

SECTION B

Answer ALL questions. Write your answers in the spaces provided.

6. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 6 & 4 \end{pmatrix}$$

(a) find the characteristic equation of the matrix $\mathbf{A}$.

(2)

(b) Hence show that $\mathbf{A}^3 = 43\mathbf{A} - 42\mathbf{I}$.

(3)

Question 6 continued

Handwriting practice area with horizontal lines.

(Total for Question 6 is 5 marks)

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7. (i) Without performing any division, explain why 8184 is divisible by 6

(2)

- (ii) Use the Euclidean algorithm to find integers a and b such that

$$27a + 31b = 1$$

(4)

Question 7 continued

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(Total for Question 7 is 6 marks)

8. A curve C is described by the equation

$$|z - 9 + 12i| = 2|z|$$

(a) Show that C is a circle, and find its centre and radius.

(4)

(b) Sketch C on an Argand diagram.

(2)

Given that w lies on C ,

(c) find the largest value of a and the smallest value of b that must satisfy

$$a \leq \operatorname{Re}(w) \leq b$$

(2)

Question 8 continued

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(Total for Question 8 is 8 marks)

9. The operation $*$ is defined on the set $S = \{0, 2, 3, 4, 5, 6\}$ by $x*y = x + y = xy \pmod{7}$

$*$	0	2	3	4	5	6
0						
2		0				
3						5
4						
5		4				
6						

- (a) (i) Complete the Cayley table shown above
(ii) Show that S is a group under the operation $*$
(You may assume the associative law is satisfied.)

(6)

- (b) Show that the element 4 has order 3

(2)

- (c) Find an element which generates the group and express each of the elements in terms of this generator.

(3)

Question 9 continued

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(Total for Question 9 is 11 marks)

Question 10 continued

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Question 10 continued

Lined area for writing the answer to Question 10.

Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS