



Oxford Cambridge and RSA

**...day June 20XX – Morning/Afternoon**

**AS Level Mathematics A**

**H230/01 Pure Mathematics and Statistics**

**SAMPLE MARK SCHEME**

**Duration:** 1 hour 30 minutes

**MAXIMUM MARK    75**



**This document consists of 16 pages**

## Text Instructions

## 1. Annotations and abbreviations

<b>Annotation in scoris</b>	<b>Meaning</b>
✓and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
<b>Other abbreviations in mark scheme</b>	<b>Meaning</b>
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

## 2. Subject-specific Marking Instructions for A Level Mathematics A

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

### **M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

### **A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

### **B**

Mark for a correct result or statement independent of Method marks.

### **E**

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

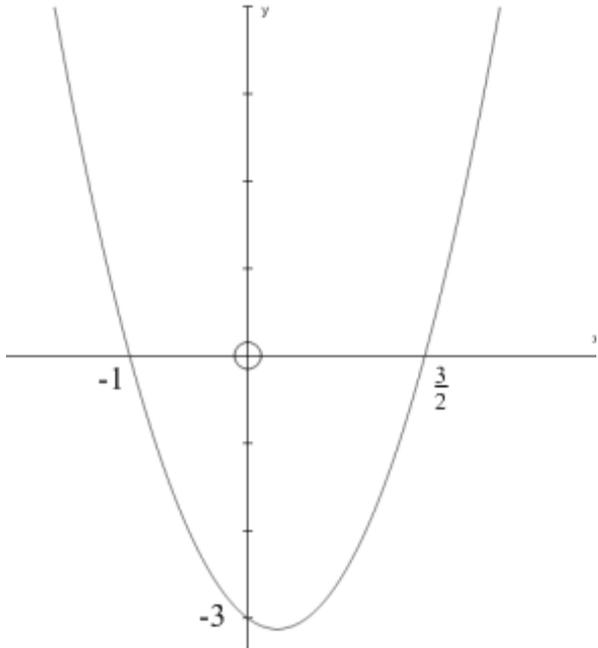
- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep\*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.  
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question		Answer	Marks	AO	Guidance	
1	(a)	$18x^2 \dots$ $-5$	<b>B1</b> <b>B1</b> [2]	<b>1.1</b> <b>1.1</b>		
1	(b)	$f''(x) = 36x$ $f''(2) = 72$	<b>M1</b> <b>A1FT</b> [2]	<b>1.1</b> <b>1.1</b>	FT their (i) FT their (i)	
2	(a)	$\left(\frac{3+9}{2}, \frac{0+8}{2}\right)$ (6, 4)	<b>M1</b> <b>A1</b> [2]	<b>1.1a</b> <b>1.1</b>	Correct working for <u>either</u> coordinate May be implied by $x = 6$ or $y = 4$	
2	(b)	Gradient of radius through B is $\frac{8-4}{9-6} = \frac{4}{3}$ Gradient of tangent is $-\frac{3}{4}$ So equation of tangent is $y = -\frac{3}{4}x + \frac{59}{4}$ oe	<b>M1</b> <b>M1</b> <b>A1</b> [3]	<b>1.1</b> <b>1.1</b> <b>2.2a</b>	FT their gradient	
3		e.g. $(2 - (-1))^2 + (10 - 6)^2$ $PQ^2 = 25, QR^2 = 162, RP^2 = 169$ $\angle PRQ = \cos^{-1} \frac{169 + 162 - 25}{2 \times 13 \times \sqrt{162}}$ $= 22.4$ to 3 sf	<b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b> [4]	<b>3.1a</b> <b>1.1</b> <b>1.1</b> <b>1.1</b>	Find at least one of $PQ^2, QR^2$ or $RP^2$ Use cosine rule to find an angle of triangle $PQR$ Accept 3 sf or better (22.38013503...)	or $PQ, QP$ or $QR$ seen

