

AS Further Mathematics Unit 2: Further Statistics A

General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations

The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only

MR = misread

PA = premature approximation

bod = benefit of doubt

oe = or equivalent

si = seen or implied

ISW = ignore subsequent working

F.T. = follow through (✓ indicates correct working following an error and ✗ indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation

A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads

When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.

This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

5. Marking codes

- 'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- 'm' marks are dependant method marks. They are only given if the relevant previous 'M' mark has been earned.
- 'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- 'S' marks are awarded for strategy
- 'E' marks are awarded for explanation
- 'U' marks are awarded for units
- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

AS Further Mathematics Unit 2: Further Statistics A

Solutions and Mark Scheme

Qu. No.	Solution	Mark	AO	Notes
1.(a)	$E(W) = E(X)E(Y) = 168$	B1	AO1	
(b)	$E(X^2) = (E(X))^2 + \text{Var}(X)$ $= 221$ $E(Y^2) = 153$ $\text{Var}(W) = E(W^2) - [E(W)]^2$ $= E(X^2)E(Y^2) - (E(X)E(Y))^2$ $= 221 \times 153 - 168^2 (= 5589)$ $\text{SD} = 74.8 \text{ (74.75961...)}$	M1 A1 A1 M1 A1 A1 [7]	AO1 AO1 AO1 AO3 AO1 AO1	
2(a)	$E(T) = \frac{1}{2500} \int_0^{10} t^2(100 - t^2) dt$ $= \frac{1}{2500} \left[\frac{100t^3}{3} - \frac{t^5}{5} \right]_0^{10}$ $= 5.33(333...)$	M1 A1 A1	AO3 AO1 AO1	
(b)(i)	$F(t) = \frac{1}{2500} \int_0^t u(100 - u^2) du$ $= \frac{1}{2500} \left[50u^2 - \frac{u^4}{4} \right]_0^t$ $= \frac{1}{2500} \left(50t^2 - \frac{t^4}{4} \right)$ (for $0 \leq t \leq 10$) $= 1$ for $t > 10$ $(F(t) = 0$ for $t < 0)$	M1 A1 A1 B1	AO3 AO1 AO1 AO1	Allow omission of $t < 0$
(ii)	$P(T > 5) = 1 - F(5)$ $= 0.563 \text{ (0.5625)}$	M1 A1	AO3 AO1	
(iii)	The median m satisfies $F(m) = 0.5$ $m^4 - 200m^2 + 5000 = 0$ $m^2 = \frac{200 \pm \sqrt{40000 - 20000}}{2}$ $(= 29.289...)$ $m = 5.41(1961...)$	M1 A1 A1 A1 [13]	AO3 AO3 AO1 AO1	

Qu. No.	Solution	Mark	AO	Notes																											
3(a)	<p>The ranks are</p> <table border="1"> <tr><td>S</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td></tr> <tr><td>H</td><td>3</td><td>6</td><td>1</td><td>8</td><td>7</td><td>2</td><td>4</td><td>5</td></tr> <tr><td>G</td><td>5</td><td>6</td><td>4</td><td>3</td><td>8</td><td>2</td><td>7</td><td>1</td></tr> </table> $\sum d^2 = 64$ $r_s = 1 - \frac{6 \times 64}{8 \times 63}$ $= 0.238(095238\dots)$	S	A	B	C	D	E	F	G	H	H	3	6	1	8	7	2	4	5	G	5	6	4	3	8	2	7	1	M1 A1 A1	AO3 AO1 AO1	Attempt to find ranks Correct values for 1 st row Correct values for 2 nd row
S	A	B	C	D	E	F	G	H																							
H	3	6	1	8	7	2	4	5																							
G	5	6	4	3	8	2	7	1																							
(b)	<p>5% 1-tail crit value = 0.6429 This suggests that there is no positive association between marks in History and marks in Geography.</p>	B1 B1	AO1 AO3																												
(c)	Because the data might not follow a bivariate normal distribution.	B1	AO2																												
		[9]																													
4(a)	The evidence suggests that good graduate prospects are associated with: strong research quality high entry standards.	B1 B1	AO2 AO2	Or The evidence suggests that good graduate prospects are not associated with student satisfaction																											
(b)	<p>Gradient $= \frac{122.72}{1.0542}$ $= 116.4(105)$</p> <p>Intercept $= \frac{2522}{7} - 116.4105 \times \frac{22.24}{7}$ $= -9.5(67)$</p> $y = 116.4(105)x - 9.5(67)$	M1 A1	AO2 AO1																												
(c)	$116.4 \times 3 - 9.6$ $= 339.6$	M1 A1	AO3 AO1	Allow for using 116.4 giving -9.5(337) FT 'their' gradient and intercept Accept 358.7(988...) if using exact values throughout																											
		[9]																													

