Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE Further Mathematics
AS Further Decision 2 Paper 8FM0_28

Summer 2019
Publications Code 8FM0_28_1906_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- | The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	Subtract each entry from a constant (e.g. 42) to covert from maximisation problem to minimisation	B1	1.1a
	Add an additional dummy row with equal values (e.g. 42, 0, etc.) to create a square array	B1	3.5c
		(2)	
(b)	e.g. $\begin{pmatrix} & P & Q & R & S \\ A & 10 & 2 & 5 & 0 \\ B & 13 & 10 & 7 & 1 \\ C & 5 & 9 & 3 & 2 \\ X & 42 & 42 & 42 & 42 \end{pmatrix}$	B1	1.1b
	No reduction for row A, reduce row B by 1, reduce row C by 2 and row X by 42 (or equivalent). No reduction of columns	B1	2.4
	Reducing rows and columns gives $\begin{pmatrix} & P & Q & R & S \\ A & 10 & 2 & 5 & 0 \\ B & 12 & 9 & 6 & 0 \\ C & 3 & 7 & 1 & 0 \\ X & 0 & 0 & 0 & 0 \end{pmatrix}$	M1	1.1b
	Two lines required to cover the zeros hence solution is not optimal (augment by 1) $\begin{pmatrix} & P & Q & R & S \\ A & 9 & 1 & 4 & 0 \\ B & 11 & 8 & 5 & 0 \\ C & 2 & 6 & 0 & 0 \\ X & 0 & 0 & 0 & 1 \end{pmatrix}$	M1	1.1b
	Three lines required to cover the zeros hence solution is not optimal (augment by 1) e.g. $\begin{pmatrix} & P & Q & R & S \\ A & 8 & 0 & 3 & 0 \\ B & 10 & 7 & 4 & 0 \\ C & 2 & 6 & 0 & 1 \\ X & 0 & 0 & 0 & 2 \end{pmatrix}$ or $\begin{pmatrix} & P & Q & R & S \\ A & 8 & 0 & 4 & 0 \\ B & 10 & 7 & 5 & 0 \\ C & 1 & 5 & 0 & 0 \\ X & 0 & 0 & 1 & 2 \end{pmatrix}$	M1	1.1b
	Four lines required to cover the zeros hence solution is optimal	B1	2.4
	A – Q, B – S, C – R, (X – P)	A1	2.2a
		(7)	
			(9 marks)

Notes
(a) **B1**: Valid statement regarding converting a maximisation problem to a minimisation problem – must imply subtracting each entry from a constant (although the value of this constant need not be stated) **B1**: Explain the need to add a valid dummy row (to create a square array) e.g. allow mention of adding an additional worker or the need to have a square array **(b)** **B1:** Mark awarded when both steps complete (a valid subtraction and addition of a correct extra row) **B1:** Correct statements regarding row and column reduction – if explicit values not stated then it must be clear that reduction is done by subtracting the least value in each row/column from each element of that row/column **M1:** Simplifying the initial matrix by reducing rows and then columns – allow one error **M1:** Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 2 lines needed to 3 lines needed **M1:** Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed (so getting to the optimal table) **B1:** Correct statement(s) regarding the minimum number of lines to cover zeros at each stage or a general statement that covers all augmentations (e.g. if we have an *n* by *n* array then if the minimum number of lines required to cover the zeros is less than *n* then the solution is not optimal but if the minimum number of lines to cover the zeros is *n* then it is optimal – as an absolute minimum allow mention that until there are 4 lines covering the zeros then the solution is not optimal (oe) – however, in all cases, there must have been two augmentations taking place (2 to 3 lines and then 3 to 4)) **A1:** CSO for the application of the Hungarian algorithm - so correct application of the algorithm from a correct initial matrix (so in (b) candidates must have scored at least B1B0M1M1M1B0) – together with the deduction of the correct allocation **Note that if array is not square or if the additional row is given entries of 'infinity' then only the second B mark in (b) can be awarded**

Question	Scheme	Marks	AOs
	$u_{n+1} = 3u_n + 2^n \quad n \geq 1$		
2(a)	(aux equation $m-3=0 \Rightarrow$) complementary function is $A(3)^n$	B1	1.1b
	Consider a trial solution of the form $u_n = k\left(2^n\right)$ so $2k\left(2^n\right) = 3k\left(2^n\right) + 2^n$	M1	1.1b
	$k = -1$	A1	1.1b
	General solution is $u_n = A(3)^n - 2^n$	A1	1.1b
		(4)	
(b)	$u_1 = u_2 \Rightarrow 3A - 2 = 9A - 4 \Rightarrow A = \ldots$	M1	3.1a
	$u_n = 3^{n-1} - 2^n$	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