Question		Answer	Marks	AO	Guidance
4	(a)	$\frac{dy}{dx} = 6x^2 + 6x - k$ <p>At <math>x = 2</math> there is a stationary point, so <math>\frac{dy}{dx} = 0</math></p> $6 \times 2^2 + 6 \times 2 - k = 0$ $k = 36$	<b>M1</b> <b>A1</b>  <b>E1</b>  <b>M1</b>  <b>A1FT</b>  <b>[5]</b>	<b>3.1a</b> <b>1.1</b>  <b>2.1</b>  <b>1.1a</b>  <b>1.1</b>	Attempt differentiation  Explain the substitution step  Substitute $x = 2$ in their $\frac{dy}{dx} = 0$ FT their $\frac{dy}{dx} = 0$
4	(b)	$\frac{d^2y}{dx^2} = 12x + 6 \text{ and } 12 \times 2 + 6 (= 30)$ $\frac{d^2y}{dx^2} > 0 \text{ hence minimum}$	<b>M1</b>   <b>A1FT</b>    <b>[2]</b>	<b>1.1</b>   <b>2.2a</b>	Attempt differentiation again and substitute $x = 2$ , FT their $\frac{dy}{dx}$ Correct conclusion FT wwww from their $\frac{d^2y}{dx^2}$ at $x = 2$  <b>OR</b> <b>M1</b> Attempt to evaluate gradient or $y$ either side <b>A1</b> Correct values and conclusion  <b>M1</b> For a complete sketch (all intercepts and both turning points identified) <b>A1</b> for conclusion given.

Question			Answer	Marks	AO	Guidance
5	(a)		$\frac{1}{4}x^4 \dots$ $-3x^2 + c$	M1	1.1a	Attempt to integrate At least one power increases by one  Correct integral including +c
				A1	1.1	
				A1	1.1	
				[3]		
5	(b)	(i)	$\frac{4}{x^2} = 4x^{-2}$ $-4x^{-1} \dots$ $-x + c$ oe	B1	1.1	soi  Attempt to integrate a power not a positive integer Correct integral including +c Penalise omission of +c only once
				M1	1.1a	
				A1	1.1	
				[3]		
5	(b)	(ii)	$\int_1^2 \left( \frac{4}{x^2} - 1 \right) dx - \int_2^5 \left( \frac{4}{x^2} - 1 \right) dx$  Area = $2\frac{4}{5}$ oe	M1	3.1a	Both M1 and A1 may be implied by correct answer  BC  SC1 for $-\frac{4}{5}$ or $\frac{4}{5}$
				A1FT	1.1	
				A1	1.1	
				[3]		

Question		Answer	Marks	AO	Guidance
6	(a)	<p><b>DR</b></p> $f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^2 + 7\left(\frac{1}{2}\right) - 5$ $= \frac{1}{2} + 1 + \frac{7}{2} - 5 = 0$ <p>Since <math>f\left(\frac{1}{2}\right) = 0</math> therefore <math>(2x - 1)</math> is a factor</p>	<p><b>*M1</b></p> <p><b>dep*E1</b></p> <p>[2]</p>	<p><b>2.1</b></p> <p><b>2.1</b></p>	<p>Must show an intermediate line of reasoning without brackets or indices</p> <p><b>OR</b></p> <p><b>*M1</b> Attempt to divide <math>f(x)</math> by <math>(2x - 1)</math></p> <p><b>dep*E1</b> State 'No remainder, hence <math>2x - 1</math> is a factor'</p>
6	(b)	<p><b>DR</b></p> <p>Substituting <math>x = \sin \theta</math> into the equation in part (i) gives the equation in part (ii)</p> <p>so since <math>x = \frac{1}{2}</math> is a solution in part (i), <math>\sin \theta = \frac{1}{2}</math> is a solution in part (ii)</p> <p>Hence <math>\theta = 30</math> or <math>150</math></p> <p>Attempt method for finding quadratic factor in terms of <math>x</math> or <math>\sin \theta</math></p> $2x^2 + 3x + 5$ <p><math>2x^2 + 3x + 5 = 0</math> has no solutions because</p> $D = 9 - 4 \times 2 \times 5 < 0$ <p>So there are no more solutions of the given equation</p>	<p><b>M1</b></p> <p><b>E1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>E1</b></p> <p>[7]</p>	<p><b>3.1a</b></p> <p><b>3.2a</b></p> <p><b>1.1</b></p> <p><b>1.1a</b></p> <p><b>1.1</b></p> <p><b>2.1</b></p> <p><b>2.4</b></p>	<p>Connect the equations given in part (i) and (ii)</p> <p>Interpret to give a solution for the equation</p> <p>for both correct with no extras</p> <p>Attempt to obtain quadratic factor by any correct method</p> <p>Attempt to solve the quadratic factor</p> <p>Explicitly use <math>b^2 - 4ac &lt; 0</math> oe</p> <p>Must be shown</p> <p>Or consider the existence of further solutions, e.g. by calculus</p>

