GCE AS MARKING SCHEME

SUMMER 2023

AS
MATHEMATICS
UNIT 1 PURE MATHEMATICS A
2300U10-1

INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE AS MATHEMATICS

UNIT 1 PURE MATHEMATICS A

SUMMER 2023 MARK SCHEME

Q	Solution	Mark	Notes
1(a)	$1 + {}^9C_1(-3x)^1 + {}^9C_2(-3x)^2$	B1	${}^9C_1(-3x)^1$
		B1	${}^9C_2(\pm 3x)^2$, oe
	$1 - 27x + 324x^2$	B1	cao Ignore extra terms
1(b)	Put $x = 0.001$	M1	sub $x = 0.001$ into either side. si
	$(1 - 3 \times 0.001)^9$		
	$= 1 - 27(0.001) + 324(0.001)^2$	A1	correct sub, ft their (a), for equivalent difficulty
	$(0.997)^9 = 1 - 0.027 + 0.000324$		
	$(0.997)^9 = 0.973(324)$		
	$(0.997)^9 = 0.973$ to 3dp	A1	cao for their expression in (a), provided 0 < answer < 1 3dp required

Q	Solution	Mark	Notes
2	$3\sin^2\theta - 5\cos^2\theta = 2\cos\theta$		
	$3(1 - \cos^2\theta) - 5\cos^2\theta = 2\cos\theta$	M1	$\sin^2\theta + \cos^2\theta = 1$
	$8\cos^2\theta + 2\cos\theta - 3 = 0$		
	$(2\cos\theta - 1)(4\cos\theta + 3) = 0$	m1	factorisation, oe
			$ax^2 + bx + c = (dx + e)(fx + g)$
			$df = a$ and $eg = c$
	$\cos\theta = \frac{1}{2}, -\frac{3}{4}$	A1	
	$\cos\theta = \frac{1}{2}$		
	$\theta = 60°$	B1	ft
	$\theta = 300°$	B1	ft
	$\cos\theta = -\frac{3}{4}$		
	$\theta = 138.59°$	B1	ft, Accept 139°
	$\theta = 221.41°$	B1	ft, Accept 221°

<u>Notes</u>

Mark each branch separately.

FT 2 branches only if different signs.

For each branch, -1 for a 3rd root in the range $0° < \theta < 360°$,

-1 for a 4th root in the range $0° < \theta < 360°$.

Ignore roots outside the range $0° < \theta < 360°$.

Q	Solution	Mark	Notes
3(a)	Gradient of *AB* = $\frac{8-5}{3-(-2)}\left(=\frac{3}{5}\right)$	B1	
	Correct method for finding the equ *AB*	M1	
	Equation of *AB* is $y - 5 = \frac{3}{5}(x-(-2))$	A1	or $y - 8 = \frac{3}{5}(x - 3)$ ft grad *AB*, any correct form. ISW
	$5y = 3x + 31$		
3(b)	Gradient *AC* = $-\frac{5}{3}$	M1	-1/grad *AB*, ft their grad *AB*
	Equation of *AC* is $y - 5 = -\frac{5}{3}(x-(-2))$	m1	correct method
	$3y + 5x = 5$		
	At *C*, $y = 0$, $5x = 5$, $x = 1$		
	C has coordinates (1, 0)	A1	Convincing
OR			
	Assuming that C is (1,0)		
	Gradient *AC* $= \frac{5-0}{-2-1} = -\frac{5}{3}$	(M1)	
	Grad *AC* × Grad *AB* = $-\frac{5}{3}\times\frac{3}{5} = -1$	(m1)	
	Hence *AC* and *AB* are perpendicular	(A1)	
OR			
	Gradient *AC* = $-\frac{5}{3}$	(M1)	-1/grad *AB*
	C has coordinates (p, 0)		
	$\frac{5-0}{-2-p} = -\frac{5}{3}$	(m1)	
	$15 = 10 + 5p$, $p = 1$	(A1)	