B1: CAO for complementary function

M1: substituting correct trial solution into the recurrence relation – allow substitution of $u_n = k\left(2^n\right)$ into $u_n = 3u_{n-1} + 2^{n-1}$ but not $u_n = 3u_{n-1} + 2^n$

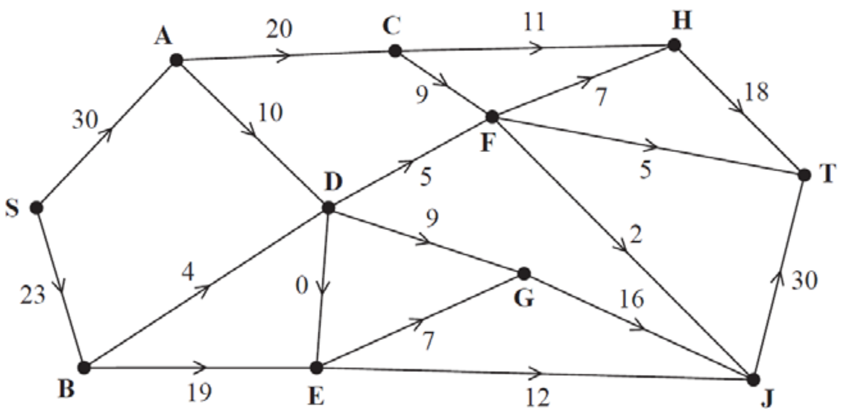
A1: CAO $k = -1$

A1: CAO for the general solution – must include $u_n = \ldots$

(b)

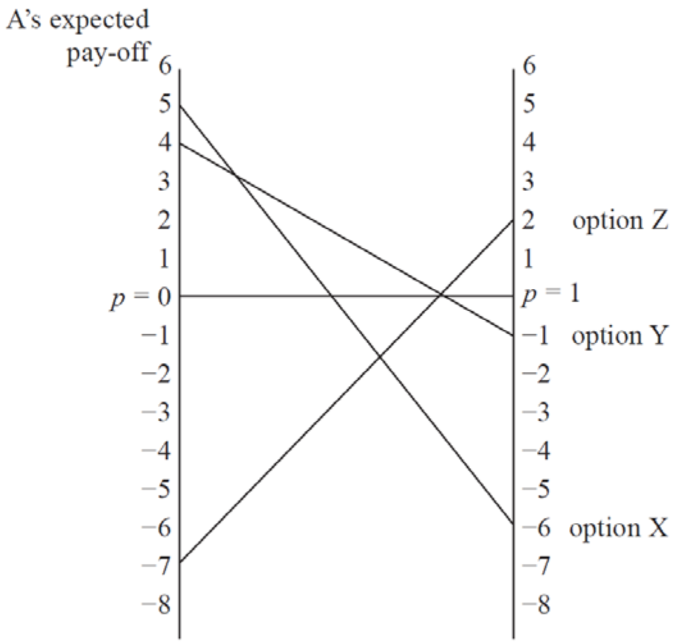
M1: using the condition $u_1 = u_2$ to calculate a value for their A $\left(= \frac{1}{3}\right)$ - this mark is dependent on the general solution being of the form $\pm\lambda(3)^n \pm \mu(2)^n$

A1: CAO for the particular solution (oe) – must include $u_n = \ldots$ - however, if neither (general nor particular) solution is given in terms of u_n then award this mark if correct expression in terms of n seen (or if both solutions are given in terms of say u_{n+1})

Question	Scheme	Marks	AOs
3(a)	45	B1	1.1b
		(1)	
(b)	e.g. the total capacity of arcs DF and DG is 5 + 11 = 16. The capacity of the two arcs leading into D are 10 (from AD) and 4 (from BD) giving a total capacity into D of 14. As 14 < 16 arcs DF and DG cannot both be full to capacity	B1	2.4
		(1)	
(c)	Value of cut = 32 + 10 + 4 + 7 + 12 = 65	B1	1.1b
		(1)	
(d)	e.g. SACFHT – 2; SADGJT – 4; SBEDFHT – 2 e.g. SACFHT – 2; SADGJT – 2; SADFHT – 2; SBEDGJT – 2 e.g. SACFHT – 2; SADGJT –4; SBEDGJT – 2	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(e)	e.g.  Alternative: DG 11, GJ 18, JT 32 DF 3, FH 5, HT 16	B1	1.1b
		(1)	
(f)	Use of max-flow min-cut theorem Identification of cut through CH, CF, AD, BD, DE, EG and EJ Value of flow = 53 Therefore by the max-flow min-cut theorem it follows that flow is maximal	M1 A1 A1	2.1 3.1a 2.2a
		(3)	
			(10 marks)

