faced with questions they have not seen before. It is certainly the case that the best candidates are distinguished from those who are merely competent by their ability to solve such problems.

Some candidates deleted work with an eraser or correcting fluid. This caused problems particularly in Question 3(b) where as a result of the scanning process it was sometimes hard or even impossible to see which crosses had or had not been deleted. Candidates are reminded that erasers or correcting fluid should not be used.


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## Question 1 (a)

**1** When a spinner is spun, the outcome is equally likely to be 1, 2 or 3. In a competition, the spinner is spun twice and the outcomes are added to give a total score $T$.

**(a)** Show that the expectation of $T$ is 4. **[3]**

This was a 'show that' question, with a given answer to be obtained, and candidates therefore needed to show details of a correct method. There were several correct possibilities: finding the expected value of one observation $X$ from $\sum rP(X=r)$, or from the formula for a uniform distribution, $(n + 1)/2$, and then doubling; or by finding the distribution of $X_1 + X_2$. However, those who confused the method for expectation with that for finding a sample mean did not gain credit; it was necessary to see probabilities.

Some candidates thought that $X_1 + X_2$ was itself a uniform distribution, with each of the possible values having probability 1/5.

## Question 1 (b)

**(b)** Find the variance of $T$. **[3]**

The two alternative correct methods were to find the variance of $X_1$ and double it, or to use the distribution of $X_1 + X_2$. However, candidates found an unexpectedly large number of ways to get this question wrong. For instance, many multiplied the variance of $T$ by 2 squared, perhaps hindered by using the wrong notation $2X$ instead of $X_1 + X_2$. (In fact most who used this notation wrote '$2T$', which is even more confusing.) Others found $E(X^2)$ and doubled it (or even used the impossible probabilities 2/3, 2/3, 2/3), then subtracted 4 squared, which is also wrong.

## Question 1 (c) (i)

A competitor pays £1.50 to enter the competition and receives £$X$, where $X = 0.3T$.

**(c)** **(i)** Find the expectation of the competitor's profit. **[1]**

Many left their answer as $E(0.3T) = 1.2$, not realising that they had to subtract 1.5. Some, perhaps worried by a negative answer, wrote 1.5 – 1.2 = 0.3 without indicating that this was a loss.

## Question 1 (c) (ii)

**(ii)** Find the variance of the competitor's profit. **[2]**

This worried some candidates, although, even though anyone who multiplied their answer to 1(b) by $0.3^2$ got full marks unless their variance was negative. Some subtracted 1.5 again. Few gave the correct unit, which is the unlikely £$^2$, although, even though this was not penalised.

## Question 2 (a)

2 On any day, the number of orders received in one randomly chosen hour by an online supplier can be modelled by the distribution Po(120).

**(a)** Find the probability that at least 28 orders are received in a randomly chosen 10-minute period. **[2]**

Most answered this correctly. Inevitably some found $1 - \mathrm{P}(\leq 28)$ instead of $1 - \mathrm{P}(\leq 27)$. Some found $1 - \mathrm{P}(= 28)$.

## Question 2 (b)

**(b)** Find the probability that in a randomly chosen 10-minute period on one day and a randomly chosen 10-minute period on the next day a total of at least 56 orders are received. **[3]**

Most realised that they had to use Po(40), and many correct answers were seen. Some thought that 0.0097 was an answer correct to 3 significant figures. A few thought that the answer was 0.0525x0.0525.

## Question 2 (c)

**(c)** State a necessary assumption for the validity of your calculation in part **(b)**. **[1]**

Few realised that the only assumption needed here was that orders on one day were independent of orders on the other day. As the use of Poisson distributions (with the same mean) for the two days is stated in the question, it is wrong to state any assumptions needed for either individual Poisson distribution (for instance 'orders are independent of one another' or 'orders occur at constant average rate'). As with the legacy syllabus there were many 'scattergun' answers; candidates need to identify which assumption or assumptions are needed, as opposed to which are definitely implied by the given scenario.

## Question 3 (a)

3 **(a)** Shula calculates the value of Spearman's rank correlation coefficient $r_s$ for 9 pairs of rankings.