Q	Solution	Mark	Notes
3(c)	$AB = \sqrt{(8-5)^2 + (3+2)^2} = \sqrt{34}$	M1	correct method for distance
	$AC = \sqrt{(0-5)^2 + (1+2)^2} = \sqrt{34}$	A1	one correct distance
	Area of *ABC* = $\frac{1}{2} \times AB \times AC$	M1	correct method for area used
	Area of *ABC* = $\frac{1}{2} \times \sqrt{34} \times \sqrt{34} = 17$	A1	cao
OR			
	Area *ABC*		
	$= \frac{1}{2}(5+8)\times(3-(-2)) - \frac{1}{2}\times3\times5 - \frac{1}{2}\times8\times2$	(M1)	
		(M1)	correct area identified
		(A1)	correct expression
	$= \frac{65}{2} - \frac{15}{2} - 8$		
	$= 17$	(A1)	cao
OR			
	Triangle *ABC* is isosceles with *AC* = *AB* and base = *BC*.		
	Midpoint of base = (2, 4)	(M1)	
	Length of base(*BC*) = $\sqrt{(3-1)^2 + (8-0)^2}$		
	$= 2\sqrt{17}$		
	Height = $\sqrt{(2 - -2)^2 + (4-5)^2} = \sqrt{17}$	(A1)	One correct length
	Area of *ABC* = $\frac{1}{2} \times$ base $\times$ height	(M1)	correct method for area used
	Area of *ABC* = $\frac{1}{2} \times 2\sqrt{17} \times \sqrt{17}$		
	$= 17$	(A1)	cao

Q	Solution	Mark	Notes
3(d)	*BC* is diameter of required circle	M1	si
	Method to find the centre	M1	
	Centre = $\left(\frac{3+1}{2}, \frac{8+0}{2}\right)$		
	Centre = (2, 4)		
	Method to find the radius	M1	from same diameter
	Radius = $\frac{1}{2}\sqrt{8^2+2^2}$		$\sqrt{4^2+1^2}$, or radius^2
	Radius = $\sqrt{17}$		
	Method for the equation of a circle	m1	Dependent on all previous 3 M1s
	$(x-2)^2+(y-4)^2=17$	A1	oe, cao, ISW
OR			
	Equation of circle is $x^2+y^2+ax+by+c=0$	(M1)	used, or $(x-p)^2+(y-q)^2=r^2$
	For $C(1, 0)$, $a+c=-1$	(A1)	one correct equation
	For $A(-2, 5)$, $-2a+5b+c=-4-25$		
	For $B(3, 8)$, $3a+8b+c=-9-64$	(A1)	3 correct equations
	Correct method for solving equations	(M1)	
	$a=-4$, $b=-8$, $c=3$	(A1)	cao
	$x^2+y^2-4x-8y+3=0$		

Q	Solution	Mark	Notes
4(a)	Attempt at long division	M1	oe, si
	$3x^2 + 11x$ (+ 34)	A1	implied by 101
	Remainder = 101	A1	cao
4(b)(i)	Attempt to use $f(-2) = 0$.	M1	
	$f(-2) = 2(-2)^3 - 3(-2)^2 + a(-2) + 6 = 0$	A1	correct equation, si
	$a = -11$	A1	

Q	Solution	Mark	Notes
4(b)(ii)	$f(x) = (x + 2)(2x^2 + px + q)$	M1	at least one of p, q correct, ft if poss. oe
	$f(x) = (x + 2)(2x^2 - 7x + 3)$	A1	
	$f(x) = (x + 2)(2x - 1)(x - 3)$		
	$x = -2$		
	$x = \frac{1}{2}$	A1	or $x = 3$
	$x = 3$	A1	all three roots
OR			
	Use of factor theorem where $x \neq -2$	(M1)	
	1st correct root $\neq -2$	(A1)	
	2nd correct root $\neq -2$	(A1)	
	All three roots	(A1)	
OR for (b)(i) and (b)(ii)			
	$2x^3 - 3x^2 + ax + 6 = (x + 2)(2x^2 + px + q)$	(M1)	
	Comparing coefficients	(M1)	
	For x^2: $-3 = 4 + p$; $p = -7$	(A1)	
	constant term $6 = 2q$; $q = 3$	(A1)	
	$f(x) = (x + 2)(2x^2 - 7x + 3)$		$(x - 3)(2x^2 + 3x - 2)$, $(2x - 1)(x^2 - x - 6)$
	$f(x) = 2x^3 - 3x^2 - 11x + 6$		
	$a = -11$	(A1)	
	$f(x) = (x + 2)(2x - 1)(x - 3)$	(A1)	
	$x = -2, \frac{1}{2}, 3$	(A1)	