Notes
(a) **B1:** CAO (45) **(b)** **B1:** CAO (as a minimum accept mention that the max flow into D is 14 and max flow out of D is 16 together with comparison of these two values – (node) D must be mentioned) **(c)** **B1:** CAO (65) **(d)** **M1:** One correct flow augmenting route found from S to T + flow value **or** two correct routes **A1:** Two correct routes + correct flow values **A1:** CSO – increasing the flow by 8 only **(e)** **B1:** CAO **–** condone more than one value on an arc only if one of these values is circled **–** mark those that have been circled only **(f)** **M1:** Construct the start of an argument based on the max-flow min-cut theorem (that is an attempt to find a genuine cut together with the value of either their cut or flow (but not re-iterating the cut given in (c) – AC, AD, BD, DE, EG, EJ)) **A1:** Use appropriate process of finding a minimum cut – must see correct cut + value correct (accept '53' and the cut either stated or drawn on either diagram) **A1:** Correct deduction that the flow is maximal by stating 'maximum flow (equal to) minimum cut' – dependent on previous A mark and the correct flow in (e)

Question	Scheme	Marks	AOs
4(a)	**(i)** Row minima: –6, –7, 3 max is 3 Column maxima: 5, 6, 3 min is 3 Row(maximin) = Col(minimax) therefore game is stable	M1 A1	1.1b 2.4
	(ii) value of the game to B is –3	B1	2.2a
		(3)	
(b)	Let A play option P with probability p and option Q with probability $1 - p$	B1	3.3
	If B plays option X, A's gains are $-6p + 5(1 - p) = 5 - 11p$ If B plays option Y, A's gains are $-p + 4(1 - p) = 4 - 5p$ If B plays option Z, A's gains are $2p + (-7)(1 - p) = -7 + 9p$	M1 A1	1.1b 1.1b



Question	Scheme	Marks	AOs
	(graph above)	M1dep A1	1.1b 1.1b
	$5 - 11p = -7 + 9p \Rightarrow p = 3/5$	A1	1.1b
	A should play option P with probability 0.6 and option Q with probability 0.4	A1ft	3.2a
		(7)	
(c)	As indicated by the graph Brendan can, for all values of p, gain more by playing either options X or Z e.g. for $0 \leq p \leq \frac{3}{5}$ Brendan would be better off playing Z and for $\frac{3}{5} < p \leq 1$ Brendan would be better off playing X than playing Y	B1	3.2a
		(1)	

Question	Scheme	Marks	AOs
(d)	**(i)** If A plays option P then B can expect to gain $-(-6q+2(1-q))$ as the values in the table are the pay-offs for A	B1	2.2a
	The value of the game to B is $-(5-11(0.6))$ = 1.6 (or equivalent calculation e.g. $-(-7+9(0.6))$)	B1	2.1
	(ii) $6q-2(1-q)=1.6 \Rightarrow q=0.45$	B1	1.1b
	B should play option X with probability 0.45 and option Z with probability 0.55	B1	3.2a
		(4)	
		(15 marks)	

Notes

(a)
M1: finding row minimums and column maximums – condone one error
A1: row maximin (3) = col minimax (3) so stable (dependent on correct row minimums and col maximums) - as a minimum accept '3 = 3 so stable'
B1: CAO (–3)
(b)
B1: defining variable *p* (must mention 'probability' – as a minimum accept 'P with probability *p* and Q with probability 1 – *p*')
M1: setting up three expressions in terms of *p* (need not be simplified)
A1: all three expressions correctly simplified
M1dep: axes correct, at least one line correctly drawn from their expressions – dependent on previous M mark in **(b)**
A1: correct graph – if no scaling on vertical axis assume 1 line = 1 unit (A0 if graph extends for $p < 0$ and/or $p > 1$)
A1: using the graph to obtain the correct probability expressions leading to the correct value of *p*
A1ft: interpret their value of *p* in the context of the question – must refer to 'play' and the associated probabilities (need not say 'probability' again) – this mark is dependent on both previous M marks in this part
(c)
B1: correct explanation in context (of playing only X and Z <u>for all</u> *p*) with specific reference to the modelling of the problem by the graph
(d)(i):
B1: correctly deducing the lhs of the given equation (with clear reasoning for the change in sign) – only allow stating $6q - 2(1 - q)$ (without seeing the change of sign) if the game is restated for player B
B1: correctly deriving the rhs of the given equation (must indicate that this is the value of the game to B although as a minimum accept V(B)) – stating ± 1.6 without any working is B0 - n**ote that for either mark in (d)(i) candidates must <u>explain</u> where the two parts of the given equation came from**
Note that candidates may explain the formulation of the given equation by considering $-6q+2(1-q)=-1.6$ (which is what player B can expect to lose if player A plays option P which is equal to the value of the game to player A)
(d)(ii):
B1: CAO for the value of *q* (must come from solving $6q - 2(1 - q) = 1.6$)
B1: CAO in context and must refer to 'play'

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom