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Examiners' report

FURTHER MATHEMATICS B (MEI)

H635
For first teaching in 2017

Y410/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y410/01 series overview

The overall standard of work achieved by the relatively small candidature for the first run of this paper was impressive. Nearly all candidates scored over half marks, and over a quarter achieved over 50 out of 60 marks. For candidates with experience of only one year's work in advanced mathematics, this was highly creditable. Even though 75 minutes is quite a short time to complete an examination, there was no evidence of candidates running out of time. The performance on many of the questions was excellent, with accurate, well presented work.

Most successful questions	*Least successful questions*
• Q1 (matrices) • Q2 (angle between vectors) • Q3 (complex numbers) • Q4 (symmetric properties of roots) • Q8 (mathematical induction)	• Q6 (invariant line of a transformation) • Q7 (difference method for summing a series) • Q10 (arrangement of planes)

Question 1

1 The matrices **A**, **B** and **C** are defined as follows:

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 0 & 3 \\ 1 & -1 & 3 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 & 3 \end{pmatrix}.$$

Calculate all possible products formed from two of these three matrices. **[4]**

The product BA and the product CB were generally calculated accurately. The product AC was omitted by a significant number of candidates.

Question 2

2 Find, to the nearest degree, the angle between the vectors $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ -3 \end{pmatrix}$. **[3]**

A misconception that there should be a second value of 112° (180 – 68) was seen in the responses from a few candidates. The angle between vectors is unique (unlike the angle between lines).

The number of candidates who misread one of the vectors – usually giving the z entry in the first vector as +2 - was well into double figures. As a result, the answer 140 was quite common. The other common error was to forget to round the final answer to the nearest degree, as requested in the question.

Question 3

3 Find real numbers a and b such that $(a-3\text{i})(5-\text{i}) = b - 17\text{i}$. **[5]**

Virtually all candidates expanded and equated real and imaginary parts. The few who lost marks did so through slips in the expansion of the brackets, most commonly getting +3 instead of –3.

Question 4

4 Find a cubic equation with real coefficients, two of whose roots are $2 - \text{i}$ and 3. **[5]**

There were no significant issues with candidates identifying the conjugate pair to find the third root of 2 + i. Thereafter, candidates divided fairly evenly between either using symmetric properties of roots or formulating the product of factors from the roots, with neither method proving more successful. The only blemish, which was quite common, was to omit the '= 0' in stating the final equation, at the expense of a mark.

Question 5(i)

5 A transformation of the x-y plane is represented by the matrix $\begin{pmatrix} \cos\theta & 2\sin\theta \\ 2\sin\theta & -\cos\theta \end{pmatrix}$, where θ is a positive acute angle.

(i) Write down the image of the point (2, 3) under this transformation. **[2]**

A few candidates attempted to find a numerical value for the image, but the majority gained full marks, albeit expressing the coordinates in vector form (which was condoned).

Question 5(ii)

(ii) You are given that this image is the point $(a, 0)$. Find the value of a. **[5]**

A regulatory requirement of the reformed qualifications is that some questions require deployment of knowledge from across a number of different topic areas. Here, candidates were required to use trigonometry from AS Mathematics to solve the equation $4\sin\theta - 3\cos\theta = 0$, and a significant minority struggled with the manipulation. Nevertheless, the majority gained full marks here.

Question 6

6 Find the invariant line of the transformation of the x-y plane represented by the matrix $\begin{pmatrix} 2 & 0 \\ 4 & -1 \end{pmatrix}$. **[4]**

The main problem that candidates had here was an inability to differentiate between invariant points and invariant lines.

Many candidates gained a method mark for calculating the image of (x, y) as ($2x$, $4x - y$). For an invariant point, this implies that $2x = x$, so $x = 0$ and $y = -y$, so $y = 0$. So, the only invariant point is the origin. However, for an invariant line, $y = mx + c$ and $4x - y = m.2x + c$, leading to $4x - mx - 2c = 2mx$. This is an identity which needs to be true for all x, so $c = 0$ and $m = 4/3$, giving an invariant line $y = 4x/3$. There is in fact another solution to the identity, namely $x = 0$, which is indeed another invariant line. However, very few spotted this, and there was no penalty for omitting this. In fact, with linear transformations such as this, it suffices to use the line $y = mx$, as the origin is necessarily invariant.