Find the largest possible value of $r_s$ that Shula can obtain that is less than 1. **[4]**

Many saw quickly that the minimum value of $\sum d^2$ had to be 2, and then quickly got the correct answer. Others spent time trying to use inequalities to obtain $\sum d^2 > 0$, after which they still had to find the number 2. Quite a lot of candidates thought that the minimum value of $\sum d^2$ could be 1.

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## Question 3 (b)

**(b)** A set of bivariate data consists of 5 pairs of values. It is known that for this data the value of Spearman's rank correlation coefficient is $-1$ but the value of Pearson's product-moment correlation coefficient is not $-1$.

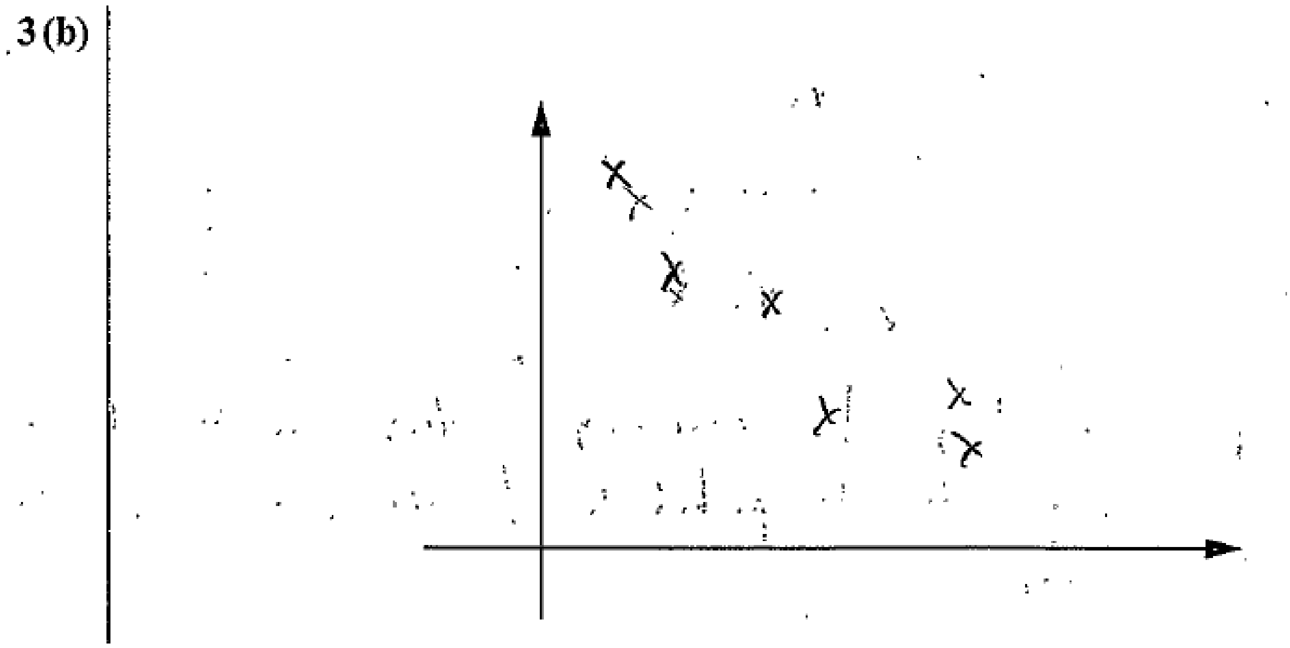
Sketch a possible scatter diagram illustrating the data. **[2]**

Many found it easy to draw such a diagram (although, even though a number of diagrams had more than 5 points, which did not matter). But others did not seem to have the proper concept, and suggested answers included scatter graphs with points either side of a line, points on a curve with a turning point, or graphs with positive gradient. This seemed to be an example of a question that identified candidates who could answer questions only if they had seen them before.

Some thought that '5 pairs' meant 10 points.

Those candidates who used dots rather than crosses, or who (as mentioned before) changed their answer using an eraser or correcting fluid, caused problems for examiners. The scanning process can often make erased objects as visible as non-erased ones. The issues can be seen from the following exemplars.