Q	Solution	Mark	Notes
5	$\sqrt[3]{512a^2} - \dfrac{a^{\frac{7}{6}} \times a^{-\frac{1}{3}}}{a^{\frac{1}{6}}}$		
	$\sqrt[3]{512a^2} = 8a^{\frac{2}{3}}$	B1	oe
	$\dfrac{a^{\frac{7}{6}} \times a^{-\frac{1}{3}}}{a^{\frac{1}{6}}} = a^{\left(\frac{7}{6} - \frac{1}{3} - \frac{1}{6}\right)}$	B1	some correct simplification of indices
	$= a^{\frac{2}{3}}$	B1	2nd term correct, oe
	$\sqrt[3]{512a^2} - \dfrac{a^{\frac{7}{6}} \times a^{-\frac{1}{3}}}{a^{\frac{1}{6}}} = 8a^{\frac{2}{3}} - a^{\frac{2}{3}}$		
	$= 7a^{\frac{2}{3}}$ or $7\sqrt[3]{a^2}$	B1	cao

Q	Solution	Mark	Notes
6	Cosine rule used correctly	M1	
	$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos B$		
	$(4+\sqrt{3})^2 = (3)^2 + (2\sqrt{5})^2 - 2(3)(2\sqrt{5})\cos B$	A1	All correct
	$19 + 8\sqrt{3} = 9 + 20 - 12\sqrt{5}\cos B$	B1	$16 + 8\sqrt{3} + 3$
		B1	9 and 20
		B1	$12\sqrt{5}$
	$12\sqrt{5}\cos B = 10 - 8\sqrt{3}$		
	$\cos B = \frac{10 - 8\sqrt{3}}{12\sqrt{5}}$		
	$\cos B = \frac{5 - 4\sqrt{3}}{6\sqrt{5}}$	A1	$a = 5$
		A1	$b = 4$
			If A0A0, award A1 for $\cos B = \frac{10 - 8\sqrt{3}}{12\sqrt{5}}$ or $\cos B = \frac{-10 + 8\sqrt{3}}{-12\sqrt{5}}$
			ISW

Q	Solution	Mark	Notes
7(a)(i)	$2x^2 + 5x - 12 = mx - 14$	M1	
	$2x^2 + (5 - m)x + 2 = 0$	A1	Allow $2x^2 + 5x - mx + 2 = 0$
	Discriminant = $(5 - m)^2 - 4\times2\times2$	m1	si
	For tangent discriminant = 0	m1	used
	$25 - 10m + m^2 - 16 = 0$		
	$m^2 - 10m + 9 = 0$	A1	convincing
7(a)(ii)	$(m - 1)(m - 9) = 0$		oe $(5 - m) = \pm4$
	$m = 1, 9$	B1B1	
	When $m = 1$ when $m = 9$		
	$2x^2+5x-12 = x-14$ or $2x^2+5x-12 = 9x-14$	B1	
	$2x^2 + 4x + 2 = 0$ or $2x^2 - 4x + 2 = 0$		
	$(x + 1)^2 = 0$ or $(x - 1)^2 = 0$	B1	si
	$x = -1$ and $x = 1$	B1	or (–1, –15) or (1, –5)
	$y = -15$ and $y = -5$		
	Points are (–1, –15) and (1, –5)	B1	2nd correct pair

OR for final 4 B1 marks

	$m = 1, \frac{dy}{dx} = 4x + 5 = 1$ $(x = -1)$	(B1)	
	$m = 9, \frac{dy}{dx} = 4x + 5 = 9$ $(x = 1)$	(B1)	
	$x = -1$ and $x = 1$	(B1)	or (–1, –15) or (1, –5)
	$y = -15$ and $y = -5$		
	Points are (–1, –15) and (1, –5)	(B1)	2nd correct pair

