Paper 1: Core Pure Mathematics Mark Scheme

Question	Scheme	Marks	AOs
1(a)	$\alpha\left(\frac{5}{\alpha}\right)\left(\alpha+\frac{5}{\alpha}-1\right)=15$	M1	1.1b
		A1	1.1b
	$\Rightarrow 5\alpha+\frac{25}{\alpha}-5=15 \Rightarrow \alpha^2-4\alpha+5=0$ $\Rightarrow \alpha=\frac{--4\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)}$ or $(\alpha-2)^2-4+5=0 \Rightarrow \alpha=...$	M1	3.1a
	$\Rightarrow \alpha = 2 \pm \mathrm{i}$	A1	1.1b
	Hence the roots of $\mathrm{f}(z)=0$ are $2+\mathrm{i}, 2-\mathrm{i}$ and 3	A1	2.2a
		(5)	
(b)	$p=-\left(\text{"}(2+\mathrm{i})\text{"}+\text{"}(2-\mathrm{i})\text{"}+\text{"}3\text{"}\right) \Rightarrow p=...$	M1	3.1a
	$\Rightarrow p=-7$ cso	A1	1.1b
		(2)	
	1(b) alternative		
	$\mathrm{f}(z)=(z-3)(z^2-4z+5) \Rightarrow p=...$	M1	3.1a
	$\Rightarrow p=-7$ cso	A1	1.1b
		(2)	
			(7 marks)

Notes:

(a)

M1: Multiplies the three given roots together and sets the result equal to 15 or -15

A1: Obtains a correct equation in α

M1: Forms a quadratic equation in α and attempts to solve this equation by either completing the square or using the quadratic formula to give $\alpha = ...$

A1: $\alpha = 2 \pm \mathrm{i}$

A1: Deduces the roots are $2+\mathrm{i}, 2-\mathrm{i}$ and 3

(b)

M1: Applies the process of finding $-\sum\left(\text{of their three roots found in part } (a)\right)$ to give $p=...$

A1: $p=-7$ by correct solution only

(b) Alternative

M1: Applies the process expanding $(z-\text{"}3\text{"})(z-(\text{their sum})z+\text{their product})$ in order to find $p=...$

A1: $p=-7$ by correct solution only

Question	Scheme	Marks	AOs
2(a)	$\mathbf{r}\bullet\begin{pmatrix}3\\-1\\2\end{pmatrix}=\begin{pmatrix}5\\-3\\-4\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\2\end{pmatrix}$	M1	1.1b
	$3x-y+2z=10$	A1	2.5
		(2)	
(b)	$\begin{pmatrix}3\\-1\\2\end{pmatrix}\bullet\begin{pmatrix}-1\\-5\\3\end{pmatrix}=8$	B1	1.1b
	$\sqrt{(3)^2+(-1)^2+(2)^2}\,.\sqrt{(-1)^2+(-5)^2+(3)^2}\,\cos\alpha="-3+5+6"$	M1	1.1b
	$\theta=90^\circ-\arccos\left(\frac{8}{\sqrt{14}.\sqrt{35}}\right)$ or $\sin\theta=\frac{8}{\sqrt{14}.\sqrt{35}}$	M1	2.1
	$\theta=21.2^\circ$ (1 dp) * cso	A1*	1.1b
		(4)	
(c)	$3(7-\lambda)-(3-5\lambda)+2(-2+3\lambda)=10\Rightarrow\lambda=...$	M1	3.1a
	$\lambda=-\frac{1}{2}$	A1	1.1b
	$\overrightarrow{OX}=\begin{pmatrix}7\\3\\-2\end{pmatrix}-\frac{1}{2}\begin{pmatrix}-1\\-5\\3\end{pmatrix}=\begin{pmatrix}...\\...\\...\end{pmatrix}$	M1	1.1b
	$X(7.5,5.5,-3.5)$	A1ft	1.1b
		(4)	
		(10 marks)	

Notes:

(a)

M1: Attempts to apply the formula $\mathbf{r.n}=\mathbf{a.n}$

A1: Correct Cartesian notation. e.g. $3x-y+2z=10$ or $-3x+y-2z=-10$

Note: Do not allow final answer given as $\mathbf{r}\bullet(3\mathbf{i}-\mathbf{j}+2\mathbf{k})=10$, o.e.

