Further Statistics 2 Mark Scheme (Section B)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>5(a)</td><td>
<table>
<tr><td>Competitor</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td></tr>
<tr><td>Judge 1's ranks</td><td>8</td><td>4</td><td>7</td><td>6</td><td>5</td><td>1</td><td>3</td><td>2</td></tr>
<tr><td>Judge 2's ranks</td><td>8</td><td>5</td><td>6</td><td>7</td><td>3</td><td>1</td><td>4</td><td>2</td></tr>
<tr><td>d^2</td><td>0</td><td>1</td><td>1</td><td>1</td><td>4</td><td>0</td><td>1</td><td>0</td></tr>
</table>
$\sum d^2 = 8$
$r_s = 1 - \frac{6 \times 8}{8(64-1)}$
</td><td>M1

M1

dM1</td><td>1.1b

1.1b

1.1b</td></tr>
<tr><td></td><td>$r_s = 0.90476\ldots$ awrt <u>0.905</u></td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(b)</td><td>H_0: $\rho_s = 0$ H_1: $\rho_s > 0$</td><td>B1</td><td>2.5</td></tr>
<tr><td></td><td>Critical value $\rho_s = 0.8333$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>$r_s = 0.905$ lies in the critical region/reject H_0</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>The two judges are in agreement.</td><td>A1</td><td>2.2b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(c)</td><td>E.g. The data is unlikely to be from a bivariate normal distribution (competitor A)/The emphasis here is on the ranks and not the individual scores.</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(d)</td><td>Both show positive correlation, but the judges agree more on the beam (since 0.952 is closer to 1)</td><td>B1</td><td>2.2b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4">(10 marks)</td></tr>
</table>

Notes:

(a)
M1: For an attempt to rank at least one row (at least four correct)
M1: For an attempt at d^2 row for their ranks
M1: Dependent on 1st M1 for use of $r_s = 1 - \frac{6 \times 8}{8(64-1)}$ with their $\sum d^2$
A1: For awrt 0.905

(b)
B1: Both hypotheses stated in terms of ρ_s
B1: For correct critical value
M1: For comparing their '0.905' with their '0.8333'
A1: For a correct contextual conclusion with no contradictions seen

(c)
B1: For a correct explanation to support the use of Spearman

(d)
B1: For a correct comparison of the correlation coefficients

Question	Scheme	Marks	AOs
6(a)	$P(X<3)=\int_1^3 \frac{1}{18}(11-2x)dx$ or area of trapezium	M1	1.1a
	$=\left[\frac{1}{18}(11x-x^2)\right]_1^3$		
	$=\frac{7}{9}$	A1	1.1b
		(2)	
(b)	Since $P(X<3)>0.75$, the upper quartile is less than 3	B1ft	2.2a
		(1)	
(c)	$E(X^2)=\int_1^4 \frac{1}{18}x^2(11-2x)dx\left[=\frac{23}{4}\right]$	M1	1.1b
	$Var(X)=\frac{23}{4}-\left(\frac{9}{4}\right)^2$	M1	1.1b
	$=\frac{11}{16}$	A1	1.1b
		(3)	
(d)	$F(4)=1 \rightarrow \frac{1}{18}(11(4)-4^2+c)=1$ or $F(1)=0 \rightarrow \frac{1}{18}(11(1)-1^2+c)=0$	M1	2.1
	$c=-10$ *	A1*cso	1.1b
		(2)	
(e)	$F(m)=0.5$	M1	1.2
	$\frac{1}{18}(11m-m^2-10)=0.5 \rightarrow m^2-11m+19=0$ and attempt to solve	M1	1.1b
	$m=\frac{11\pm\sqrt{11^2-4(19)}}{2}$ [= 2.1458 or 8.8541…]		
	m = 2.1458… **2.15** (only)	A1	2.2a
		(3)	
			(11 marks)

Notes:

(a)

M1: For integrating f(x) with correct limits **or** for finding area of trapezium

A1: For $\frac{7}{9}$ (allow awrt 0.778)

(b)

B1ft: For comparison of their (a) with 0.75 and concluding that the upper quartile is less than 3

(c)

