Paper 2 Option A

Further Pure Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	$\sec x - \tan x = \dfrac{1}{\dfrac{1-t^2}{1+t^2}} - \dfrac{2t}{1-t^2}$	M1	2.1
	$= \dfrac{1+t^2}{1-t^2} - \dfrac{2t}{1-t^2} = \dfrac{1-2t+t^2}{1-t^2}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1-t)(1+t)} = \dfrac{1-t}{1+t}$ *	A1*	2.1
		(3)	
(b)	$\dfrac{1-\sin x}{1+\sin x} = \dfrac{1-\dfrac{2t}{1+t^2}}{1+\dfrac{2t}{1+t^2}}$	M1	1.1a
	$= \dfrac{1+t^2-2t}{1+t^2+2t}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1+t)^2} = \left(\dfrac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2$ *	A1*	2.1
		(3)	
			(6 marks)

Notes:

(a)

M1: Uses $\sec x = \dfrac{1}{\cos x}$ and the t-substitutions for both $\cos x$ and $\tan x$ to obtain an expression in terms of t

M1: Sorts out the $\sec x$ term, and puts over a common denominator of $1-t^2$

A1*: Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer

(b)

M1: Uses the t-substitution for $\sin x$ in both numerator and denominator

M1: Multiples through by $1+t^2$ in numerator and denominator

A1*: Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result

Question	Scheme	Marks	AOs
2	£300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$; half an hour after purchase $\Rightarrow t_2 = 1.5$, so step h required is 0.25	B1	3.3
	$t_0 = 1,\ V_0 = 3,\ \left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$	M1	3.4
	$V_1 \approx V_0 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 = 3 + 0.25 \times 2 = \ldots$	M1	1.1b
	$= 3.5$	A1ft	1.1b
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$	M1	1.1b
	$V_2 \approx V_1 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963\ldots$, so £396 (nearest £)	A1	3.2a
		(6)	
			(6 marks)

Notes:

B1: Identifies the correct initial conditions and requirement for h

M1: Uses the model to evaluate $\frac{\mathrm{d}V}{\mathrm{d}t}$ at t_0, using their t_0 and V_0

M1: Applies the approximation formula with their values

A1ft: 3.5 or exact equivalent. Follow through their step value

M1: Attempt to find $\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1$ with their 3.5

A1: Applies the approximation and interprets the result to give £396

Question	Scheme	Marks	AOs
3	$\frac{1}{x} < \frac{x}{x+2}$		
	$\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$	M1	2.1
	$\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$	M1	1.1b
	At least two correct critical values from $-2, -1, 0, 2$	A1	1.1b
	All four correct critical values $-2, -1, 0, 2$	A1	1.1b
	$\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$	M1 A1	2.2a 2.5
		(6)	
			(6 marks)

Notes:

M1: Gathers terms on one side and puts over common denominator, or multiply by $x^2(x+2)^2$ and then gather terms on one side

M1: Factorise numerator or find roots of numerator or factorise resulting in equation into 4 factors

A1: At least 2 correct critical values found

A1: Exactly 4 correct critical values

M1: Deduces that the 2 “outsides” and the “middle interval” are required. May be by sketch, number line or any other means

A1: Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set e.g. accept $\mathbb{R} - ([-2, -1] \cup [0, 2])$ or $\{x \in \mathbb{R} : x < -2 \text{ or } -1 < x < 0 \text{ or } x > 2\}$

Question	Scheme	Marks	AOs
4(a)	Identifies glued face is triangle ABC and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert$	M1	3.1a
	$\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert = \frac{1}{2}\lvert(-2\mathbf{i}+3\mathbf{j}+\mathbf{k})\times(-\mathbf{i}+\mathbf{j}+2\mathbf{k})\rvert$	M1	1.1b
	$=\frac{1}{2}\lvert 5\mathbf{i}+3\mathbf{j}+\mathbf{k}\rvert$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\,(\mathrm{m}^2)$	A1	1.1b
		(4)	
	Alternative		
	Identifies glued face is triangle ABC and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{\lvert\mathbf{AB}\rvert^2\lvert\mathbf{AC}\rvert^2-(\mathbf{AB.AC})^2}$	M1	3.1a
	$\lvert\mathbf{AB}\rvert^2=4+9+1=14,\quad \lvert\mathbf{AC}\rvert^2=1+1+4=6$ and $\mathbf{AB.AC}=2+3+2=7$	M1	1.1b
	So area of glue is = $\frac{1}{2}\sqrt{('14')('6')-('7')^2}$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\ (\mathrm{m}^2)$	A1	1.1b
		(4)	
(b)	Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC}.(\mathbf{OA}\times\mathbf{OB}))$	M1	3.1a
	$=\frac{1}{6}(\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}\times(3\mathbf{j}+\mathbf{k}))$	M1	1.1b
	$=\frac{10}{6}=\frac{5}{3}$	A1	1.1b
	Volume of parallelepiped is 6 × volume of tetrahedron (= 10), so volume of glass is difference between these, viz. $10-\frac{5}{3}=\ldots$	M1	3.1a
	Volume of glass $=\frac{25}{3}(\mathrm{m}^3)$	A1	1.1b
		(5)	

Question	Scheme	Marks	AOs
	4(b) Alternative		
	$-\mathbf{j}+3\mathbf{k}$ is perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$	M1	3.1a
	Area $AOB = \frac{1}{2}\times\lvert\mathbf{OA}\rvert\times\lvert\mathbf{OB}\rvert = \frac{1}{2}\times 2\times\sqrt{10} = \sqrt{10}$	A1	1.1b
	$\mathbf{i}+\mathbf{j}+2\mathbf{k}-p(-\mathbf{j}+3\mathbf{k}) = \mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k}) \Rightarrow p=\frac{1}{2}$ and so height of tetrahedron is $h=\frac{1}{2}\lvert-\mathbf{j}+3\mathbf{k}\rvert = \frac{1}{2}\sqrt{10}$	M1	3.1a
	Volume of glass is $V = 5 \times$ Volume of tetrahedron $= 5\times\frac{1}{3}\sqrt{10}\times\frac{1}{2}\sqrt{10}$	M1	1.1b
	$= \frac{25}{3}\left(\text{m}^3\right)$	A1	1.1b
		(5)	
(c)	The glued surfaces may distort the shapes / reduce the volume of concrete Measurements in m may not be accurate The surface of the concrete tetrahedron may not be smooth Pockets of air may form when the concrete is being poured	B1	3.2b
		(1)	
			(10 marks)

Question 4 notes:

Accept use of column vectors throughout

(a)

M1: Shows an understanding of what is required via an attempt at finding the area of triangle ABC

M1: Any correct method for the triangle area is fine

M1: Finds $\mathbf{AB}$ and $\mathbf{AC}$ or any other appropriate pair of vectors to use in the vector product and attempts to use them

A1: Correct procedure for the vector product with at least 1 correct term $\frac{1}{2}\sqrt{35}$ or exact equivalent

(a) Alternative

M1: Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides

M1: May use different sides to those shown

M1: Correct full method to find the area of the triangle using their two sides

A1: $\frac{1}{2}\sqrt{35}$ or exact equivalent

Question 4 notes continued:
(b) **M1:** Attempts volume of concrete by finding volume of tetrahedron with appropriate method **M1:** Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted **A1:** Correct value for the volume of concrete **M1:** Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped **A1:** $\frac{25}{3}$ only, ignore reference to units
(b) **Alternative** **M1:** Notes (or works out using scalar products) that $-\mathbf{j}+3\mathbf{k}$ is a vector perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$ **A1:** Finds (using that **OA** and **OB** are perpendicular), area of $AOB = \sqrt{10}$ **M1:** Solves $\mathbf{i}+\mathbf{j}+2\mathbf{k} - p(-\mathbf{j}+3\mathbf{k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j}+\mathbf{k})$ to get the height of the tetrahedron $\left[(\mu=\lambda=)\ p=\frac{1}{2},\ \text{so } h=\frac{1}{2}\lvert-\mathbf{j}+3\mathbf{k}\rvert=\frac{1}{2}\sqrt{10}\right]$ **M1:** Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference) **A1:** $\frac{25}{3}$ only, ignore reference to units
(c) **B1:** Any acceptable reason in context