(b)

B1: $\overline{OA}\bullet\mathbf{n}=8$

M1: An attempt to apply the correct dot product formula between **n** and **d**

M1: Depends on previous M mark. Applies the dot product formula to find the angle between Π and l

A1*: 21.2° cso

Question 2 notes continued:
(c) **M1:** Substitutes l into Π and solves the resulting equation to give $\lambda = ...$ **A1:** $\lambda = -\frac{1}{2}$ o.e. **M1:** Depends on previous M mark. Substitutes their λ into l and finds at least one of the coordinates **A1ft:** $(7.5, 5.5, -3.5)$ but follow through on their value of λ

Question	Scheme	Marks	AOs
3	x = value of savings account, y = value of property bond account, z = value of share dealing account	M1	3.1b
	$x+y+z=5000$ $x+400=y$ $0.015x+0.035y-0.025z=79$ **or** $1.015x+1.035y+0.975z=5079$	A1	1.1b
	Let $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.015 & 0.035 & -0.025 \end{pmatrix}$ or $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1.015 & 1.035 & 0.975 \end{pmatrix}$		
	e.g. $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.015 & 0.035 & -0.025 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ -400 \\ 79 \end{pmatrix}$	M1	3.1a
		A1	1.1b
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.015 & 0.035 & -0.025 \end{pmatrix}^{-1}\begin{pmatrix} 5000 \\ -400 \\ 79 \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1800 \\ 2200 \\ 1000 \end{pmatrix}$	A1	1.1b
	Tyler invested £1800 in the savings account, £2200 in the property bond account and £1000 in the share dealing account	A1ft	3.2a
			(7 marks)

Notes:

M1: Attempts to set up 3 equations with 3 unknowns

A1: At least 2 equations are correct with the appropriate variables defined

M1: Sets up a matrix equation of the form, e.g. $\begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$, where "…" are numerical values

A1: Correct matrix equation (or equivalent)

M1: Depends on previous M mark. Applies $(\text{their } \mathbf{A})^{-1}\begin{pmatrix} 5000 \\ \text{their } "-400" \\ \text{their } "79" \end{pmatrix}$ and obtains at least one value of x, y or z

A1: Correct answer

A1ft: Correct follow through answer in context

Question	Scheme	Marks	AOs
4	$\{w = x-1 \Rightarrow\}\ x = w+1$	B1	3.1a
	$(w+1)^3 + 3(w+1)^2 - 8(w+1) + 6 = 0$	M1	3.1a
	$w^3 + 3w^2 + 3w + 1 + 3(w^2 + 2w + 1) - 8w - 8 + 6 = 0$		
	$w^3 + 6w^2 + w + 2 = 0$	M1	1.1b
		A1	1.1b
		A1	1.1b
		(5)	
	Alternative		
	$\alpha + \beta + \gamma = -3,\ \alpha\beta + \beta\gamma + \alpha\gamma = -8,\ \alpha\beta\gamma = -6$	B1	3.1a
	sum roots $= \alpha - 1 + \beta - 1 + \gamma - 1$	M1	3.1a
	$= \alpha + \beta + \gamma - 3 = -3 - 3 = -6$		
	pair sum $= (\alpha-1)(\beta-1) + (\alpha-1)(\gamma-1) + (\beta-1)(\gamma-1)$		
	$= \alpha\beta + \alpha\gamma + \beta\gamma - 2(\alpha+\beta+\gamma) + 3$		
	$= -8 - 2(-3) + 3 = 1$		
	product $= (\alpha - 1)(\beta - 1)(\gamma - 1)$		
	$= \alpha\beta\gamma - (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha+\beta+\gamma) - 1$		
	$= -6 - (-8) - 3 - 1 = -2$		
	$w^3 + 6w^2 + w + 2 = 0$	M1	1.1b
		A1	1.1b
		A1	1.1b
		(5)	
	(5 marks)		

