Mark Scheme (Results)

Summer 2022

Pearson Edexcel GCE
In Further Mathematics (8FM0)
Paper 28 Decision Mathematics 2

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	e.g. $\begin{array}{cc} & \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{pmatrix} 54 & 48 & 51 & 52 \\ 55 & 51 & 53 & 58 \\ 52 & 100 & 53 & 54 \\ 67 & 63 & 68 & 100 \end{pmatrix} \end{array}$	B1	1.1b
		(1)	
(b)	Reduce row A by 48, reduce row B by 51, reduce row C by 52 and row D by 63 (or equivalent). No reduction for columns P and Q, reduce R by 1 and column S by 2	B1	2.4
	Reducing rows and columns gives e.g. $\begin{array}{cc} & \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{pmatrix} 6 & 0 & 3 & 4 \\ 4 & 0 & 2 & 7 \\ 0 & 48 & 1 & 2 \\ 4 & 0 & 5 & 37 \end{pmatrix} \end{array}$ followed by $\begin{array}{cc} & \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{pmatrix} 6 & 0 & 2 & 2 \\ 4 & 0 & 1 & 5 \\ 0 & 48 & 0 & 0 \\ 4 & 0 & 4 & 35 \end{pmatrix} \end{array}$	M1	1.1b
	Two lines required to cover the zeros hence solution is not optimal (augment by 1) e.g. $\begin{array}{cc} & \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{pmatrix} 5 & 0 & 1 & 1 \\ 3 & 0 & 0 & 4 \\ 0 & 49 & 0 & 0 \\ 3 & 0 & 3 & 34 \end{pmatrix} \end{array}$	M1	1.1b
	Three lines required to cover the zeros hence solution is not optimal (augment by 1) e.g. $\begin{array}{cc} & \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{pmatrix} 4 & 0 & 0 & 0 \\ 3 & 1 & 0 & 4 \\ 0 & 50 & 0 & 0 \\ 2 & 0 & 2 & 33 \end{pmatrix} \end{array}$ or e.g. $\begin{array}{cc} & \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{pmatrix} 4 & 0 & 1 & 0 \\ 2 & 0 & 0 & 3 \\ 0 & 50 & 1 & 0 \\ 2 & 0 & 3 & 33 \end{pmatrix} \end{array}$	M1	1.1b
	Four lines required to cover the zeros hence solution is optimal	B1	2.4
	A – S, B – R, C – P, D – Q	A1	2.2a
		(6)	
			(7 marks)

Notes for Question 1

(a)

B1: Replace the blanks in cells CQ and DS with values > 68

(b)

B1: Correct statements regarding row and column reduction

M1: Simplifying the initial matrix by reducing rows and then columns – allow 2 independent slips

M1: Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 2 lines needed to 3 lines needed

M1: Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed (so getting to the optimal table)

B1: Dependent on two augmentations taking place (2 to 3 lines and then 3 to 4). Either a correct statement(s) regarding the minimum number of lines to cover the zeros at each stage **or** a general statement that covers all augmentations.

In the first case, at each stage, they must state the number of lines (not just shown on the diagram), state whether it is optimal or not (so must use the word 'optimal') and mention 'zeros' at least once.

In the second case, they must state that until 4 lines cover the zeros then the solution is not optimal (or equivalent e.g. if there are 4 lines covering the zeros then the solution is optimal) – in this case they must show the lines.

Accept a hybrid of the two e.g. at each stage they could say whether it requires four lines or not but they would still have to mention 'zeros' at least once and make it clear at each augmentation whether it is optimal or not.

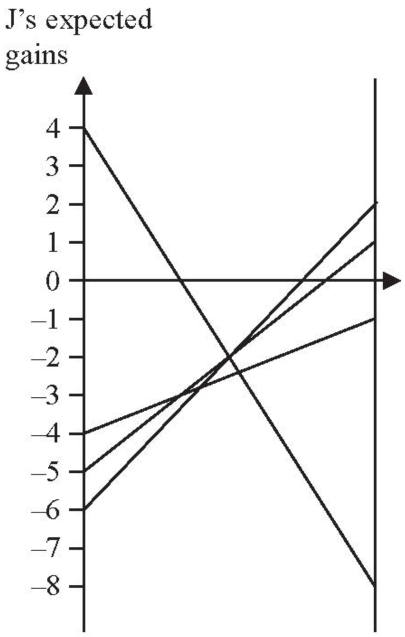
To award this mark we must see mention at least one mention of 'zeros' and the word 'optimal' being used.