Qu. No.	Solution	Mark	AO	Notes																			
5(a)	<p>H_0 : The data can be modelled by the Poisson distribution with mean 2. H_1 : The data cannot be modelled by the Poisson distribution with mean 2.</p>	B1	AO3																				
(b)	<p>The expected frequencies are</p> <table border="1"> <thead> <tr> <th>Goals scored</th> <th>0</th> <th>1</th> <th>2</th> <th>3</th> <th>4 or more</th> </tr> </thead> <tbody> <tr> <td>Obs</td> <td>6</td> <td>11</td> <td>15</td> <td>10</td> <td>8</td> </tr> <tr> <td>Exp</td> <td>6.767</td> <td>13.534</td> <td>13.534</td> <td>9.022</td> <td>7.144</td> </tr> </tbody> </table>	Goals scored	0		1	2	3	4 or more	Obs	6	11	15	10	8	Exp	6.767	13.534	13.534	9.022	7.144	B1 B1	AO3 AO3	For at least 1 correct For all correct
Goals scored	0	1	2		3	4 or more																	
Obs	6	11	15		10	8																	
Exp	6.767	13.534	13.534		9.022	7.144																	
	<p>Use of χ^2 stat = $\sum \frac{O^2}{E} - N$</p> $= \frac{6^2}{6.767} + \frac{11^2}{13.534} + \dots + \frac{8^2}{7.144} - 50$ $= 0.93$ <p>DF = 4 5% crit val = 9.488 Since $0.93 < 9.488$ (Accept H_0) We conclude that the data can be modelled by the Poisson distribution with mean 2.</p>	M1 A1 A1 B1 B1 B1 B1	AO3 AO2 AO1 AO1 AO1 AO2 AO3																				
		[10]																					
6(a)	<p>The number of arrivals X is Poi(7.5)</p> $P(X = 5) = \frac{e^{-7.5} \times 7.5^5}{5!}$ $= 0.109(3745\dots)$	B1 M1 A1	AO3 AO1 AO1	Or straight from the calculator																			
(b)(i)	<p>$P(T > t) = P(\text{No customers arrive between 11am and } t \text{ mins after 11am})$ $= e^{-0.5t}$</p>	B1	AO2																				
(ii)	<p>The cumulative distribution function of T is</p> $F(t) = P(T \leq t)$ $= 1 - P(T > t) = 1 - e^{-0.5t}$ <p>Let $f(t)$ denote the probability density function of T</p> $f(t) = F'(t)$ $= 0.5e^{-0.5t}$	M1 A1 M1 A1	AO3 AO2 AO2 AO1																				
(iii)	<p>This is the exponential distribution. Therefore mean = standard deviation = $1/0.5 = 2$</p>	B1 B1 [10]	AO2 AO2																				

Qu. No.	Solution	Mark	AO	Notes																
7(a)(i)	<p>H_0: There is no association between parents knowing their child's social media passwords and age of child</p> <p>H_1: There is an association between parents knowing their child's social media passwords and age of child</p>	B1	AO3	<p>Or H_0: Parents knowing their child's social media passwords is independent of age</p> <p>H_1: Parents knowing their child's social media passwords is not independent of age</p>																
(ii)	<p>Expected values</p> <table border="1"> <thead> <tr> <th></th> <th colspan="3">Age (years)</th> </tr> <tr> <th>Parent knows password</th> <th>13</th> <th>14</th> <th>15</th> </tr> </thead> <tbody> <tr> <th>Yes</th> <td>62.79</td> <td>78.71</td> <td>76.50</td> </tr> <tr> <th>No</th> <td>79.21</td> <td>99.29</td> <td>96.50</td> </tr> </tbody> </table> <p>$142 \times \frac{275}{493} = 79.21$ OR $275 \times \frac{142}{493} = 79.21$</p>		Age (years)			Parent knows password	13	14	15	Yes	62.79	78.71	76.50	No	79.21	99.29	96.50	M1 A1	AO2 AO1	Or any equivalent correct method
	Age (years)																			
Parent knows password	13	14	15																	
Yes	62.79	78.71	76.50																	
No	79.21	99.29	96.50																	
(iii)	<p>Chi-squared contributions</p> <table border="1"> <thead> <tr> <th></th> <th colspan="3">Age (years)</th> </tr> <tr> <th>Parent knows password</th> <th>13</th> <th>14</th> <th>15</th> </tr> </thead> <tbody> <tr> <th>Yes</th> <td>2.779</td> <td>0.175</td> <td>1.180</td> </tr> <tr> <th>No</th> <td>2.203</td> <td>0.139</td> <td>0.935</td> </tr> </tbody> </table>		Age (years)			Parent knows password	13	14	15	Yes	2.779	0.175	1.180	No	2.203	0.139	0.935	M1 A1	AO2 AO1	M1A0 for one correct χ^2 contribution FT 'their observed values'
	Age (years)																			
Parent knows password	13	14	15																	
Yes	2.779	0.175	1.180																	
No	2.203	0.139	0.935																	
(iv)	2 degrees of freedom from $(3 - 1) \times (2 - 1)$	B1	AO1																	
(v)	<p>Since p-value < 0.05, Reject H_0</p> <p>Strong evidence to suggest there is an association between parents knowing their child's social media passwords and age</p>	B1 B1	AO1 AO2	B1 for < 0.05 B1 for Reject H_0																
		B1 B1	AO2 AO3	B1 for strong evidence B1 for relating back to hypothesis																
(b)	<p>Largest contribution for 13-year-olds especially for yes</p> <p>It seems more parents than expected know passwords for their 13-year-old children.</p>	E1 E1	AO2 AO2																	
		[12]																		