Exemplar 1

6 $\begin{pmatrix} 2 & 0 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} x' \\ mx'+c \end{pmatrix}$

$2x = x'$

$4x - mx - c = mx' + c$

$4x - mx ~~c~~ = 2mx ~~+c~~$

$4x - mx = 2mx$

$m = \frac{4}{3}$ A0

$y = \frac{4}{3}x$

Some candidates, as in this response, incorrectly cancelled c's rather than finding $2c = 0$ and hence $c = 0$.

Question 7(i)

7 (i) Express $\frac{1}{2r-1} - \frac{1}{2r+1}$ as a single fraction. **[2]**

This straightforward combining of fractions was negotiated successfully by virtually the whole candidature.

Question 7(ii)

(ii) Find how many terms of the series

$$\frac{2}{1\times3}+\frac{2}{3\times5}+\frac{2}{5\times7}+\ldots+\frac{2}{(2r-1)(2r+1)}+\ldots$$

are needed for the sum to exceed 0.999 999. **[7]**

The difference method for summing the series was attempted by most candidates, but a significant minority used invalid methods based on the results for standard series.

Those who used the valid difference method usually got to a sum of 1 – 1/(2n + 1); if they did not, it was due to muddling n's and r's and getting 1 – 1/(2r + 1), which lost a couple of marks.

The final three marks were then open to candidates, and many gained all three, though those who solved the inequality (rather than the equality) sometimes lost the final 'A' mark through errors in handling inequality signs.

Question 8

8 Prove by induction that $\begin{pmatrix}1 & 1\\ 0 & 2\end{pmatrix}^n = \begin{pmatrix}1 & 2^n-1\\ 0 & 2^n\end{pmatrix}$ for all positive integers n. **[6]**

The induction step in this question was a relatively straightforward one. However, it is important that candidates show some working to justify this step. Errors and omissions here proved quite costly, as the final two marks were dependent on this. These errors, though rare, were of two types: one was to add M to M^k (where M is the given matrix); another involved manipulating handling indices, such as writing 2^{k-1} instead of $2^k - 1$, or $2\times 2^k = 4^k$.

In order to gain full marks for this question, it was important that candidates used precise language that showed a clear mathematical justification of the induction process. For example, in their concluding statement, if they wrote 'so, as it is true for n = 1, n = k and n = k + 1, it is true for all n', they missed out on the final two marks; whereas if they wrote 'so, as it is true for n = 1, and **if** true for n = k, then true for n = k + 1, it is true for all n', they got full marks (assuming the induction step was negotiated without errors). Although many candidates are well schooled in the logic of induction, the use of precise language to convey this logic is required, and it is important that candidates understand that the truth for n = k is an *assumption*, not a given.

Exemplar 2

8

For $n=1$:

$$\begin{pmatrix}1 & 1\\0 & 2\end{pmatrix}^1 = \begin{pmatrix}1 & 1\\0 & 2\end{pmatrix}$$

and

$$\begin{pmatrix}1 & 2^1-1\\0 & 2^1\end{pmatrix} = \begin{pmatrix}1 & 1\\0 & 2\end{pmatrix}$$

same ∴ true

Assume true for $n=k$

$$\begin{pmatrix}1 & 1\\0 & 2\end{pmatrix}^k = \begin{pmatrix}1 & 2^k-1\\0 & 2^k\end{pmatrix}$$

For $n=k+1$

$$\begin{pmatrix}1 & 1\\0 & 2\end{pmatrix}^{k+1} = \begin{pmatrix}1 & 1\\0 & 2\end{pmatrix}^k \begin{pmatrix}1 & 1\\0 & 2\end{pmatrix} = \begin{pmatrix}1 & 2^k-1\\0 & 2^k\end{pmatrix}\begin{pmatrix}1 & 1\\0 & 2\end{pmatrix} =$$

$$\begin{pmatrix}1 & 1+2\times2^k-2\\0 & 2\times2^k\end{pmatrix} = \begin{pmatrix}1 & 1+2^{k+1}-2\\0 & 2^{k+1}\end{pmatrix}$$

$$= \begin{pmatrix}1 & 2^{k+1}-1\\0 & 2^{k+1}\end{pmatrix}$$

Same as $\begin{pmatrix}1 & 2^{k+1}-1\\0 & 2^{k+1}\end{pmatrix}$ same ∴ true

B0

If true for $n=1$, $n=k$, $n=k+1$, true for all positive integer values of n

∴ $\begin{pmatrix}1 & 1\\0 & 2\end{pmatrix}^n = \begin{pmatrix}1 & 2^n-1\\0 & 2^n\end{pmatrix}$ is true

In this response, the candidate loses a mark in summarising the induction step for the above reason.