A1: CSO on final table + deduction of the correct allocation

Question	Scheme	Marks	AOs
2(a)	50	B1	1.1b
		(1)	
(b)	Saturated arcs: AB, BC, SD, BE, DT, GE, GH and GT	B1	1.1b
		(1)	
(c)	e.g., the capacity of arc EH is 37. The three arcs that flow into E are BE, FE and GE. The total capacity of these three arcs is 10 + 19 + 5 = 34 and as 37 > 34, EH cannot be full to capacity.	B1	2.4
		(1)	
(d)	**(i)** Value of cut $C_1 = 10 + 6 + 6 + 13 + 23 = 58$	B1	1.1b
	(ii) Value of cut $C_2 = 37 + 6 + 12 + 13 + 23 = 91$	B1	1.1b
		(2)	
(e)	e.g. SACBFEHT	B1	1.1b
		(1)	
(f)	Use of max-flow min-cut theorem Identification of cut through BE, FE, GE, GH, GT, DT Value of flow = 53 Therefore it follows that flow is maximal	M1 A1 A1	2.1 3.1a 2.2a
		(3)	
			(9 marks)

Notes for Question 2

(a)
B1: cao (50)
(b)
B1: cao (AB, BC, SD, BE, DT, GE, GH, GT)
(c)
B1: Correct reasoning for why EH cannot be full to capacity (e.g., EH has capacity 37 but the total capacity of the arcs that flow into E is only 34 so EH cannot be full to capacity). Must be numerical.
(d)(i) B1: cao (58) **(ii) B1:** cao (91)
(e)
B1: One correct flow-augmenting route only (SACBFEHT)
(f)
M1: Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through saturated arcs – must contain source on one side and sink on the other) – allow cut shown on the diagram
A1: Use appropriate process of finding a minimum cut – cut (BE, FE, GE, GH, GT, DT) and value of flow through the network stated correctly (53)
A1: Correct deduction that the flow is maximal – must use all four words ‘maximum’, ‘flow’, ‘minimum’ and ‘cut’ (allow abbreviations for maximum and minimum) – dep on first A mark

Question	Scheme	Marks	AOs
3(a)	In a 'zero-sum' game each participant's gain or loss (of utility) is exactly balanced by the losses or gains (of the utility) of the other participant.	B1	1.2
		(1)	
(b)	Row minima: 1, –2, –1, –4 so max is 1 Column maxima: 8, 6 so min is 6	M1	1.1b
	Row(maximin) $\neq$ Col(minimax) therefore game is not stable	A1	2.4
		(2)	
(c)	$\begin{pmatrix} -1 & 2 & 1 & -8 \\ -4 & -6 & -5 & 4 \end{pmatrix}$	B1	1.1b
		(1)	
(d)	**(i)** Let June play X with probability p and Y with probability $1-p$	B1	3.3
	If Terry plays A June's gains are $-p+(-4)(1-p)=3p-4$ If Terry plays B June's gains are $2p+(-6)(1-p)=8p-6$ If Terry plays C June's gains are $p+(-5)(1-p)=6p-5$ If Terry plays D June's gains are $-8p+4(1-p)=-12p+4$	M1 A1	1.1b 1.1b
		M1	1.1b
	$3p-4=-12p+4 \Rightarrow p=8/15$	M1	1.1b
	June should play X with probability 8/15 and play Y with probability 7/15	A1	3.2a
	(ii) Value of the game to Terry is 2.4	A1	3.4
		(7)	

(e)	$-t+(-8)(1-t)=-4t+4(1-t)$ (oe e.g., $t+8(1-t)=4t-4(1-t)$) or $-t+(-8)(1-t)=-\frac{12}{5}$ (oe) or $-4t+4(1-t)=-\frac{12}{5}$ (oe)	M1	3.1a
	$t=\frac{4}{5}$	A1	1.1b
	Terry should play option A with probability 0.8, never play options B and C, and play option D with probability 0.2	A1ft	3.2a
		(3)	
			(14 marks)

Notes for Question 3

(a)
B1: cao (give bod but must get the idea across that one person's losses are equal to the other's gains)

(b)
M1: finding row minimums and column maximums – condone one error
A1: row maximin (1) $\neq$ col minimax (6) (so not stable) – dependent on all correct 6 values

(c)
B1: cao

(d)(i)
B1: defining variable p (or any other letter e.g., q) – must use the word 'probability' (do not have to mention 'June' so as a minimum accept 'X with probability p and Y with probability $1-p$')
M1: setting up four expressions in terms of their p
A1: all four expressions correctly simplified
M1: at least three lines correctly drawn for their expressions – if values on at least one vertical axis not given then lines must be in the right position relative to each other
M1: using their graph to obtain their correct probability equation leading to a value of p (dependent on both previous M marks)
A1: interpret the correct value of p in the context of the question – must refer to 'play' and the associated probabilities (need not say 'probability' again). This mark is dependent on a completely correct graph (so if no scaling on the vertical axis assume that 1 line = 1 unit, and lines must not extend past $p<0$ and/or $p>1$)

(d)(ii)
A1: cao (2.4) oe (dependent on all previous M marks)