### Exemplar 1




Here it is hard for the examiner to see what has been deleted. The examiner will give benefit of doubt where possible, but it is not always possible.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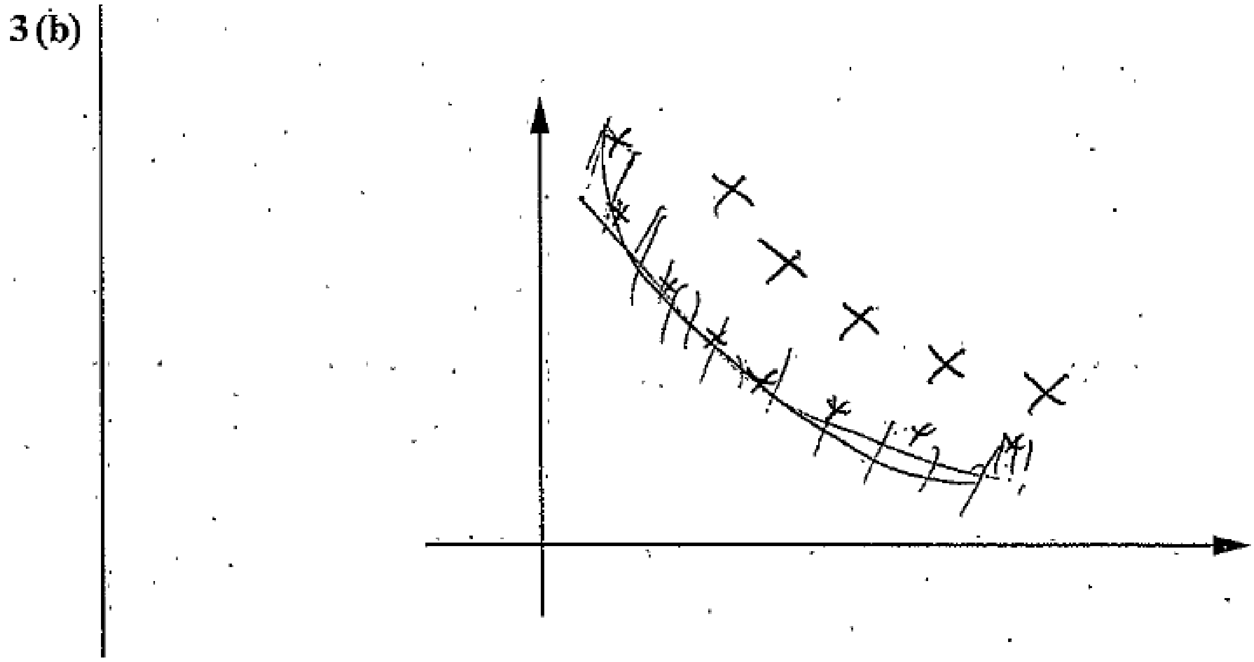
## Exemplar 2




| |
|---|
| This is an excellent way of ensuring that the examiner can see clearly what is and what is not deleted. |

## Question 4 (a)

**4** The members of a team stand in a random order in a straight line for a photograph. There are four men and six women.

**(a)** Find the probability that all the men are next to each other. **[3]**

| |
|---|
| This is a relatively standard question and most candidates should have seen it before. There were many correct answers, but also many scripts where candidates attempted to improvise (although, even though most realised that there was no point in attempting to multiply probabilities). |

## Question 4 (b)

**(b)** Find the probability that no two men are next to one another. **[4]**

This is certainly a harder question, but it is also standard. Question 4(i) of the specimen paper was similar, and other examples have appeared on the practice papers. There is a standard way of answering this type of question, which is to put one gap between each woman and then to find how many ways the men can be placed in the gap. Here there are 6 women (who can be arranged in 6! ways). There are 7 gaps (including one at each end), so 4 men can be placed in these in 7P4 ways, and the answer is simply 6!x7P4/10!.

Subtraction methods do not simplify the problem; most of the terms that have to be subtracted require the method described above, and there is the additional issue of two separate pairs of men to be considered. Nobody tried to use the Principle of Inclusion and Exclusion – wisely enough, as it does not work at all easily here.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## Question 5 (a)

**5** Sixteen candidates took an examination paper in mechanics and an examination paper in statistics.

**(a)** For all sixteen candidates, the value of the product moment correlation coefficient $r$ for the marks on the two papers was 0.701 correct to 3 significant figures.