Q	Solution	Mark	Notes

<u>Alternative solution for Q7 (using the gradient function)</u>

7(a)(i)	At point of intersection		
	$2x^2 + 5x - 12 = mx - 14$	(M1)	
	Gradient of curve = $\frac{dy}{dx} = 4x + 5$	(m1)	
	When line is tangent, $4x + 5 = m$	(A1)	
	$x = \frac{m-5}{4}$		
	$2\left(\frac{m-5}{4}\right)^2 + 5\left(\frac{m-5}{4}\right) - 12 = m\left(\frac{m-5}{4}\right) - 14$	(A1)	
	$m^2 - 10m + 9 = 0$	(A1)	convincing
7(a)(ii)	$2x^2 + 5x - 12 = mx - 14$	(M1)	
	At point of contact, $m = 4x + 5$	(A1)	
	$2x^2 + 5x - 12 = (4x + 5)x - 14$	(m1)	
	$2x^2 - 2 = 0$		
	$(x + 1)(x - 1) = 0$	(m1)	or $x^2 = 1$
	$x = -1, 1$	(A1)	one correct pair
	$y = -15, -5$	(A1)	all correct
7(b)	For 2 distinct points of intersection		
	Discriminant > 0	M1	used, si
	$(m - 1)(m - 9) > 0$		OR $5 - m > 4$ or $5 - m < -4$
	$m < 1$ or $m > 9$	A1	condone ‘,’, or nothing A0 for ‘and’ A0 for non-strict inequality Mark final answer

Q	Solution	Mark	Notes
8	$n = 3$	M1	correct value of n (e.g. 5, 7, 8)
	$n^2 + 1 = 3^2 + 1 = 10$	A1	correct value (e.g. 26, 50, 65)
	10 (= 2 × 5) is not a prime number,		
	hence the statement is false.	A1	concluding statement Condone one of ‘statement is false’ or e.g. ‘10 is not a prime number’

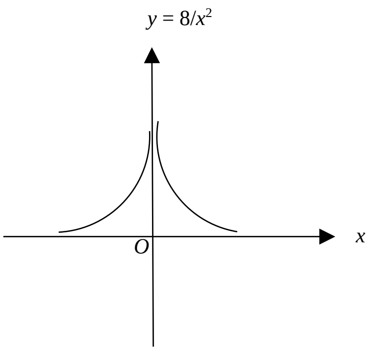
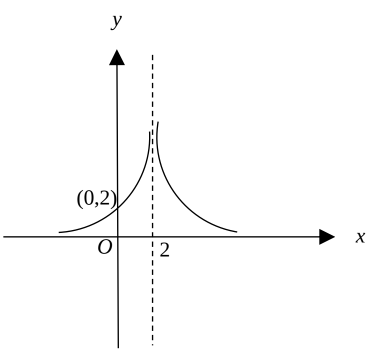
Q	Solution	Mark	Notes
9(a)	$y + \delta y = (x + \delta x)^2 - 3(x + \delta x)$	B1	
	$y + \delta y = x^2 + 2x(\delta x) + (\delta x)^2 - 3x - 3\delta x$		
	Subtract $y = x^2 - 3x$ from $y + \delta y$	M1	
	$\delta y = 2x\delta x + (\delta x)^2 - 3\delta x$	A1	
	$\frac{\delta y}{\delta x} = 2x + \delta x - 3$		
	$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$	M1	$\frac{dy}{dx} = \lim_{\delta x \to 0}(2x + \delta x - 3)$
	$\frac{dy}{dx} = 2x - 3$	A1	All correct

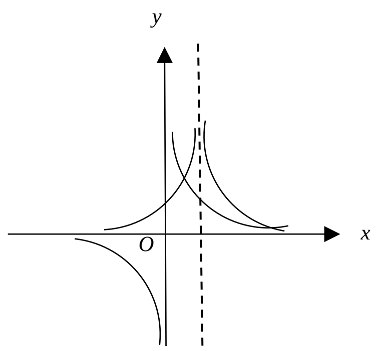
OR

	Solution	Mark	Notes
	$f(x + h) = (x + h)^2 - 3(x + h)$	(B1)	
	$f(x + h) = x^2 + 2xh + h^2 - 3x - 3h$		
	$f(x + h) - f(x) = 2xh + h^2 - 3h$	(M1A1)	
	$\frac{f(x+h)-f(x)}{h} = 2x + h - 3$		
	$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$	(M1)	$f'(x) = \lim_{h \to 0}(2x + h - 3)$
	$f'(x) = 2x - 3$	(A1)	All correct