Question 9(i)

9 Fig. 9 shows a sketch of the region OPQ of the Argand diagram defined by

$$\{z : |z| \leqslant 4\sqrt{2}\} \cap \{z : \tfrac{1}{4}\pi \leqslant \arg z \leqslant \tfrac{1}{3}\pi\}.$$

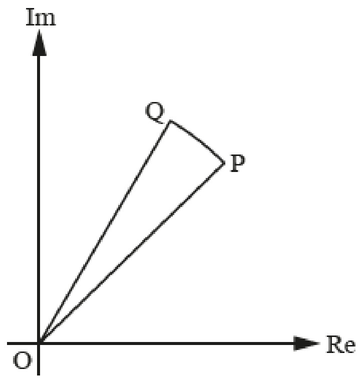

Fig. 9

(i) Find, in modulus-argument form, the complex number represented by the point P. **[2]**

This was well answered. Most correctly identified the modulus and argument of P as 4√2 and $\pi/4$, but a few did not express the point in modulus-argument form as 4√2(cos$\pi/4$ + isin$\pi/4$).

Question 9(ii)

(ii) Find, in the form $a + ib$, where a and b are exact real numbers, the complex number represented by the point Q. **[3]**

Again, the majority of candidates found the correct modulus and argument, but there were occasional errors in finding *a* and *b*.

Question 9(iii)

(iii) **In this question you must show detailed reasoning.**

Determine whether the points representing the complex numbers

- $3 + 5i$
- $5.5(\cos 0.8 + i \sin 0.8)$

lie within this region. **[4]**

This question was, on the whole, well answered. Most compared the modulus of 3+5i to 4√2 and concluded it was outside the region. Some, instead, compared the imaginary part of 3+5i with that of Q to reach the same conclusion. The second point was handled appropriately by most, though occasionally they went round in a circle by first expressing the point in *x* + i*y* form and then finding the modulus and argument of their *x* + i*y* to get 5.5 and 0.8 (or approximations to these).

Question 10(i)

10 Three planes have equations

$$\begin{aligned} -x + 2y + z &= 0 \\ 2x - y - z &= 0 \\ x + y &= a \end{aligned}$$

where a is a constant.

(i) Investigate the arrangement of the planes:

- when $a = 0$;
- when $a \neq 0$. **[6]**

This question proved to be more demanding, and even the best candidates lost marks here. While some concluded correctly that in the case a = 0 the planes formed a sheaf, and in the case $a \neq 0$ they formed a prismatic intersection, complete solutions demanded a systematic investigation to eliminate the other possible arrangements.

The following strategy might help for future questions on this topic:

1. Is the determinant of the matrix of coefficients zero? If yes, we eliminate a unique point of intersection.

2. Are the coefficients of x, y and z for any two equations multiples of each other? If no, this eliminates parallel planes.

3. Are the equations consistent, so that there are solutions? In the case a = 0, they are, so the planes form a sheaf.

4. In the case $a \neq 0$, the equations are inconsistent and there are no solutions, so they form a prismatic intersection.

In the case a = 0, some candidates attempted to give solutions to the equations, but quite often gave the solutions in the form of a plane rather than a line.

Question 10(ii)

(ii) Chris claims that the position vectors $-\mathbf{i}+2\mathbf{j}+\mathbf{k}$, $2\mathbf{i}-\mathbf{j}-\mathbf{k}$ and $\mathbf{i}+\mathbf{j}$ lie in a plane. Determine whether or not Chris is correct. **[2]**

Candidates who recognised the three vectors as normal to the planes gained a mark. However, many wrongly concluded that Chris was incorrect. If the correct arrangement of the planes was given, then it suffices to state that the normal must be coplanar for a sheaf or a prism. The other approach was to show that the given vectors are linearly dependent, as the third is the sum of the first two. Some tried to find a normal to each of the three planes, with some success on occasions.

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