Test whether there is evidence, at the 5% significance level, of association between the marks on the two papers. **[4]**

The mark scheme of the Specimen Paper made it clear that hypotheses had to be stated in terms of the population parameter rho, and that verbal statements such as ‘$H_0$: there is no association between the marks on the papers’ would not gain credit. Candidates should also state clearly that rho is the population correlation coefficient. Exemplar 3 indicates an acceptable explanation.

Many chose the wrong critical value, typically 0.4259 (the one-tailed value) instead of the correct 0.4973. However, comparisons and conclusions were often stated clearly and carefully. As usual, conclusions to hypothesis tests need to be contextualised (here requiring phrases like ‘association between candidates’ marks’) and with acknowledgement of uncertainly (so not ‘there is association …’ but ‘there is significant evidence of association …’).

Some gave the conclusion as ‘insufficient evidence of no association’, which is wrong.

## Question 5 (b) (i)

**(b)** A teacher decided to omit the marks of the candidates who were in the top three places in mechanics and the candidates who were in the bottom three places in mechanics. The marks for the remaining 10 candidates can be summarised by

$$n = 10,\ \Sigma x = 750,\ \Sigma y = 690,\ \Sigma x^2 = 57\,690,\ \Sigma y^2 = 49\,676,\ \Sigma xy = 50\,829.$$

**(i)** Calculate the value of $r$ for these 10 candidates. **[2]**

Almost always right. Most candidates showed a good quantity of working.


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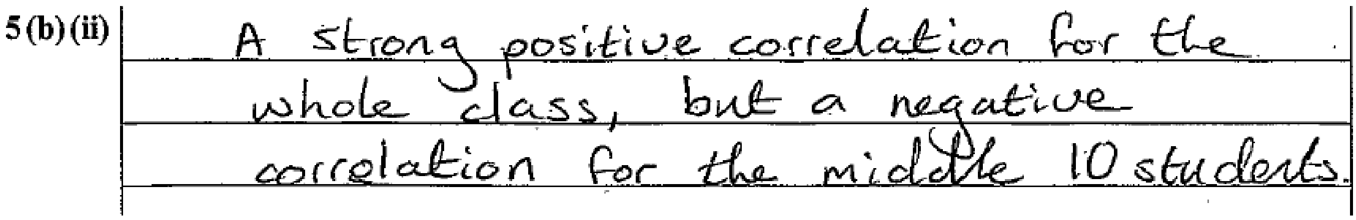
## Question 5 (b) (ii)

**(ii)** What do the two values of $r$, in parts **(a)** and **(b)(i)**, tell you about the scores of the sixteen candidates? **[1]**

The wording of this question required an inference about the scores of the candidates, not about correlation or association, so that answers such as Exemplar 3 did not gain the mark. Nor does it make sense to reject 6 candidates out of 16 as outliers. Good answers showed good statistical literacy, drawing inferences from the information, for example 'the best candidates did well on both papers, the worst did badly on both the others tended to do worse on one paper the better they did on the other'.

Some answers referred to 'a large range of results'; whether or not the use of the word 'range' here is intended to be technical, this cannot be inferred from the values of $r$. Answers such as 'the results are very varied' revealed a limited understanding of the concepts. Some candidates were unable to engage with the question and either left it blank or regurgitated irrelevant answers such as 'the data must be bivariate normal' or 'correlation is not causality'.

### Exemplar 3



**5(b)(ii)** A strong positive correlation for the whole class, but a negative correlation for the middle 10 students.

This answer refers merely to correlation and does not draw any inferences about the scores.

## Question 6 (a)

**6** A bag contains a mixture of blue and green beads, in unknown proportions. The proportion of green beads in the bag is denoted by $p$.

**(a)** Sasha selects 10 beads at random, with replacement.

Write down an expression, in terms of $p$, for the variance of the number of green beads Sasha selects. **[1]**

Many wrote down the formula for the variance of a geometric, rather than a binomial, distribution.