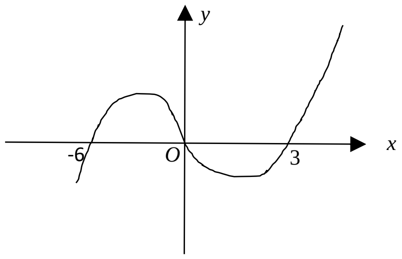
Q	Solution	Mark	Notes
9(b)(i)	$f(x) = 4x^{\frac{3}{2}} + \frac{6}{\sqrt{x}}$		
	$f'(x) = 4 \times \frac{3}{2} \times x^{\frac{1}{2}} + 6 \times (-\frac{1}{2}) \times x^{-\frac{3}{2}}$	B1	one correct term
		B1	second correct term ISW
	$f'(x) = 6x^{\frac{1}{2}} - 3x^{-\frac{3}{2}}$		
9(b)(ii)	$f'(x) > 0$		
	$6x^{\frac{1}{2}} - 3x^{-\frac{3}{2}} > 0$		
	Multiplying by $x^{\frac{3}{2}}$: $6x^2 - 3x^0 > 0$	M1	oe eg $3x^{\frac{1}{2}}(2 - x^{-2})$ FT similar expression Allow $\le$, $<$, $=$, $\ge$
	$x^2 > 0.5$	A1	Allow $\le$, $<$, $=$, $\ge$, but must be same as in previous M1 FT similar expression
	For increasing function $f'(x) > 0$	M1	used Allow $f'(x) \ge 0$
	$x > (0.5)^{\frac{1}{2}} = 0.707106\ldots..$		
	$k = 0.71$	A1	cao needs 2 dp Condone $x = 0.71$

Q	Solution	Mark	Notes
10(a)	$2x + 5 = e^3$	M1	Correctly removing ln
	$x = \frac{1}{2}(e^3 - 5)$ (= 7.5427…..)	A1	ISW, Accept 7.54 Answer only, M0
10(b)	$(2x + 1)\ln 5 = \ln 14$	M1	oe $2x\ln 5 = \ln\left(\frac{14}{5}\right)$
	$2x = \frac{\ln 14}{\ln 5} - 1$	A1	isolating x term
	$x = \frac{1}{2}\left(\frac{\ln 14}{\ln 5} - 1\right)$ (= 0.31(98….))	A1	ISW, Accept 0.32 Answer only, M0
OR			
	$2x + 1 = \log_5 14$	(M1)	
	$2x = \log_5 14 - 1$	(A1)	isolating x term
	$x = \frac{1}{2}(\log_5 14 - 1)$ (= 0.31(98….))	(A1)	ISW, Accept 0.32 Answer only, M0
10(c)	$\log_7\left(\frac{8x^3 \times x}{8x^2}\right) = 4$	B1	one use power law
		B1	one use addition law
		B1	one use subtraction law
		B1	$\log_3 81 = 4$, si
	$\log_7 x^2 = 4$, $2\log_7 x = 4$		
	$\log_7 x = 2$	B1	$x^2 = 7^4$
	$x = 49$	B1	B0 for ±49

Q	Solution	Mark	Notes
11(a)		B2	B1 each branch
11(b)		M1	ft shift entire graph to the right
		B1	(0, 2) cao
		A1	x = 2 as **asymptote**

Q	Solution	Mark	Notes
11(c)			
		B1	correct curve $y = \frac{8}{x}$, both branches. May be seen in (b).
	Equation has two solutions	B1	award only if both graphs correct in first quadrant.

