AS Further Mathematics Unit 3: Further Mechanics A
General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations
The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only
MR = misread
PA = premature approximation
bod = benefit of doubt
oe = or equivalent
si = seen or implied
ISW = ignore subsequent working

F.T. = follow through ($\surd$ indicates correct working following an error and $\not\surd$ indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation
A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads
When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.
This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

5. Marking codes

- 'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- 'm' marks are dependant method marks. They are only given if the relevant previous 'M' mark has been earned.
- 'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- 'S' marks are awarded for strategy
- 'E' marks are awarded for explanation
- 'U' marks are awarded for units
- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

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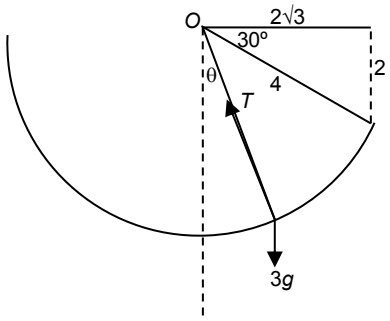
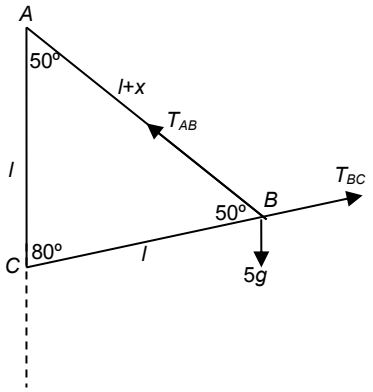
Solutions and Mark Scheme

Question Number	Solution	Mark	AO	Notes
1. (a)	Conservation of momentum	M1	AO3	Dimensionally correct
	$12 \times 600 = 1600 \times v$	A1	AO2	
	$v = \frac{9}{2}$ (ms^{-1})	A1	AO1	allow -ve
(b)	Energy considerations	M1	AO3	
	$E = 0.5 \times 12 \times 600^2 + 0.5 \times 1600 \times 4.5^2$	A1	AO2	both expressions correct, Ft v in (a)
	$E = 2160000 + 16200$ $E = \underline{2176200}$ (J)	A1	AO1	cao
	Energy dissipated by eg sound of cannon firing ignored.	B1	AO3	oe
	In actual fact, quite a lot of energy would be dissipated as sound or heat or in overcoming the friction in the barrel of the cannon.	E1	AO3	
(c)	Work-energy principle $F \times d = E$	M1	AO3	Used
	$F \times 1.2 = 16200$ $F = \underline{13500}$ (N)	A1	AO2	cao
	We have not taken into account friction or other resistance to motion which would stop the recoiling part anyway even if no external force is applied.	E1	AO3	
	So the calculated force is an over estimate of the force required.	B1	AO3	
		[12]		

Question Number	Solution	Mark	AO	Notes
2.				
(a)	For *P* falling freely $v^2 = u^2 + 2as,\ u = 0,\ s = 2,\ a = g$ $v^2 = 2g \times 2$ $v = 2\sqrt{g}$	M1 A1	AO3 AO1	
	When string tightens, *P* has vertical speed $2\sqrt{g}$. The component of this along the string is destroyed and *P* begins to move in a vertical circle with initial speed $2\sqrt{g}\cos 30°$ $= 2\sqrt{g} \times \frac{\sqrt{3}}{2} = \sqrt{3g}$	 m1 A1	 AO3 AO1	
(b) (i)	Conservation of energy $\frac{1}{2}mv^2 = \frac{1}{2}m \times 3g + mg \times 4\,(\cos 45° - \sin 30°)$ $v^2 = 3g - 4g + 8g\cos 45°$ $v^2 = 45.63717(165)$ $v = 6.76\ \text{ms}^{-1}$ $(6.7555289\ldots)$	M1 A1 A1 A1	AO3 AO1 AO1 AO1	KE and PE in dim correct equation KE PE
(ii)	N2L towards centre $T - mg\cos 45° = \frac{mv^2}{4}$ $T = 3g\left(\frac{1}{\sqrt{2}}\right) + \frac{3}{4}(45.63717)$ $T = 55.02$ (55.0168….)	M1 A1 m1 A1 **[12]**	AO3 AO2 AO1 AO1	dim correct equation substitute for v^2

Question Number	Solution	Mark	AO	Notes
3.	$\mathbf{r} = \mathbf{p} + t\mathbf{v}$	M1	AO3	Used
	$\mathbf{r}_A = (1 + 2t)\mathbf{i} + 5t\mathbf{j} - 4t\mathbf{k}$ $\mathbf{r}_B = (3 + t)\mathbf{i} + 3t\mathbf{j} - 5t\mathbf{k}$	A1	AO2	either correct, any form
	$\mathbf{r}_B - \mathbf{r}_A = (2 - t)\mathbf{i} - 2t\mathbf{j} - t\mathbf{k}$	M1	AO3	
	$AB^2 = x^2 + y^2 + z^2$	M1	AO1	
	$AB^2 = (2 - t)^2 + 4t^2 + t^2$ $(AB^2 = 6t^2 - 4t + 4)$	A1	AO1	cao
	Differentiate	M1	AO2	at least 1 power reduced
	$\frac{dAB^2}{dt} = 2(2 - t)(-1) + 10t \ (= 12t - 4)$			
	$-4 + 2t + 10t = 0$	m1	AO2	equating to 0
	$t = \frac{1}{3}$	A1	AO1	cao
	(least distance)^2 = $(2 - \frac{1}{3})^2 + 5(\frac{1}{3})^2$			
	least distance = $\sqrt{\frac{10}{3}}$ = 1.83 (m)	A1 **[9]**	AO1	cao

