

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Sample Question Paper

Version 2

Date – Morning/Afternoon

Time allowed: 2 hours

You must have:

- Printed Answer Booklet

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient Rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{1}{2}\theta^2, \tan \theta \approx \theta \text{ where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2} h \{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B) \quad \text{or} \quad P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Sample variance

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^n C_r p^r q^{n-r}$ where $q = 1 - p$

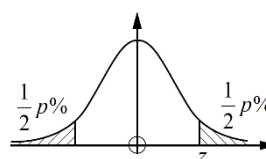
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2$$

$$s = \frac{1}{2} (u + v) t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2} at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2} \mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2} (\mathbf{u} + \mathbf{v}) t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2} \mathbf{a}t^2$$

Answer **all** the questions

Section A (23 marks)

- 1 **Fig. 1** shows a sector of a circle of radius 7 cm. The area of the sector is 5 cm^2 .

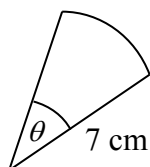


Fig. 1

Find the angle θ in radians.

[2]

- 2 A geometric series has first term 3. The sum to infinity of the series is 8.
Find the common ratio.

[3]

- 3 Solve the inequality $|2x - 1| \geq 4$.

[4]

- 4 Differentiate the following.

(a) $\sqrt{1 - 3x^2}$

[3]

(b) $\frac{x^2}{3x + 2}$

[3]

- 5 A woman is pulling a loaded sledge along horizontal ground. The only resistance to motion of the sledge is due to friction between it and the ground.



Fig. 5

At first, she pulls with a force of 100 N inclined at 32° to the horizontal, as shown in **Fig.5**, but the sledge does not move.

- (a) Determine the frictional force between the ground and the sledge.
Give your answer correct to 3 significant figures. [2]
- (b) Next she pulls with a force of 100 N inclined at a smaller angle to the horizontal. The sledge still does not move.

Compare the frictional force in this new situation with that in part (a), justifying your answer. [2]

6 Fig. 6 shows a partially completed spreadsheet.

This spreadsheet uses the trapezium rule with four strips to estimate $\int_0^{\frac{1}{2}\pi} \sqrt{1 + \sin x} \, dx$.

	A	B	C	D	E
1		x	$\sin x$	y	
2	0	0.0000	0.0000	1.0000	0.5000
3	0.125	0.3927	0.3827	1.1759	1.1759
4	0.25	0.7854	0.7071	1.3066	1.3066
5	0.375	1.1781	0.9239	1.3870	1.3870
6	0.5	1.5708	1.0000	1.4142	0.7071
7					5.0766
8					

Fig. 6

(a) Show how the value in cell B3 is calculated. [1]

(b) Show how the values in cells D2 to D6 are used to calculate the value in cell E7. [1]

(c) Complete the calculation to estimate $\int_0^{\frac{1}{2}\pi} \sqrt{1 + \sin x} \, dx$.

Give your answer to 3 significant figures. [2]

Answer **all** the questions

Section B (77 marks)

7 In this question take $g = 10$.

A small stone is projected from a point O with a speed of 26 m s^{-1} at an angle θ above the horizontal. The initial velocity and part of the path of the stone are shown in **Fig. 7**.

You are given that $\sin \theta = \frac{12}{13}$.

After t seconds the horizontal displacement of the stone from O is x metres and the vertical displacement is y metres.

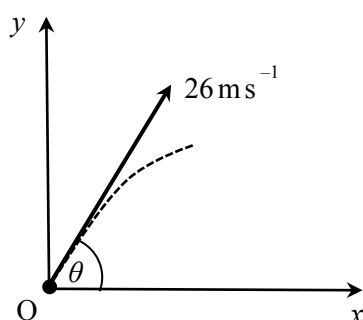


Fig. 7

(a) Using the standard model for projectile motion,

- show that $y = 24t - 5t^2$,
- find an expression for x in terms of t .

[4]

The stone passes through a point A. Point A is 16 m above the level of O.

(b) Find the two possible horizontal distances of A from O.

[4]

A toy balloon is projected from O with the same initial velocity as the small stone.

(c) Suggest two ways in which the standard model could be adapted.

[2]

8 Find $\int x^2 e^{2x} dx$. [7]

- 9 In an experiment, a small box is hit across a floor. After it has been hit, the box slides without rotation.

The box passes a point A. The distance the box travels after passing A before coming to rest is S metres and the time this takes is T seconds.

The only resistance to the box's motion is friction due to the floor. The mass of the box is m kg and the frictional force is a constant F N.

- (a) (i) Find the equation of motion for the box while it is sliding.

(ii) Show that $S = kT^2$ where $k = \frac{F}{2m}$. [4]

- (b) Given that $k = 1.4$, find the value of the coefficient of friction between the box and the floor. [4]

- 10** In a certain region, the populations of grey squirrels, P_G and red squirrels P_R , at time t years are modelled by the equations:

$$P_G = 10\,000(1 - e^{-kt})$$

$$P_R = 20\,000e^{-kt}$$

where $t \geq 0$ and k is a positive constant.

- (a) (i) On the axes in your Printed Answer Book, sketch the graphs of P_G and P_R on the same axes.
- (ii) Give the equations of any asymptotes. [4]
- (b) What does the model predict about the long term population of
- grey squirrels
 - red squirrels? [2]

Grey squirrels and red squirrels compete for food and space. Grey squirrels are larger and more successful than red squirrels.