(e)
M1: Setting up a linear equation in t (with possibly their value from (d)(ii)), using only the two valid options from their graph in (d)(i) (so if the graph was correct in (d) they must be using Terry's options A and D) – allow sign slips only (allow any other choice of letter)
A1: cao (0.8)
A1ft: interpret their value of t in the context of the question – do not penalise a lack of 'play' twice. Must also include the two options that are never played

Question	Scheme	Marks	AOs
4(a)	(aux equation $m+3=0 \Rightarrow$) complementary function is $A(-3)^n$	B1	2.1
	Particular solution try $u_n = an+b$ and substitute into recurrence relation	M1	1.1b
	$a(n+1)+b+3(an+b)=n+k$ and by comparing linear and constant terms gives $a+3a=1$ $a+b+3b=k$	dM1	1.1b
	$a=\frac{1}{4}, 4b+\frac{1}{4}=k \Rightarrow b=\frac{4k-1}{16}$	ddM1	1.1b
	$(u_n =)A(-3)^n+\frac{1}{4}n+\frac{4k-1}{16}$ or $(u_{n+1} =)A(-3)^{n-1}+\frac{1}{4}(n-1)+\frac{4k+7}{16}$	A1	1.1b
	$u_0=1 \Rightarrow A+\frac{4k-1}{16}=1$	dddM1	3.4
	$u_n=\left(\frac{17-4k}{16}\right)(-3)^n+\frac{1}{4}n+\frac{4k-1}{16}$	A1	1.1b
		(7)	
(b)	Setting $\frac{17-4k}{16}=0$ and solving for k	M1	3.1a
	$k=\frac{17}{4} \Rightarrow u_{100}=\frac{1}{4}(100)+1$	dM1	1.1b
	$u_{100}=26$	A1	1.1b
		(3)	
			(10 marks)

Notes for Question 4

(a)

B1: cao (or equivalent e.g. $A(-3)^{n-1}$) – condone $A(-3^n)$ in (a) for all but the final A mark

M1: correct form for particular solution e.g. $an+b$, $a(n-1)+b$, etc. (so anything that is a constant times n + a constant) and substituted into recurrence relation

dM1: compares coefficients and setting up both equations in a, b, k (so one equation in a only and one equation in a, b and k (although a may have already been found from the first equation)) – dependent on previous M mark

ddM1: solve for a and b (with b in terms of k) – dependent on both previous M marks

A1: a correct general solution (in terms of k) – ignore labelling of left-hand side

dddM1: use correct initial condition correctly to form an equation in their A and k – dependent on all three previous M marks

A1: correct particular solution (in terms of k) – must have correct left-hand side

(b)

M1: Setting the coefficient of the exponential term equal to zero and solving for k – their solution from (a) must be of the form $\alpha(\beta)^n + \gamma n + \delta$ (where $\alpha, \beta, \gamma, \delta$ are constants and α, δ are in terms of k only)

dM1: Substituting their value of k to obtain an expression for u_n which is linear and substituting $n = 100$ (dependent on previous M mark)

A1: cao (26) – from correct working including a correct expression for u_n in (a)

Additional guidance:

Those candidates who re-write $u_{n+1} + 3u_n = n + k$ as $u_n + 3u_{n-1} = (n-1) + k$ can score full marks. Their solution will usually begin: CF is $A(-3)^n$ then a PS of the form $an + b$ leading to $4an + 4b - 3a - n + 1 - k = 0$ and so $a = \frac{1}{4}, b = \frac{4k-1}{16}$ and then their general solution should be as in the main scheme (although for the final mark in (a) do look out for those who call the left-hand side u_{n+1})

It is common for candidates to re-write $u_{n+1} + 3u_n = n + k$ as $u_n + 3u_{n-1} = n + k$ which is incorrect. This can score all B and M marks in both parts only

Slightly less common is to have a CF of the form $A(-3)^{n-1}$ and a PS of the form $a(n-1) + b$ this leads to

$a(n-1) + a + 3(a(n-2) + b) = n + k$ so $4an - 7a + 4b = n + k$ therefore $a = \frac{1}{4}$ and $b = \frac{4k+7}{16}$

So, giving as a general solution $(u_{n+1} =) A(-3)^{n-1} + \frac{1}{4}(n-1) + \frac{4k+7}{16}$

Now to use the initial condition correctly $u_0 = 1 \Rightarrow u_1 = k - 3$ (oe) and this leads to $A = \frac{153 - 36k}{16}$

So, $u_{n+1} = \left(\frac{153-36k}{16}\right)(-3)^{n-1} + \frac{1}{4}(n-1) + \frac{4k+7}{16}$ which when re-written in terms of u_n gives the form as in the main mark scheme

Of course, any CF of the form $A(-3)^{n \pm k_1}$ and any PS of the form $a(n \pm k_2) + b$ where k_1, k_2 are constants will work. So, award the M marks for the correct methods as illustrated in the notes in mark scheme and the first A mark in (a) for a correct general solution (ignoring the labelling of the left-hand side) and the second A mark in (a) for a fully correct expression with correct left-hand side