**GCE**

**MATHEMATICS**

**UNIT 3: PURE MATHEMATICS B**

**SAMPLE ASSESSMENT MATERIALS**

**(2 hour 30 minutes)**

## ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

• a 12 page answer book;
• a Formula Booklet;
• a calculator.

## INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. Find a small positive value of $x$ which is an approximate solution of the equation.

$$\cos x - 4\sin x = x^2.$$ [4]

2. Air is pumped into a spherical balloon at the rate of 250 cm$^3$ per second. When the radius of the balloon is 15 cm, calculate the rate at which the radius is increasing, giving your answer to three decimal places [3]

3. (a) Sketch the graph of $y = x^2 + 6x + 13$, identifying the stationary point. [2]

   (b) The function $f$ is defined by $f(x) = x^2 + 6x + 13$ with domain $(a, b)$.

   (i) Explain why $f^{-1}$ does not exist when $a = -10$ and $b = 10$. [1]

   (ii) Write down a value of $a$ and a value of $b$ for which the inverse of $f$ does exist and derive an expression for $f^{-1}(x)$. [5]

4. (a) Expand $(1-x)^{-\frac{1}{2}}$ in ascending power of $x$ as far as the term in $x^2$. State the range of $x$ for which the expansion is valid. [2]

   (b) By taking $x = \dfrac{1}{10}$, find an approximation for $\sqrt{10}$ in the form $\dfrac{a}{b}$, where $a$ and $b$ are to be determined.

   [2]

5. Aled decides to invest £1000 in a savings scheme on the first day of each year. The scheme pays 8% compound interest per annum, and interest is added on the last day of each year. The amount of savings, in pounds, at the end of the third year is given by

$$1000 \times 1\cdot08 + 1000 \times 1\cdot08^2 + 1000 \times 1\cdot08^3$$

   Calculate, to the nearest pound, the amount of savings at the end of thirty years. [5]

6. The lengths of the sides of a fifteen-sided plane figure form an arithmetic sequence. The perimeter of the figure is 270 cm and the length of the largest side is eight times that of the smallest side. Find the length of the smallest side. [4]

7. The curve $y = ax^4 + bx^3 + 18x^2$ has a point of inflection at (1, 11).

(a) Show that $2a + b + 6 = 0$. [2]

(b) Find the values of the constants $a$ and $b$ and show that the curve has another point of inflection at (3, 27). [8]

(c) Sketch the curve, identifying all the stationary points including their nature. [6]

8. (a) Integrate

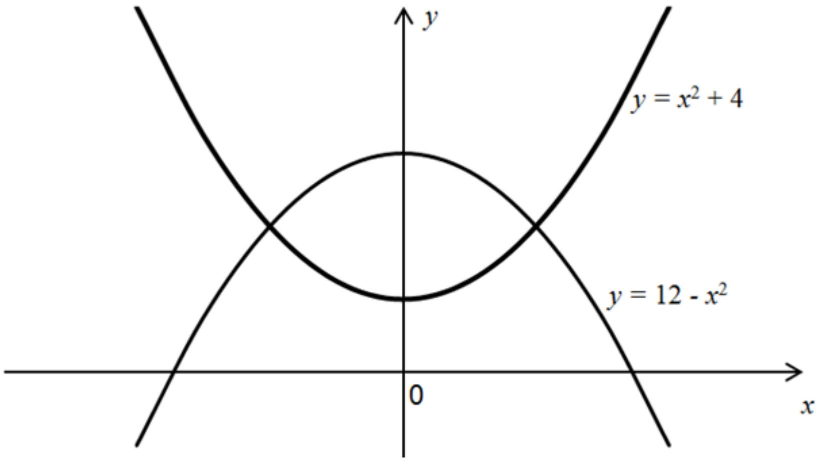
(i) $e^{-3x+5}$ [2]

(ii) $x^2 \ln x$ [4]

(b) Use an appropriate substitution to show that

$$\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx = \frac{\pi}{12} - \frac{\sqrt{3}}{8}.$$ [8]

9.




The diagram above shows a sketch of the curves $y = x^2 + 4$ and $y = 12 - x^2$.

Find the area of the region bounded by the two curves. [6]

10. The equation

$$1+5x-x^4=0$$

has a positive root $\alpha$.

(a) Show that $\alpha$ lies between 1 and 2. [2]

(b) Use the iterative sequence based on the arrangement

$$x=\sqrt[4]{1+5x}$$

with starting value 1.5 to find $\alpha$ correct to two decimal places. [3]

(c) Use the Newton-Raphson method to find $\alpha$ correct to six decimal places. [6]

11. (a) The curve *C* is given by the equation

$$x^4+x^2y+y^2=13.$$

Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point (-1, 3). [4]

(b) Show that the equation of the normal to the curve $y^2=4x$ at the point $P(p^2,2p)$ is

$$y+px=2p+p^3.$$

Given that $p\neq 0$ and that the normal at *P* cuts the $x$-axis at $B(b,0)$, show that $b>2$. [7]

12. (a) Differentiate $\cos x$ from first principles. [5]

(b) Differentiate the following with respect to $x$, simplifying your answer as far as possible.

(i) $\dfrac{3x^2}{x^3+1}$ [2]

(ii) $x^3\tan 3x$ [2]

13. (a) Solve the equation

$$\text{cosec}^2 x + \cot^2 x = 5$$

for $0^\circ \le x \le 360^\circ$. [5]

(b) (i) Express $4\sin\theta + 3\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0^\circ \le \alpha \le 90^\circ$. [4]

(ii) Solve the equation

$$4\sin\theta + 3\cos\theta = 2$$

for $0^\circ \le \theta \le 360^\circ$, giving your answer correct to the nearest degree. [3]

14. (a) A cylindrical water tank has base area 4 $\text{m}^2$. The depth of the water at time $t$ seconds is $h$ metres. Water is poured in at the rate 0.004 $\text{m}^3$ per second. Water leaks from a hole in the bottom at a rate of 0.0008$h$ $\text{m}^3$ per second. Show that

$$5000\frac{\mathrm{d}h}{\mathrm{d}t} \equiv 5 - h.$$ [2]

[Hint: the volume, $V$, of the cylindrical water tank is given by $V = 4h$.]

(b) Given that the tank is empty initially, find $h$ in terms of $t$. [7]

(c) Find the depth of the water in the tank when $t = 3600$ s, giving your answer correct to 2 decimal places. [1]

15. Prove by contradiction the following proposition.

When $x$ is real and positive,

$$4x + \frac{9}{x} \ge 12.$$

The first line of the proof is given below.

*Assume that there is a positive and a real value of* $x$ *such that*

$$4x + \frac{9}{x} < 12.$$ [3]