## Question 6 (b) (i)

Freda selects one bead at random from the bag, notes its colour, and replaces it in the bag. She continues to select beads in this way until a green bead is selected. The first green bead is the $X$th bead that Freda selects.

**(b)** Assume that $p = 0.3$.

Find

**(i)** $P(X \geqslant 5)$, **[2]**

Very often correct, with the usual errors involving 0.7^5 or 1 – 0.7^4 rather than 0.7^4.

## Question 6 (b) (ii)

**(ii)** $\text{Var}(X)$. **[1]**

Often this was confused with the formula for the variance of a geometric distribution. Otherwise, usually correct, although, even though it was necessary to substitute $n$ = 10 into the formula $np(1 – p)$.

## Question 6 (c)

**(c)** In fact, on the basis of a large number of observations of $X$, it is found that $P(X = 3) = \frac{4}{25} \times P(X = 1)$.

Estimate the value of $p$. **[5]**

This question, expected to be challenging, was well done, although a few used the binomial or even the Poisson distributions. Most could get the correct quadratic equation and solve it correctly, and a large number explicitly rejected the solution $p$ = 1.4. (This is AO2, third bullet point.) However, those who used the easiest approach of solving $(1 – p)^2 = 4/25$ often omitted to consider the possibility $(1 – p) = -2/5$.

## Question 7 (a)

7 In a standard model from genetic theory, the ratios of types $a$, $b$, $c$ and $d$ of a characteristic from a genetic cross are predicted to be 9:3:3:1. Andrei collects 120 specimens from such a cross, and the numbers corresponding to each type of the characteristic are given in the table.

| Type | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| Frequency | 51 | 33 | 30 | 6 |

Andrei tests, at the 1% significance level, whether the observed frequencies are consistent with the standard model.

**(a)** State appropriate hypotheses for the test. **[1]**

Many seemed unfamiliar with this type of question and said that the null hypothesis was 'the underlying ratios are <u>not</u> 9:3:3:1'. Some said 'the <u>observed</u> frequencies are in the ratio 9:3:3:1', which is wrong.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## Question 7 (b)

**(b)** Carry out the test. **[6]**

By contrast, most could correctly calculate the expected frequencies and the chi-squared statistic, although there were mistakes in selecting the correct critical value. Conclusions were generally well stated.

## Question 7 (c)

**(c)** State with a reason which one of the frequencies is least consistent with the standard model. **[1]**

Roughly half the candidates knew that the answer to this question was determined by the biggest contribution to the value of the chi-squared statistic, namely *b*. Many simply chose *a* as giving the largest (absolute) difference between observed and expected frequencies.

## Question 7 (d)

**(d)** Suggest a different, improved model by changing exactly two of the ratio values. **[1]**

This needed to be answered in conjunction with both the values of chi-squared and the raw data. The answer to 7(c) makes it clear that *b* needs to be changed, and then either *a* or *c*. Comparison of observed and expected values shows that the ratio for *b* is too low, that for *a* is too high and that for *c* is too low. So acceptable answers were to increase *b* and either to decrease *a* (for instance getting 7:5:3:1) or to increase *c* as well (for instance getting 9:5:5:1). Again roughly half the candidates gave satisfactory answers.

## Question 8

**8** Alex claims that he can read people's minds. A volunteer, Jane, arranges the integers 1 to $n$ in an order of Jane's own choice and Alex tells Jane what order he believes was chosen.

They agree that Alex's claim will be accepted if he gets the order completely correct or if he gets the order correct apart from two numbers which are the wrong way round.

They use a value of $n$ such that, if Alex chooses the order of the integers at random, the probability that Alex's claim will be accepted is less than 1%.

Determine the smallest possible value of $n$. **[7]**

This was deliberately a test of problem-solving skills. It requires use of combinations. Only a small number correctly realised that the number of ways of interchanging two ranks was ${}^{n}C_{2}$. Spearman's correlation coefficient is irrelevant here as we are interested in <u>any</u> two numbers being the wrong way round, not two consecutive ones. Most sensible attempts ended with an inequality involving factorials, and this had to be solved by numerical trial and improvement. Despite the challenges, quite a few correct solutions were seen.


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