Question		Answer	Marks	AO	Guidance
7	(a)	$x = \frac{3}{2}, x = -1$ 	<b>B1</b>  <b>B1</b>  <b>B1</b>  <b>[3]</b>	<b>1.1</b>  <b>1.1</b>  <b>1.1</b>	BC Correct roots  Good curve: <ul style="list-style-type: none"> <li>• Correct shape, symmetrical positive quadratic</li> <li>• FT Minimum point in the correct quadrant for their roots</li> <li>• FT their <math>x</math> intercepts correctly labelled</li> </ul> $y$ intercept at $(0, -3)$  Must have a curve
7	(b)	$x \in \left(-1, \frac{3}{2}\right)$	<b>M1</b>  <b>A1FT</b>  <b>[2]</b>	<b>1.1</b>  <b>1.1</b>	Choosing the interval between their $x$ intercepts This interval identified clearly FT their $x$ values in part (i)  Other clear notation is acceptable

Question		Answer	Marks	AO	Guidance	
7	(c)	<p>No real roots implies that the discriminant is negative</p> $b^2 - 4ac = 1^2 - 4 \times 2 \times -(3+k) < 0$ $25 + 8k < 0$ $k < -\frac{25}{8}$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>[3]</b></p>	<p><b>3.1a</b></p> <p><b>1.1</b></p> <p><b>3.2a</b></p>		<p><b>OR</b></p> <p><b>M1</b> Attempt to find turning point and use <math>k &lt; y_{\min}</math></p> <p><b>A1</b> Turning point at <math>(\frac{1}{4}, -\frac{25}{8})</math></p>
8	(a)	<p>E.g. Members who attend may be of a particular type</p> <p>E.g. Absent members cannot be included</p>	<p><b>B1</b></p> <p><b>[1]</b></p>	<p><b>2.5</b></p>	<p>Any correct explanation</p> <p>Sample is not random <b>B0</b></p>	
8	(b)	<p>156, 248</p> <p>73, 181</p>	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>[2]</b></p>	<p><b>1.1</b></p> <p><b>1.1</b></p>	<p>Allow 073</p>	<p>965 must be discarded</p> <p>In <i>this</i> context do not accept a repeat of 156</p>

Question		Answer	Marks	AO	Guidance
9		$0.6 \times 0.3$ or $0.6 \times 0.1$ or $0.3 \times 0.1$  $0.6 \times 0.3 + 0.6 \times 0.1 + 0.3 \times 0.1$ oe $= 0.27$	<b>M1</b>  <b>M1</b> <b>A1</b> <b>[3]</b>	<b>3.1a</b>  <b>1.1</b> <b>1.1</b>	Any correct product seen, oe  <b>OR</b> <b>M1</b> $0.6^2 + 0.3^2 + 0.1^2 (= 0.46)$ <b>M1</b> $0.5 \times (1 - '0.46')$
10	(a)	$y^7 + 7xy^6 + 21x^2y^5 + 35x^3y^4$	<b>B2</b>  <b>[2]</b>	<b>1.1</b> <b>1.1</b>	<b>B1</b> for three terms correct
10	(b)	$21x^2y^5 = 35x^3y^4$  $\frac{x}{y} = \frac{3}{5}$ or 0.6	<b>M1</b> <b>A1</b>  <b>[2]</b>	<b>3.1a</b> <b>1.1</b>	Equate their terms in $x^2y^5$ and $x^3y^4$
10	(c)	$P(L = k) = {}_7C_k \left(\frac{3}{8}\right)^k \left(\frac{5}{8}\right)^{7-k}$  $P(L = 2) = {}_7C_2 \left(\frac{3}{8}\right)^2 \left(\frac{5}{8}\right)^5$ and $P(L = 3) = {}_7C_3 \left(\frac{3}{8}\right)^3 \left(\frac{5}{8}\right)^4$  So $P(L = 2) = 21 \times \frac{3^2 \times 5^5}{8^7} = 7 \times \frac{3^3 \times 5^5}{8^7}$ and $P(L = 3) = 35 \times \frac{3^3 \times 5^4}{8^7} = 7 \times \frac{3^3 \times 5^5}{8^7}$ so they are equal	<b>M1</b>  <b>M1</b>  <b>E1</b>  <b>[3]</b>	<b>3.3</b>  <b>3.4</b> <b>2.1</b>	Seen or implied  Attempt to find the probabilities for each case  For both values and a conclusion