Notes:

B1: Selects the method of making a connection between x and w by writing $x = w+1$

M1: Applies the process of substituting their $x = w+1$ into $x^3 + 3x^2 - 8x + 6 = 0$

M1: Depends on previous M mark. Manipulating their equation into the form $w^3 + pw^2 + qw + r = 0$

A1: At least two of p, q, r are correct

A1: Correct final equation

Alternative

B1: Selects the method of giving three correct equations each containing α, β and γ

M1: Applies the process of finding sum roots, pair sum and product

M1: Depends on previous M mark. Applies $w^3 - (\text{their sum roots})w^2 + (\text{their pair sum})w - \text{their } \alpha\beta\gamma = 0$

A1: At least two of p, q, r are correct

A1: Correct final equation

Question	Scheme	Marks	AOs
5(a)	$\det(\mathbf{M}) = (1)(1) - (\sqrt{3})(-\sqrt{3})$	M1	1.1a
	M is non-singular because $\det(\mathbf{M}) = 4$ and so $\det(\mathbf{M}) \neq 0$	A1	2.4
		(2)	
(b)	$\text{Area}(S) = 4(5) = 20$	B1ft	1.2
		(1)	
(c)	$k = \sqrt{(1)(1) - (\sqrt{3})(-\sqrt{3})}$	M1	1.1b
	$= 2$	A1ft	1.1b
		(2)	
(d)	$\cos\theta = \frac{1}{2}$ or $\sin\theta = \frac{\sqrt{3}}{2}$ or $\tan\theta = \sqrt{3}$	M1	1.1b
	$\theta = 60°$ or $\frac{\pi}{3}$	A1	1.1b
		(2)	
			(7 marks)

Notes:

(a)

M1: An attempt to find $\det(\mathbf{M})$.

A1: $\det(\mathbf{M}) = 4$ **and** reference to zero, e.g. $4 \neq 0$ **and** conclusion.

(b)

B1ft: 20 or a correct ft based on their answer to part (a).

(c)

M1: $\sqrt{(\text{their } \det\mathbf{M})}$

A1ft: 2

(d)

M1: **Either** $\cos\theta = \frac{1}{(\text{their } k)}$ **or** $\sin\theta = \frac{\sqrt{3}}{(\text{their } k)}$ **or** $\tan\theta = \sqrt{3}$

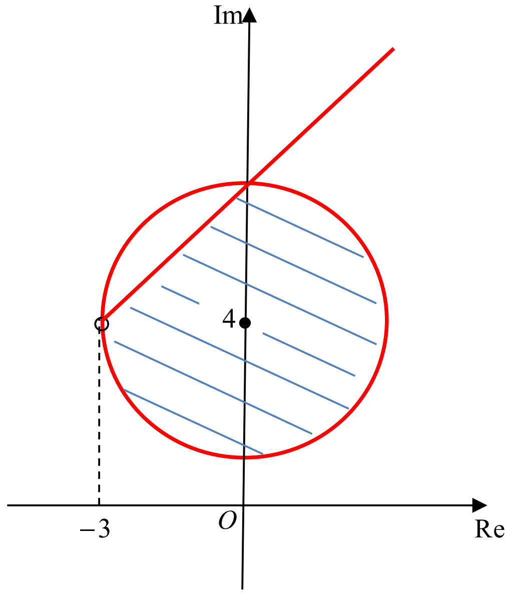
A1: $\theta = 60^{\circ}$ or $\frac{\pi}{3}$. Also accept any value satisfying $360n + 60^{\circ}$, $n \in \mathbb{Z}$, o.e.