- (c) Comment on the validity of the model given by the equations, giving a reason for your answer. [1]
- (d) Show that, according to the model, the rate of decrease of the population of red squirrels is always double the rate of increase of the population of grey squirrels. [4]
- (e) When $t = 3$, the numbers of grey and red squirrels are equal. Find the value of k . [4]

11 Fig. 11 shows the curve with parametric equations

$$x = 2 \cos \theta, \quad y = \sin \theta, \quad 0 \leq \theta \leq 2\pi.$$

The point P has parameter $\frac{1}{4}\pi$. The tangent at P to the curve meets the axes at A and B.

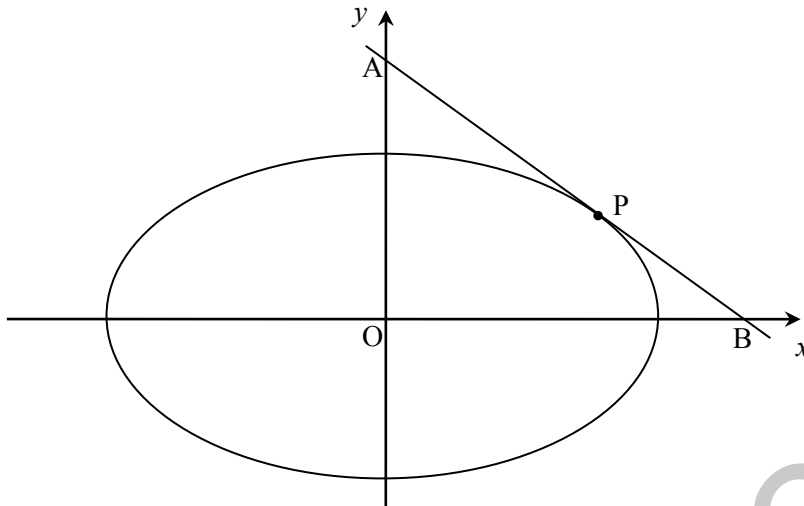


Fig. 11

(a) Show that the equation of the line AB is $x + 2y = 2\sqrt{2}$. [6]

(b) Determine the area of the triangle AOB. [3]

12 A model boat has velocity $\mathbf{v} = ((2t-2)\mathbf{i} + (2t+2)\mathbf{j}) \text{ m s}^{-1}$ for $t \geq 0$, where t is the time in seconds. \mathbf{i} is the unit vector east and \mathbf{j} is the unit vector north. When $t = 3$, the position vector of the boat is $(3\mathbf{i} + 14\mathbf{j}) \text{ m}$.

(a) Show that the boat is never instantaneously at rest. [2]

(b) Determine any times at which the boat is moving directly northwards. [2]

(c) Determine any times at which the boat is north-east of the origin. [5]

13 In this question you must show detailed reasoning.

Determine the values of k for which part of the graph of $y = x^2 - kx + 2k$ appears below the x -axis.

[4]

- 14** Blocks A and B are connected by a light rigid horizontal bar and are sliding on a rough horizontal surface.

A light horizontal string exerts a force of 40 N on B.

This situation is shown in **Fig. 14**, which also shows the direction of motion, the mass of each of the blocks and the resistances to their motion.

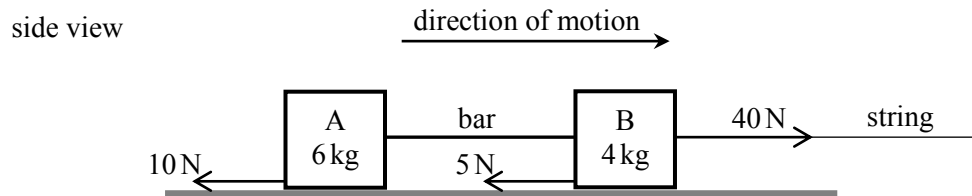


Fig. 14

- (a) Calculate the tension in the bar. [4]

The string breaks while the blocks are sliding. The resistances to motion are unchanged.

- (b) Determine [5]
- the magnitude of the new force in the bar,
 - whether the bar is in tension or in compression.

- 15** **Fig. 15** shows a uniform shelf AB of weight W N. The shelf is 180 cm long and rests on supports at points C and D. Point C is 30 cm from A and point D is 60 cm from B.

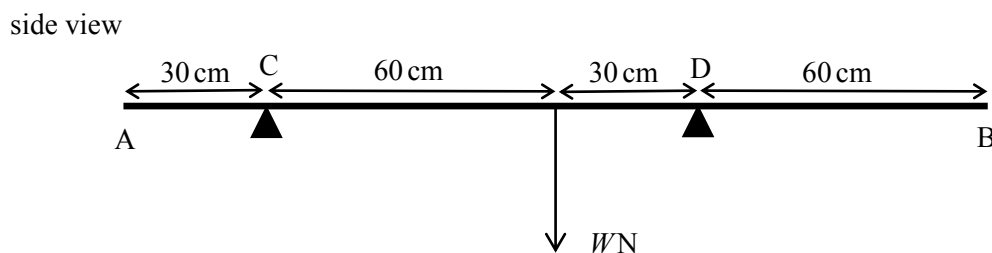


Fig. 15

Determine the range of positions a point load of $3W$ could be placed on the shelf without the shelf tipping. [6]

END OF QUESTION PAPER

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Specimen

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