Question		Answer	Marks	AO	Guidance
11	(a)		<b>B1</b>	<b>2.2b</b>	At least the three with solid rings. No extras other than those in the dashed ring.
			[1]		
11	(b)	<p>e.g. the dotted ringed group are “metropolitan districts” which have good infrastructure, so they have high proportions of travelling by bus. The solid ringed group are probably large “unitary authorities” which are not urban, so they don’t have good bus services. The unringed points are a mix of small “unitary authorities” and “non-metropolitan districts” which are difficult to tell apart with these data.</p>	<b>B1</b>	<b>2.2b</b>	For identifying (not necessarily using the diagram) the two subpopulations shown as being one in which there is a positive correlation between the two variables, and one in which larger populations do not appear to lead to increases in the proportion travelling by bus.
			<b>E1</b>	<b>1.2</b>	For identifying two distinct subpopulations in terms of the structure of the large data set
			<b>E1</b>	<b>2.3</b>	For explaining why it might be difficult to tell the others apart.
			[3]		

Question		Answer	Marks	AO	Guidance	
12	(a)	$H_0 : p = 0.097$ $H_1 : p > 0.097$ where $p$ is the proportion of patients experiencing side effects within a year $X \sim B(450, 0.097)$ and $X = 50$  $P(X \geq 51) = 1 - 0.862 = 0.138$ (3 s.f.) Comparison with 0.1 Do not reject $H_0$ No evidence (at 10% level) that proportion under new treatment greater than under standard treatment	<b>B1</b> <b>B1</b> <b>M1</b> <b>A1</b> <b>A1</b> <b>M1</b> <b>A1</b>	<b>1.1</b> <b>2.5</b> <b>3.3</b> <b>3.4</b> <b>1.1</b> <b>1.1</b> <b>2.2b</b>	Must be stated in terms of parameters Undefined $p$ <b>B1B0</b> Stated or implied BC In context, not definite, e.g. Proportion not greater <b>A0</b>	Only 0.138 seen without parameters/distribution <b>M1AO</b>  FT their 0.138, but not comparison with 0.1
12	(b)	E.g. The patients could be treated together so they are not independent, so the binomial model is not valid. E.g. The 450 patients are not a random sample from the population, so the binomial model is not valid. E.g. It is not known whether the proportion of patients experiencing side effects under the standard treatment is 9.7%, so the binomial model used may not be valid.	<b>B1</b>	<b>3.5a</b>	In context, referring to independence or random sampling. Must include a comment on appropriateness.	
			[7]			
			[1]			

Question			Answer	Marks	AO	Guidance
13	(a)		She has assumed that any car has exactly two people in it: one passenger and the driver.	<b>B1</b>	<b>2.2b</b>	Must refer to “Driving a car or van”, or equivalent
			Subtract the value in “Passenger in a car or van” from the value in “Driving a car or van” to get the number of people driving alone.	<b>B1</b>	<b>2.2a</b>	
13	(b)	(i)	The proportion using individual motorised transport in region B (56.2) is greater than region A (49.3)	<b>B1</b>	<b>2.3</b>	Or other valid reason taken from data
				[1]		
13	(b)	(ii)	The proportion using no motorised transport in region B (23.1) is greater than region A (21.5)	<b>B1</b>	<b>2.3</b>	Or other valid reason taken from data
				[1]		