Question	Scheme	Marks	AOs
6(a)	$n=1,\ \sum_{r=1}^{1} r^2 = 1$ and $\frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}(1)(2)(3) = 1$	B1	2.2a
	Assume general statement is true for $n = k$ So assume $\sum_{r=1}^{k} r^2 = \frac{1}{6}k(k+1)(2k+1)$ is true	M1	2.4
	$\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$	M1	2.1
	$= \frac{1}{6}(k+1)(2k^2+7k+6)$	A1	1.1b
	$= \frac{1}{6}(k+1)(k+2)(2k+3) = \frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$	A1	1.1b
	Then the general result is <u>true for $n = k+1$</u> As the general result has been shown to be <u>true for $n = 1$,</u> then the general result <u>is true for all n</u> $\in \mathbb{Z}^+$	A1	2.4
		(6)	
(b)	$\sum_{r=1}^{n} r(r+6)(r-6) = \sum_{r=1}^{n} (r^3 - 36r)$		
	$= \frac{1}{4}n^2(n+1)^2 - \frac{36}{2}n(n+1)$	M1 A1	2.1 1.1b
	$= \frac{1}{4}n(n+1)\left[n(n+1) - 72\right]$	M1	1.1b
	$= \frac{1}{4}n(n+1)(n-8)(n+9)$ * cso	A1*	1.1b
		(4)	
(c)	$\frac{1}{4}n(n+1)(n-8)(n+9) = \frac{17}{6}n(n+1)(2n+1)$	M1	1.1b
	$\frac{1}{4}(n-8)(n+9) = \frac{17}{6}(2n+1)$	M1	1.1b
	$3n^2 - 65n - 250 = 0$	A1	1.1b
	$(3n+10)(n-25) = 0$	M1	1.1b
	(As n must be a positive integer,) $n = 25$	A1	2.3
		(5)	
			(15 marks)

Question 6 notes:
(a) **B1:** Checks $n=1$ works for both sides of the general statement **M1:** Assumes (general result) true for $n=k$ **M1:** Attempts to add $(k+1)^{\text{th}}$ term to the sum of k terms **A1:** Correct algebraic work leading to **either** $\frac{1}{6}(k+1)(2k^2+7k+6)$ **or** $\frac{1}{6}(k+2)(2k^2+5k+3)$ **or** $\frac{1}{6}(2k+3)(k^2+3k+2)$ **A1:** Correct algebraic work leading to $\frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$ **A1:** cso leading to a correct induction statement conveying all three underlined points
(b) **M1:** Substitutes at least one of the standard formulae into their expanded expression **A1:** Correct expression **M1:** Depends on previous M mark. Attempt to factorise at least $n(n+1)$ having used **A1*:** Obtains $\frac{1}{4}n(n+1)(n-8)(n+9)$ by cso
(c) **M1:** Sets their part (a) answer equal to $\frac{17}{6}n(n+1)(2n+1)$ **M1:** Cancels out $n(n+1)$ from both sides of their equation **A1:** $3n^2-65n-250=0$ **M1:** A valid method for solving a 3 term quadratic equation **A1:** Only one solution of $n=25$

