OCR
Oxford Cambridge and RSA

Accredited

AS Level Further Mathematics A

Y531 Pure Core

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You may use:
- a scientific or graphical calculator

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INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

The equation $x^2 + 2x + 5 = 0$ has roots α and β. The equation $x^2 + px + q = 0$ has roots α^2 and β^2. Find the values of p and q. **[3]**

2 **In this question you must show detailed reasoning.**

Given that $z_1 = 3 + 2\text{i}$ and $z_2 = -1 - \text{i}$, find the following, giving each in the form $a + b\text{i}$.

(i) $z_1^* z_2$ **[2]**

(ii) $\dfrac{z_1 + 2z_2}{z_2}$ **[2]**

3 **(i)** You are given two matrices, **A** and **B**, where

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}.$$

Show that $\mathbf{AB} = m\mathbf{I}$, where m is a constant to be determined. **[2]**

(ii) You are given two matrices, **C** and **D**, where

$$\mathbf{C} = \begin{pmatrix} 2 & 1 & 5 \\ 1 & 1 & 3 \\ -1 & 2 & 2 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} -4 & 8 & -2 \\ -5 & 9 & -1 \\ 3 & -5 & 1 \end{pmatrix}.$$

Show that $\mathbf{C}^{-1} = k\mathbf{D}$ where k is a constant to be determined. **[2]**

(iii) The matrices **E** and **F** are given by $\mathbf{E} = \begin{pmatrix} k & k^2 \\ 3 & 0 \end{pmatrix}$ and $\mathbf{F} = \begin{pmatrix} 2 \\ k \end{pmatrix}$ where k is a constant.

Determine any matrix **F** for which $\mathbf{EF} = \begin{pmatrix} -2k \\ 6 \end{pmatrix}$. **[5]**

4 Draw the region of the Argand diagram for which $|z - 3 - 4\text{i}| \leq 5$ and $|z| \leq |z - 2|$. **[4]**

5 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$.

(i) The diagram in the Printed Answer Booklet shows the unit square *OABC*. The image of the unit square under the transformation represented by **M** is $OA'B'C'$. Draw and clearly label $OA'B'C'$. **[3]**

(ii) Find the equation of the line of invariant points of this transformation. **[3]**

(iii) (a) Find the determinant of **M**. **[1]**

(b) Describe briefly how this value relates to the transformation represented by **M**. **[2]**

6 At the beginning of the year John had a total of £2000 in three different accounts. He has twice as much money in the current account as in the savings account.

- The current account has an interest rate of 2.5% per annum.
- The savings account has an interest rate of 3.7% per annum.
- The supersaver account has an interest rate of 4.9% per annum.

John has predicted that he will earn a total interest of £92 by the end of the year.

(i) Model this situation as a matrix equation. **[2]**

(ii) Find the amount that John had in each account at the beginning of the year. **[2]**

(iii) In fact, the interest John will receive is £92 **to the nearest pound**. Explain how this affects the calculations. **[2]**

7 **In this question you must show detailed reasoning.**

It is given that $\mathrm{f}(z) = z^3 - 13z^2 + 65z - 125$.

The points representing the three roots of the equation $\mathrm{f}(z) = 0$ are plotted on an Argand diagram.

Show that these points lie on the circle $|z| = k$, where *k* is a real number to be determined. **[9]**

8 Prove that $n! > 2^n$ for $n \geq 4$. **[5]**

9 **(i)** Find the value of k such that $\begin{pmatrix}1\\2\\1\end{pmatrix}$ and $\begin{pmatrix}-2\\3\\k\end{pmatrix}$ are perpendicular. **[2]**

Two lines have equations $l_1 : \mathbf{r} = \begin{pmatrix}3\\2\\7\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\3\end{pmatrix}$ and $l_2 : \mathbf{r} = \begin{pmatrix}6\\5\\2\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$.

(ii) Find the point of intersection of l_1 and l_2. **[4]**

(iii) The vector $\begin{pmatrix}1\\a\\b\end{pmatrix}$ is perpendicular to the lines l_1 and l_2.

Find the values of a and b. **[5]**

END OF QUESTION PAPER