Question Number	Solution	Mark	AO	Notes
4. (a)	$\mathbf{v} = \dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$	M1	AO2	
	$\mathbf{v} = 2\mathrm{e}^{2t}\,\mathbf{i} + 2\cos(2t)\,\mathbf{j} - 2\sin(2t)\,\mathbf{k}$	A1	AO1	correct differentiation of any one term
		A1	AO1	all correct
	$\mathbf{v}.\mathbf{r} = 2\mathrm{e}^{4t} + 2\cos(2t)\sin(2t) - 2\sin(2t)\cos(2t)$	M1	AO2	
		A1	AO1	correct dot product
	$\mathbf{v}.\mathbf{r} = 2\mathrm{e}^{4t}$ which is never 0 Hence $\mathbf{v}$ and $\mathbf{r}$ are never perpendicular to each other	E1	AO2	
(b)	$v^2 = (2\mathrm{e}^{2t})^2 + (2\cos(2t))^2 + (-2\sin(2t))^2$	M1	AO2	
	$v^2 = 4\mathrm{e}^{4t} + 4\cos^2(2t) + 4\sin^2(2t)$ $v^2 = 4\mathrm{e}^{4t} + 4$	A1	AO1	
(c)	KE = $0.5 \times 0.4 \times (4\mathrm{e}^{4t} + 4)$ KE = $0.8(\mathrm{e}^{4t} + 1)$	B1	AO1	
(d)	WD = change in KE	M1	AO1	
	WD = $0.8(\mathrm{e}^4 + 1) - 0.8(1 + 1)$ WD = $0.8(\mathrm{e}^4 - 1) = 42.9$ (J)	A1	AO1	
(e)	Rate of work = $\dfrac{\mathrm{d}}{\mathrm{d}t}$(KE)	M1	AO2	
	Rate of work = $\dfrac{\mathrm{d}}{\mathrm{d}t}(0.8(\mathrm{e}^{4t} + 1))$			
	Rate of work = $3.2\,\mathrm{e}^{4t}$ (W)	A1	AO1	
		[13]		

Question Number	Solution	Mark	AO	Notes
5.	Resolve vertically $T\cos\theta = mg$ N2L towards centre $T\sin\theta = \frac{mv^2}{r}$ $T\sin\theta = \frac{m\times 4.8^2}{2}$ $\tan\theta = \frac{4.8^2}{2\times 9.8}$ $\theta = 49.61(2371\ldots)^\circ$	M1 A1 M1 A1 m1 A1 **[6]**	AO3 AO2 AO3 AO2 AO1 AO1	
6.				
(a)	$AB = 2\times 2\cos 50^\circ$ Hooke's Law $T_{AB} = \frac{\lambda}{2}(4\cos 50^\circ - 2) = \lambda(2\cos 50^\circ - 1)$ $T_{AB} = 0.286\lambda$ (N)	B1 M1 A1	AO3 AO2 AO1	
(b)	$EE = \frac{1}{2}\frac{\lambda\left(4\cos 50^\circ - 2\right)^2}{2}$ $EE = 0.0816\lambda$ (J)	M1 A1	AO1 AO1	
(c)	For vertical equilibrium $T_{AB}\cos 50^\circ + T_{BC}\cos 80^\circ = mg$ $T_{BC}\cos 80^\circ = 49 - \lambda 0.286\cos 50^\circ$ $T_{BC} = 282.180 - 1.057\lambda$ (N) OR For horizontal equilibrium $T_{AB}\sin 50^\circ = T_{BC}\sin 80^\circ$ $T_{BC} = \lambda(2\cos 50^\circ - 1)\times\frac{\sin 50^\circ}{\sin 80^\circ}$ $T_{BC} = 0.222\lambda$ (N)	M1 A1 m1 A1 (M1) (A1) (m1) (A1) **[9]**	AO3 AO2 AO1 AO1 (AO3) (AO2) (AO1) (AO1)	resolve vertically Resolve horizontally

Question Number	Solution	Mark	AO	Notes
7.	N2L	M1	AO3	dim correct, all forces
	$T - mg\sin\alpha - R = ma$	A1	AO2	correct equation
	$T = \frac{P}{v}$	B1	AO3	used si
	$\frac{5P}{16} - 6000 \times 9.8 \times \frac{6}{49} - R = 6000 \times 2$	A1	AO1	correct equation in P & R
	$\frac{5P}{16} - R = 19200$			
	N2L with a = 0	M1	AO3	dim correct, all forces
	$T - mg\sin\alpha - R = 0$	A1	AO2	correct equation
	$\frac{3P}{16} - 6000 \times 9.8 \times \frac{6}{49} - R = 0$	A1	AO1	correct equation in P & R
	$\frac{3P}{16} - R = 7200$			
	Solving simultaneously	m1	AO1	eliminating one variable, Dep. on both M's
	$\frac{2P}{16} = 12000$			
	$P = 96000;\ R = 10800$	A1 **[9]**	AO1	both answers cao