Question	Scheme		Marks	AOs
7(a)	Depth = 0.16 (m)		B1	2.2b
			(1)	
(b)	$y = 1 + kx^2 \Rightarrow 1.16 = 1 + k(0.2)^2 \Rightarrow k = ...$		M1	3.3
	$\Rightarrow k = 4$ cao $\{\text{So } y = 1 + 4x^2\}$		A1	1.1b
			(2)	
(c)	$\frac{\pi}{4}\int (y-1)\mathrm{d}y$	$\frac{\pi}{4}\int y\,\mathrm{d}y$	B1ft	1.1a
	$= \left\{\frac{\pi}{4}\right\}\int_1^{1.16} (y-1)\mathrm{d}y$	$= \left\{\frac{\pi}{4}\right\}\int_0^{0.16} y\mathrm{d}y$	M1	3.3
	$= \left\{\frac{\pi}{4}\right\}\left[\frac{y^2}{2} - y\right]_1^{1.16}$	$= \left\{\frac{\pi}{4}\right\}\left[\frac{y^2}{2}\right]_0^{0.16}$	M1 A1	1.1b 1.1b
	$= \frac{\pi}{4}\left(\left(\frac{1.16^2}{2} - 1.16\right) - \left(\frac{1}{2} - 1\right)\right) \{= 0.0032\pi\}$	$= \frac{\pi}{4}\left(\left(\frac{0.16^2}{2}\right) - (0)\right) \{= 0.0032\pi\}$		
	$V_{\text{cylinder}} = \pi(0.2)^2(1.16)\ \{= 0.0464\pi\}$		B1	1.1b
	$\text{Volume} = 0.0464\pi - 0.0032\pi\ \{= 0.0432\pi\}$		M1	3.4
	$= 0.1357168026... = 0.136(\text{m}^3)$ (3sf)		A1	1.1b
			(7)	
(d)	Any one of e.g. the measurements may not be accurate the inside surface of the bowl may not be smooth there may be wastage of concrete when making the bird bath		B1	3.5b
			(1)	
(e)	Some comment consistent with their values. We do need a reason e.g. $\left[\left(\frac{0.136 - 0.127}{0.127}\right) \times 100 = 7.0866...\right]$ so not a good estimate because the volume of concrete needed to make the bird bath is approximately 7% lower than that predicted by the model **or** We might expect the actual amount of concrete to exceed that which the model predicts due to wastage, so the model does not look suitable since it predicts more concrete than was used		B1ft	3.5a
			(1)	
			(12 marks)	

Question 7 notes:
(a) **B1:** Infers that the maximum depth of the bird bath could be 0.16 (m)
(b) **M1:** Substitutes $y = 1.16$ and $x = 0.2$ or $x = -0.2$ into $y = 1 + kx^2$ and rearranges to give $k = ...$ **A1:** $k = 4$ cao
(c) **B1ft:** Uses the model to obtain either $\frac{\pi}{(\text{their } k)}\int (y-1)\,\mathrm{d}y$ **or** $\frac{\pi}{(\text{their } k)}\int y\,\mathrm{d}y$ **M1:** Chooses limits that are appropriate to their model **M1:** Integrates y (with respect to y) to give $\pm\lambda y^2$, where $\lambda \neq 0$ is a constant **A1:** Uses their model correctly to give either $y - 1 \to \frac{y^2}{2} - y$ or $y \to \frac{y^2}{2}$ **B1:** $V_{\text{cylinder}} = \pi(0.2)^2(1.16)$ or 0.0464π **or** $\frac{29}{625}\pi$, o.e. **M1:** Depends on **both** previous M marks Uses the model to find $V_{\text{their cylinder}}$ – their integrated volume **A1:** 0.136 cao
(d) **B1:** States an acceptable limitation of the model
(e) **B1ft:** Compares the actual volume with their answer to (c). Makes an assessment of the model. E.g. evaluates the percentage error and uses this to make a sensible comment about the model with a reason

Question	Scheme	Marks	AOs
8(a)		M1 A1 M1 A1 M1 A1	1.1b 1.1b 1.1b 2.2a 3.1a 1.1b
		(6)	
(b)	$(\arg w)_{\max} = \frac{\pi}{2} + \arcsin\left(\frac{3}{4}\right)$	M1	3.1a
	$= 2.42\ (2\text{dp})$ **cao**	A1	1.1b
		(2)	
			(8 marks)

Notes:

(a)
M1: Circle
A1: Centre (0, 4) and above the real axis
M1: Half-line
A1: $(-3, 4)$ positioned correctly and the half-line intersects the top of the circle on the y-axis
M1: Depends on **both** previous M marks Shades in a region inside the circle and below the half-line
A1: cso

Note: Final A1 mark is dependent on all previous marks being scored in part (a)