M1: For an attempt to find $E(X^2)$

M1: For use of $Var(X)=E(X^2)-\left(\frac{9}{4}\right)^2$

A1: For $\frac{11}{16}$ (allow awrt 0.688)(M1 marks may be implied by a correct answer)

Question 6 notes continued:
(d) **M1:** For use of F(4) = 1 or F(1) = 0 **A1*cso:** For a fully correct solution leading to given answer with no errors seen
(e) **M1:** For use of F(m) = 0.5 **M1:** For setting up quadratic and attempt to solve **A1:** For 2.15 and rejecting the other solution

Question	Scheme	Marks	AOs
7(a)	$r = \dfrac{284.4 - \frac{251(12)}{10}}{\sqrt{10.36 \times 40.9}}$	M1	1.1b
	$r = -0.79671...$ awrt **–0.797**	A1	1.1b
		(2)	
(b)	$b = \dfrac{'-16.4'}{10.36}$	M1	3.3
	$a = \dfrac{251}{10} - 'b' \dfrac{12}{10}$	M1	1.1b
	$y = 27.0 - 1.58x$	A1	1.1b
		(3)	
(c)	$y = [27.0 - 1.58(2)] = 23.84$ awrt **23.8**	B1ft	3.4
		(1)	
(d)	$\text{RSS} = 40.9 - \dfrac{(-16.4)^2}{10.36}$	M1	1.1b
	RSS = 14.938... awrt **14.9**	A1	1.1b
		(2)	
(e)	$\sum$residuals = 0 → –0.63 + (–0.32) + … + f + (–1.88) = 0	M1	3.1a
	f = **–1.04**	A1	1.1b
		(2)	
(f)	The residuals should be randomly scattered above and below zero so linear model may not be appropriate	B1	3.5b
		(1)	
			(11 marks)

Notes:

(a)
M1: For a complete correct method for finding r
A1: For awrt –0.797

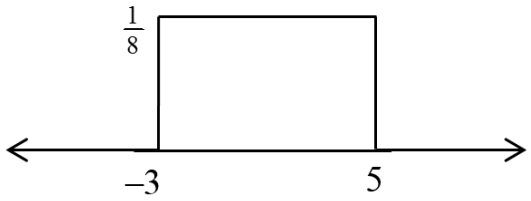
(b)
M1: For use of a correct model i.e. a correct expression for b (ft their S_{xy})
M1: For use of a correct model i.e. a correct (ft) expression for a
A1: For $y = 27.0 - 1.58x$ [a correct answer here can imply both method marks]

(c)
B1: For awrt 23.8 (evaluating their model found in part (b) with $x = 2$)

(d)
M1: For a correct expression for RSS
A1: For awrt 14.9

(e)
M1: For use of $\sum$residuals = 0 [Use of regression equation needs correct sign]
A1: For –1.04

(f)
B1: For identifying that the residuals are not randomly scattered above and below zero and concluding the linear regression model may not be appropriate

Question	Scheme	Marks	AOs
8(a)		B1 (shape) B1 (labels)	1.1b 1.1b
		(2)	
(b)	$P(X < 2(k - X)) = P(X < \frac{2}{3}k)$	M1	3.1a
	$\frac{\frac{2}{3}k - (-3)}{5 - (-3)} = 0.25$	M1	1.1b
	$k = -\frac{3}{2}$	A1	1.1b
		(3)	
(c)	$E(X^3) = \int_{-3}^{5} \frac{1}{5-(-3)} x^3 dx$	M1	2.1
	$= \left[\frac{1}{32}x^4\right]_{-3}^{5} = \frac{1}{32}(5^4 - (-3)^4)$	dM1	1.1b
	$= 17$*	A1*cso	1.1b
		(3)	
			(8 marks)

Notes:

(a)
B1: For correct shape
B1: For correct labels

(b)
M1: For simplifying to $P(X < \frac{2}{3}k)$
M1: For equating probability expression to 0.25
A1: For $-\frac{3}{2}$

Another method for part (b) is:
M1: For understanding $2[k - x] = -1$ and $x = -1$
M1: For substitution and attempt to solve
A1: For $-\frac{3}{2}$

(c)
B1: For integrating x^3 f(x)
M1: For use of correct limits (dependent on previous M1)
A1*: For fully correct solution leading to the given answer with no errors seen