GCE AS/A LEVEL

2305U10-1

S23-2305U10-1

MONDAY, 15 MAY 2023 – AFTERNOON

FURTHER MATHEMATICS – AS unit 1
FURTHER PURE MATHEMATICS A

1 hour 30 minutes

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Answers without working may not gain full credit.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 70.
The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

Additional Formulae for 2023

Laws of Logarithms

$$\log_a x + \log_a y \equiv \log_a (xy)$$
$$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y}\right)$$
$$k \log_a x \equiv \log_a \left(x^k\right)$$

Sequences

General term of an arithmetic progression:

$$u_n = a + (n-1)d$$

General term of a geometric progression:

$$u_n = ar^{n-1}$$

Mensuration

For a circle of radius, r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A :

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Calculus and Differential Equations

Differentiation

Function	Derivative
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f\left(g(x)\right)$	$f'\left(g(x)\right)g'(x)$

Integration

Function	Integral
$f'\left(g(x)\right)g'(x)$	$f\left(g(x)\right) + c$

Area under a curve $= \int_a^b y \,\mathrm{d}x$

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. The complex number z is given by $z = 3 + \lambda \mathrm{i}$, where λ is a positive constant. The complex conjugate of z is denoted by $\bar{z}$.

Given that $z^2 + \bar{z}^2 = 2$, find the value of λ. [4]

2. The matrices $\mathbf{A}$ and $\mathbf{B}$ are such that $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 4 & -7 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 0 & 9 \\ 4 & -20 & 13 \end{bmatrix}$.

(a) Find the inverse of $\mathbf{A}$. [2]

(b) Hence, find the matrix $\mathbf{X}$, where $\mathbf{AX} = \mathbf{B}$. [3]

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3. Given that $5 - \mathrm{i}$ is a root of the equation $x^4 - 10x^3 + 10x^2 + 160x - 416 = 0$,

(a) write down another root of the equation, [1]

(b) find the remaining roots. [5]

4. The transformation *T* in the plane consists of a translation in which the point (x, y) is transformed to the point $(x + 2, y - 2)$, followed by a reflection in the line $y = x$.

(a) Determine the 3×3 matrix which represents *T*. [4]

(b) Determine how many invariant points exist under the transformation *T*. [3]

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5. The points *A* and *B* have coordinates (3, 4, –2) and (–2, 0, 7) respectively.
The equation of the plane Π is given by $2x+3y+3z=27$.

(a) Show that the vector equation of the line *AB* may be expressed in the form

$$\mathbf{r} = (3-5\lambda)\mathbf{i} + (4-4\lambda)\mathbf{j} + (-2+9\lambda)\mathbf{k}.$$ [3]

(b) Find the coordinates of the point of intersection of the line *AB* and the plane Π. [3]

6. The complex number z is represented by the point $P(x, y)$ in an Argand diagram.
Given that

$$|z-3+\mathrm{i}| = 2|z-5-2\mathrm{i}|,$$

show that the locus of *P* is a circle and write down the coordinates of its centre. [6]

7. Using mathematical induction, prove that

$$\begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix}^n = \begin{bmatrix} 2^n & 2^{n-1} \times 5n \\ 0 & 2^n \end{bmatrix}$$

for all positive integers n. [7]

8. The roots of the cubic equation $x^3 + 5x^2 + 2x + 8 = 0$ are denoted by α, β, γ.

Determine the cubic equation whose roots are $\dfrac{\alpha}{\beta\gamma}, \dfrac{\beta}{\gamma\alpha}, \dfrac{\gamma}{\alpha\beta}$.

Give your answer in the form $ax^3 + bx^2 + cx + d = 0$, where a, b, c, d are constants to be determined. [9]

9. The complex numbers z and w are represented by the points $P(x, y)$ and $Q(u, v)$ respectively, in Argand diagrams, and $w = 1 - z^2$.

(a) Find expressions for u and v in terms of x and y. [4]

(b) The point P moves along the line $y = 4x$. Find the equation of the locus of Q. [4]

(c) Find the perpendicular distance of the point corresponding to $z = 2 + 5\mathrm{i}$ in the (u, v)-plane, from the locus of Q. [4]

10. Gareth is investigating a series involving cube numbers. His series is

$$1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3 + 7^3 - \dots$$

Gareth continues his series and **ends with an odd number**.

Find and simplify an expression for the sum of Gareth's series in terms of k, where k is the number of odd numbers in his series. [8]

END OF PAPER

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