(b)
M1: Uses trigonometry to give an expression for an angle in the range $\left(\frac{\pi}{2}, \pi\right)$ or $(90^\circ, 180^\circ)$
A1: 2.42 cao

Question	Scheme	Marks	AOs
9(a)	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\left\{ \overrightarrow{OF} \bullet \overrightarrow{AB} = 0 \Rightarrow \right\} \begin{pmatrix} -3 + 12\lambda \\ 1 + 3\lambda \\ -7 + 18\lambda \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = 0$ $\Rightarrow -36 + 144\lambda + 3 + 9\lambda - 126 + 324\lambda = 0 \Rightarrow 477\lambda - 159 = 0$	dM1	1.1b
	$\Rightarrow \lambda = \frac{1}{3}$	A1	1.1b
	$\left\{ \overrightarrow{OF} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and minimum distance $= \sqrt{(1)^2 + (2)^2 + (-1)^2}$	dM1	3.1a
	$= \sqrt{6}$ **or** $2.449\ldots$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F	A1ft	3.2a
		(7)	

Question	Scheme	Marks	
	9(a) Alternative 1		
	$\overrightarrow{AB}=\begin{pmatrix}9\\4\\11\end{pmatrix}-\begin{pmatrix}-3\\1\\-7\end{pmatrix}\left\{=\begin{pmatrix}12\\3\\18\end{pmatrix}\right\}$ or $\mathbf{d}=\begin{pmatrix}4\\1\\6\end{pmatrix}$	M1	3.1a
	$\left\{\overrightarrow{OA}=\begin{pmatrix}-3\\1\\-7\end{pmatrix}\text{ and }\overrightarrow{AB}=\begin{pmatrix}12\\3\\18\end{pmatrix}\Rightarrow\right\}\begin{pmatrix}-3\\1\\-7\end{pmatrix}\bullet\begin{pmatrix}12\\3\\18\end{pmatrix}$	M1	1.1b
	$\cos\theta\left\{=\dfrac{\overrightarrow{OA}\bullet\overrightarrow{AB}}{\lvert\overrightarrow{OA}\rvert.\lvert\overrightarrow{AB}\rvert}\right\}=\dfrac{\pm\left(\begin{pmatrix}-3\\1\\-7\end{pmatrix}\bullet\begin{pmatrix}12\\3\\18\end{pmatrix}\right)}{\sqrt{(-3)^2+(1)^2+(-7)^2}.\sqrt{(12)^2+(3)^2+(18)^2}}$	dM1	1.1b
	$\left\{\cos\theta=\dfrac{-36+3-126}{\sqrt{59}.\sqrt{477}}=\dfrac{-159}{\sqrt{59}.\sqrt{477}}\right\}$		
	$\theta=161.4038029...$ or $18.59619709...$ or $\sin\theta=0.3188964021...$	A1	1.1b
	minimum distance $=\sqrt{(-3)^2+(1)^2+(-7)^2}\sin(18.59619709...)$	dM1	3.1a
	$=\sqrt{6}$ or $2.449\ldots$	A1	1.1b
	>2, so the octopus is not able to catch the fish F	A1ft	3.2a
		(7)	
	9(a) Alternative 2		
	$\overrightarrow{AB}=\begin{pmatrix}9\\4\\11\end{pmatrix}-\begin{pmatrix}-3\\1\\-7\end{pmatrix}\left\{=\begin{pmatrix}12\\3\\18\end{pmatrix}\right\}$ or $\mathbf{d}=\begin{pmatrix}4\\1\\6\end{pmatrix}$	M1	3.1a
	$\left\{\overrightarrow{OF}=\mathbf{r}=\right\}\begin{pmatrix}-3\\1\\-7\end{pmatrix}+\lambda\begin{pmatrix}12\\3\\18\end{pmatrix}$	M1	1.1b
	$\lvert\overrightarrow{OF}\rvert^2=(-3+12\lambda)^2+(1+3\lambda)^2+(-7+18\lambda)^2$	dM1	1.1b
	$=9-72\lambda+144\lambda^2+1+6\lambda+9\lambda^2+49-252\lambda+324\lambda^2$		
	$=477\lambda^2-318\lambda+59$	A1	1.1b
	$=53(3\lambda-1)^2+6$	dM1	3.1a
	minimum distance $=\sqrt{6}$ or $2.449\ldots$	A1	1.1b
	>2, so the octopus is not able to catch the fish F	A1ft	3.2a
		(7)	