Question	Scheme	Marks	AOs
5(a)	$y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$ $\Rightarrow$ $P(4p^2, 8p)$ is a general point on C	B1	2.2a
		(1)	
(b)	$y^2 = 16x$ gives $a = 4$, or $2y\dfrac{dy}{dx} = 16$ so $\dfrac{dy}{dx} = \dfrac{8}{y}$	M1	2.2a
	$l: y - 8p = \left(\dfrac{8}{8p}\right)\left(x - 4p^2\right)$	M1	1.1b
	leading to $py = x + 4p^2$ *	A1*	2.1
		(3)	
(c)	$B\left(-4, \dfrac{10}{3}\right)$ into l $\Rightarrow$ $\dfrac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \dfrac{3}{2}$ and l cuts x-axis when $\dfrac{3}{2}(0) = x + 4\left(\dfrac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \dfrac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \dfrac{1}{2}(9 - -9)(12) - \displaystyle\int_0^9 4x^{\frac{1}{2}}dx$	M1	2.1
	$\displaystyle\int 4x^{\frac{1}{2}}dx = \dfrac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} (+c)$ or $\dfrac{8}{3}x^{\frac{3}{2}} (+c)$	M1 A1	1.1b 1.1b
	$\text{Area}(R) = \dfrac{1}{2}(18)(12) - \dfrac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *	A1*	1.1b
		(8)	

Question	Scheme	Marks	AOs
	5(c) Alternative 1		
	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ into l gives $\frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = \frac{3}{2}y - 9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12} \left(\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)\right) \mathrm{d}y$	M1	2.1
	$\int \left(\frac{1}{16}y^2 - \frac{3}{2}y + 9\right) \mathrm{d}y = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y \ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \left(\frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12)\right) - (0)$ $= 36 - 108 + 108 = 36$ *	A1*	1.1b
		(8)	
	5(c) Alternative 2		
	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts px-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12)$ and $x = 0$ in $l: y = \frac{2}{3}x + 6$ gives $y = 6$ $\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9 \left(\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right) \mathrm{d}x$	M1	2.1
	$\int \left(\frac{2}{3}x + 6 - 4x^{\frac{1}{2}}\right) \mathrm{d}x = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}} \ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = 27 + \left(\left(\frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}\left(9^{\frac{3}{2}}\right)\right) - (0)\right)$ $= 27 + (27 + 54 - 72) = 27 + 9 = 36$ *	A1*	1.1b
		(8)	
		(12 marks)	

Question 5 notes:

(a)

B1: Substitutes $y_P = 8p$ into y^2 to obtain $64p^2$ and substitutes $x_p = 4p^2$ into $16x$ to obtain $64p^2$ and concludes that P lies on C

(b)

M1: Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it

M1: Applies $y - 8p = m(x - 4p^2)$, with their tangent gradient m, which is in terms of p. Accept use of $8p = m(4p^2) + c$ with a clear attempt to find c

A1*: Obtains $py = x + 4p^2$ by **cso**

(c)

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into l

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$

M1: Substitutes their p (which must be positive) and $y = 0$ into l and solves to give $x = ...$

A1: Finds that l cuts the x-axis at $x = -9$

M1: Fully correct method for finding the area of R

i.e. $\frac{1}{2}(\text{their } x_P - "-9")(\text{their } y_P) - \int_0^{\text{their } x_P} 4x^{\frac{1}{2}}\,dx$

M1: Integrates $\pm\lambda x^{\frac{1}{2}}$ to give $\pm\mu x^{\frac{3}{2}}$, where $\lambda, \mu \neq 0$

A1: Integrates $4x^{\frac{1}{2}}$ to give $\frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

(c) Alternative 1

M1: Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into l

M1: Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
Substitutes their p (which must be positive) into l and rearranges to give $x = ...$

M1: Finds l as $x = \frac{3}{2}y - 9$

A1: Fully correct method for finding the area of R

M1: i.e. $\int_0^{\text{their } y_P} \left(\frac{1}{16}y^2 - \text{their}\left(\frac{3}{2}y - 9\right)\right)dy$

M1: Integrates $\pm\lambda y^2 \pm \mu y \pm \nu$ to give $\pm\alpha y^3 \pm \beta y^2 \pm \nu y$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$

A1: Integrates $\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)$ to give $\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y$, simplified or un-simplified

A1*: Fully correct proof leading to a correct answer of 36

Question 5 notes continued:	
(c)	**Alternative 2**
M1:	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into l
M1:	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1:	Substitutes their p (which must be positive) and $y = 0$ into l and solves to give $x = ...$
A1:	Finds that l cuts the x-axis at $x = -9$
M1:	Fully correct method for finding the area of R i.e. $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_P} \left(\text{their}\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right)\mathrm{d}y$
M1:	Integrates $\pm\lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$ to give $\pm\alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$
A1:	Integrates $\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)$ to give $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified
A1*:	Fully correct proof leading to a correct answer of 36