

A Level Further Mathematics A

Y541 Pure Core 2

Sample Question Paper

Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 30 minutes

You must have:

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:

- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1 Find $\sum_{r=1}^n (r+1)(r+5)$. Give your answer in a fully factorised form. [4]

2 **In this question you must show detailed reasoning.**

The finite region R is enclosed by the curve with equation $y = \frac{8}{\sqrt{16+x^2}}$, the x -axis and the lines $x=0$ and $x=4$. Region R is rotated through 360° about the x -axis. Find the exact value of the volume generated. [4]

- 3 (i) Find $\sum_{r=1}^n \left(\frac{1}{r} - \frac{1}{r+2} \right)$. [3]

(ii) What does the sum in part (i) tend to as $n \rightarrow \infty$? Justify your answer. [1]

- 4 It is given that $\frac{5x^2+x+12}{x^3+kx} \equiv \frac{A}{x} + \frac{Bx+C}{x^2+k}$ where k , A , B and C are positive integers. Determine the set of possible values of k . [5]

5 **In this question you must show detailed reasoning.**

Evaluate $\int_0^\infty 2xe^{-x} dx$.

[You may use the result $\lim_{x \rightarrow \infty} xe^{-x} = 0$.] [4]

- 6 The equation of a plane Π is $x - 2y - z = 30$.

(i) Find the acute angle between the line $\mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 3 \\ 2 \end{pmatrix}$ and Π . [4]

(ii) Determine the geometrical relationship between the line $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ and Π . [4]

- 7 (i) Use the Maclaurin series for $\sin x$ to work out the series expansion of $\sin x \sin 2x \sin 4x$ up to and including the term in x^5 . [4]

(ii) Hence find, in exact surd form, an approximation to the least positive root of the equation $2\sin x \sin 2x \sin 4x = x$. [3]

- 8 The equation of a curve is $y = \cosh^2 x - 3 \sinh x$. Show that $\left(\ln \left(\frac{3 + \sqrt{13}}{2} \right), -\frac{5}{4} \right)$ is the only stationary point on the curve. [8]

- 9 A curve has equation $x^4 + y^4 = x^2 + y^2$, where x and y are not both zero.

(i) Show that the equation of the curve in polar coordinates is $r^2 = \frac{2}{2 - \sin^2 2\theta}$. [4]

- (ii) Deduce that no point on the curve $x^4 + y^4 = x^2 + y^2$ is further than $\sqrt{2}$ from the origin. [2]

- 10 Let $C = \sum_{r=0}^{20} \binom{20}{r} \cos r\theta$. Show that $C = 2^{20} \cos^{20} \left(\frac{1}{2} \theta \right) \cos 10\theta$. [8]

- 11 During an industrial process substance X is converted into substance Z . Some of the substance X goes through an intermediate phase, and is converted to substance Y , before being converted to substance Z . The situation is modelled by

$$\frac{dy}{dt} = 0.3x - 0.2y \quad \text{and} \quad \frac{dz}{dt} = 0.2y + 0.1x$$

where x , y and z are the amounts in kg of X , Y and Z at time t hours after the process starts.

Initially there is 10 kg of substance X and nothing of substances Y and Z . The amount of substance X decreases exponentially. The initial rate of decrease is 4 kg per hour.

- (i) Show that $x = Ae^{-0.4t}$, stating the value of A . [3]
- (ii) (a) Show that $\frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} = 0$. [2]
- (b) Comment on this result in the context of the industrial process. [2]
- (iii) Express y in terms of t . [5]
- (iv) Determine the maximum amount of substance Y present during the process. [3]
- (v) How long does it take to produce 9 kg of substance Z ? [2]

END OF QUESTION PAPER

Copyright Information:

OCR is committed to seeking permission to reproduce all third-party content that it uses in the assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact OCR, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.