GCE

FURTHER MATHEMATICS

UNIT 1: FURTHER PURE MATHEMATICS A

SAMPLE ASSESSMENT MATERIALS

(1 hour 30 minutes)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

• a 12 page answer book;
• a Formula Booklet;
• a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1.   Use mathematical induction to prove that $4^n + 2$ is divisible by 6 for all positive integers n. [7]

2.   Solve the equation $2z + i\bar{z} = \dfrac{-1+7i}{2+i}$.

    (a)   Give your answer in Cartesian form [7]

    (b)   Give your answer in modulus-argument form. [4]

3.   Find an expression, in terms of n, for the sum of the first n terms of the series

$$1.2.4 + 2.3.5 + 3.4.6 + \ldots + n(n+1)(n+3) + \ldots$$

Express your answer as a product of linear factors. [6]

4.   The roots of the equation
$$x^3 - 4x^2 + 14x - 20 = 0$$

are denoted by α, β, γ.

    (a)   Show that
$$\alpha^2 + \beta^2 + \gamma^2 = -12.$$

Explain why this result shows that exactly one of the roots of the above cubic equation is real. [3]

    (b)   Given that one of the roots is $1+3i$, find the other two roots.
Explain your method for each root. [4]

5.   The complex number z is represented by the point $P(x, y)$ in an Argand diagram and

$$|z - 3| = 2|z + i|.$$

Show that the locus of P is a circle and determine its radius and the coordinates of its centre.

[9]

6.   The transformation T in the plane consists of a reflection in the line $y = x$, followed by a translation in which the point (x, y) is transformed to the point $(x + 1, y - 2)$, followed by an anticlockwise rotation through $90°$ about the origin.

    (a)   Find the 3×3 matrix representing T. [6]

    (b)   Show that T has no fixed points. [3]

7.   The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u,v)$ in Argand diagrams and

$$w = z(1+z)$$

(a)   Show that
$$v = y(1+2x)$$

and find an expression for u in terms of x and y.   [4]

(b)   The point P moves along the line $y = x + 1$. Find the Cartesian equation of the locus of Q, giving your answer in the form $v = au^2 + bu$, where a and b are constants whose values are to be determined.   [5]

8.   The line L passes through the points A(1, 2, 3) and B(2, 3, 1).

(a)   (i)   Find the vector **AB**.

       (ii)   Write down the vector equation of the line L.   [3]

(b)   The plane Π has equation $x + 3y - 2z = 5$.

       (i)   Find the coordinates of the point of intersection of L and Π.

       (ii)   Find the acute angle between L and Π.   [9]