Q	Solution	Mark	Notes
12(a)	**AB** = **b** – **a**	M1	used
	AB = 8**i** + 4**j**	A1	any notation ISW
12(b)(i)	$\lvert\mathbf{a}\rvert = \sqrt{(-3)^2 + 4^2} = 5$	B1	si
	Unit vector = $-\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$	B1	oe
12(b)(ii)	Position vector of *C* is $7(-\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j})$	B1	oe, ft from (b)(i), provided vector is not **a**, **b** or **AB**.
	$= (-\frac{21}{5}\mathbf{i} + \frac{28}{5}\mathbf{j})$		
12(c)	$AOB = 180° - \tan^{-1}\left(\frac{8}{5}\right) - \tan^{-1}\left(\frac{4}{3}\right)$	M1	oe
	$AOB = 180° - 57.99° - 53.13°$	B1	any correct relevant angle, si
	$AOB = 68.9°$ (68.875…)	A1	
OR			
	angle $AOB = \tan^{-1}\left(\frac{5}{8}\right) + \tan^{-1}\left(\frac{3}{4}\right)$	(M1)	oe
	angle $AOB = 32.01° + 36.87°$	(B1)	any correct relevant angle, si
	angle $AOB = 68.9°$ (68.875…)	(A1)	
OR			
	$OA = \sqrt{(-3)^2 + 4^2} = \sqrt{25}$		
	$OB = \sqrt{5^2 + 8^2} = \sqrt{89}$		
	$AB = \sqrt{8^2 + 4^2} = \sqrt{80}$	(B1)	all correct
	$80 = 25 + 89 - 2\times 5 \times \sqrt{89}\cos\theta$	(M1)	correct use of cosine rule with their distances
	$\cos\theta = \frac{25 + 89 - 80}{10\sqrt{89}} = 0.3603992792$		
	angle AOB == 68.9° (68.875 ...)	(A1)	

Q	Solution	Mark	Notes
13(a)	$4\frac{x^{\frac{1}{3}}}{\frac{1}{3}} + \frac{5}{4}x^4 + 7x + C$	B3	B1 each term ISW –1 if no +C
13(b)			
	Curve cuts x-axis when $x = -6, 0, 3$	B1	maybe seen on sketch, may be implied by limits
	$f(x) = x^3 + 3x^2 - 18x$	B1	
	$A_1 = \int_{-6}^{0}(x^3 + 3x^2 - 18x)\,\mathrm{d}x$	M1	attempt to integrate, limits not required. Or $\int_{0}^{3}(x^3 + 3x^2 - 18x)\,\mathrm{d}x$
	$= \left[\frac{x^4}{4} + x^3 - 9x^2\right]_{-6}^{0}$	A1	correct integration, ft similar expression, limits not required
	$= (0) - \left(\frac{(-6)^4}{4} + (-6)^3 - 9 \times (-6)^2\right)$	m1	correct use of limits, either –6 and 0, or 0 and 3
	$= 216$	A1	Must be from –6 to 0 Only FT for $f(x) = x^3 - 3x^2 - 18x$ $\left(\int_{-6}^{0} f(x)\mathrm{d}x = -216\right)$ or $f(x) = x^3 + 3x^2 + 18x$ $\left(\int_{-6}^{0} f(x)\mathrm{d}x = -432\right)$

Q	Solution	Mark	Notes
13(b)	(continued)		
	$A_2 = \left[\frac{x^4}{4} + x^3 - 9x^2\right]_0^3$		
	$= \left(\frac{3^4}{4} + 3^3 - 9 \times 3^2\right) - (0)$		
	$= -\frac{135}{4} = -33.75$	A1	allow (+)33.75, Only FT for $f(x) = x^3 - 3x^2 - 18x$ $\left(\int_0^3 f(x)\mathrm{d}x = -87.75\right)$ or $f(x) = x^3 + 3x^2 + 18x$ $\left(\int_0^3 f(x)\mathrm{d}x = 128.25\right)$
	Total area $= 216 + \frac{135}{4}$	m1	si
	Total area $= \frac{999}{4} = 249.75$	A1	cao

Note:

Must be supported by workings.

If M0, award SC1 for sight of 216 **and** ±33.75, OR SC2 for 249.75

Q	Solution	Mark	Notes
14(a)	$y = Ae^{-kx}$ or $y = Ae^{kx}$	B1	oe Accept numerical values for $A \neq 0$, and/or $k \neq 0$.
14(b)(i)	$Y = 5e^{-kt}$	B1	for $A = 5$
	$1.25 = 5e^{-4k}$		
	$e^{-4k} = 0.25$		
	$k = -\frac{1}{4}\ln(0.25) = 0.3465(735903)$	B1	Convincing, answer given Allow verification
14(b)(ii)	$0.6 = 5e^{-0.3466t}$	M1	
	$e^{-0.3466t} = 0.12$		
	$t = \frac{\ln(0.12)}{-0.3466}$ (= 6.12 (hours))	A1	
	Additional time (= 6.12 – 4) = 2.12 (hours)	A1	oe e.g. hours and minutes, ISW Award A1 for "their 6.12" – 4, provided "their 6.12" > 4