Question	Scheme	Marks	AOs
9(b)	e.g. Fish F may not swim in an exact straight line from A to B Fish F may hit an obstacle whilst swimming from A to B Fish F may deviate his path to avoid being caught by the octopus	B1	3.5b
		(1)	
(c)	e.g. Octopus is effectively modelled as a particle – so we may need to look at where the octopus's mass is distributed Octopus may during the fish F's motion move away from its fixed location at O	B1	3.5b
		(1)	
			(9 marks)

Question 9 notes:

(a)

M1: Attempts to find $\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OA} - \overrightarrow{OB}$ **or** the direction vector **d**

M1: Applies $\overrightarrow{OA} + \lambda(\text{their } \overrightarrow{AB} \text{ or their } \overrightarrow{BA} \text{ or their } \mathbf{d})$ or equivalent

M1: Depends on previous M mark. Writes down $(\text{their } \overrightarrow{OF} \text{ which is in terms of } \lambda)\bullet(\text{their } \overrightarrow{AB}) = 0$. Can be implied

A1: Lambda is correct. e.g. $\lambda = \frac{1}{3}$ for $\overrightarrow{AB} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$ or $\lambda = 1$ for $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$

M1: Depends on previous M mark. Complete method for finding $\left|\overrightarrow{OF}\right|$

A1: $\sqrt{6}$ **or** awrt 2.4

A1ft : Correct follow through conclusion, which is in context with the question

Alternative 1

(a)

M1: Attempts to find $\overrightarrow{OB} - \overrightarrow{OA}$ **or** $\overrightarrow{OA} - \overrightarrow{OB}$ **or** the direction vector **d**

M1: Realisation that the dot product is required between $\overrightarrow{OA}$ and their $\overrightarrow{AB}$. (o.e.)

M1: Depends on previous M mark. Applies dot product formula between $\overrightarrow{OA}$ and their $\overrightarrow{AB}$ (o.e.)

A1: $\theta =$ awrt 161.4 or awrt 18.6 or $\sin\theta =$ awrt 0.319

M1: Depends on previous M mark. $(\text{their } OA)\sin(\text{their } \theta)$

A1: $\sqrt{6}$ or awrt 2.4

A1ft : Correct follow through conclusion, which is in context with the question

Question 9 notes continued:
Alternative 2 **(a)** **M1:** Attempts to find $\overrightarrow{OB}-\overrightarrow{OA}$ **or** $\overrightarrow{OA}-\overrightarrow{OB}$ **or** the direction vector **d** **M1:** Applies $\overrightarrow{OA}+\lambda(\text{their } \overrightarrow{AB} \text{ or their } \overrightarrow{BA} \text{ or their } \mathbf{d})$ or equivalent **M1:** Depends on previous M mark. Applies Pythagoras by finding $\left\lvert\overrightarrow{OF}\right\rvert^2$, o.e. **A1:** $\left\lvert\overrightarrow{OF}\right\rvert^2 = 477\lambda^2 - 318\lambda + 59$ **M1:** Depends on previous M mark. Method of completing the square or differentiating their $\left\lvert\overrightarrow{OF}\right\rvert^2$ w.r.t. λ **A1:** $\sqrt{6}$ or awrt 2.4 **A1ft :** Correct follow through conclusion, which is in context with the question
(b) **B1:** An acceptable criticism for fish F, which is in context with the question
(c) **B1:** An acceptable criticism for the octopus, which is in context with the question