

AS Level Further Mathematics A

Y531 Pure Core

Sample Question Paper

Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:

- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You may use:

- a scientific or graphical calculator



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INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 In this question you must show detailed reasoning.

The equation $x^2 + 2x + 5 = 0$ has roots α and β . The equation $x^2 + px + q = 0$ has roots α^2 and β^2 . Find the values of p and q .

[3]

2 In this question you must show detailed reasoning.

Given that $z_1 = 3 + 2i$ and $z_2 = -1 - i$, find the following, giving each in the form $a + bi$.

(i) $z_1^* z_2$ [2]

(ii) $\frac{z_1 + 2z_2}{z_2}$ [2]

3 (i) You are given two matrices, \mathbf{A} and \mathbf{B} , where

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}.$$

Show that $\mathbf{AB} = m\mathbf{I}$, where m is a constant to be determined. [2]

(ii) You are given two matrices, \mathbf{C} and \mathbf{D} , where

$$\mathbf{C} = \begin{pmatrix} 2 & 1 & 5 \\ 1 & 1 & 3 \\ -1 & 2 & 2 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} -4 & 8 & -2 \\ -5 & 9 & -1 \\ 3 & -5 & 1 \end{pmatrix}.$$

Show that $\mathbf{C}^{-1} = k\mathbf{D}$ where k is a constant to be determined. [2]

(iii) The matrices \mathbf{E} and \mathbf{F} are given by $\mathbf{E} = \begin{pmatrix} k & k^2 \\ 3 & 0 \end{pmatrix}$ and $\mathbf{F} = \begin{pmatrix} 2 \\ k \end{pmatrix}$ where k is a constant.

Determine any matrix \mathbf{F} for which $\mathbf{EF} = \begin{pmatrix} -2k \\ 6 \end{pmatrix}$. [5]

4 Draw the region of the Argand diagram for which $|z - 3 - 4i| \leq 5$ and $|z| \leq |z - 2|$. [4]

5 The matrix \mathbf{M} is given by $\mathbf{M} = \begin{pmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$.

(i) The diagram in the Printed Answer Booklet shows the unit square $OABC$. The image of the unit square under the transformation represented by \mathbf{M} is $OA'B'C'$. Draw and clearly label $OA'B'C'$. [3]

(ii) Find the equation of the line of invariant points of this transformation. [3]

(iii) (a) Find the determinant of \mathbf{M} . [1]

(b) Describe briefly how this value relates to the transformation represented by \mathbf{M} . [2]

6 At the beginning of the year John had a total of £2000 in three different accounts. He has twice as much money in the current account as in the savings account.

- The current account has an interest rate of 2.5% per annum.
- The savings account has an interest rate of 3.7% per annum.
- The supersaver account has an interest rate of 4.9% per annum.

John has predicted that he will earn a total interest of £92 by the end of the year.

(i) Model this situation as a matrix equation. [2]

(ii) Find the amount that John had in each account at the beginning of the year. [2]

(iii) In fact, the interest John will receive is £92 **to the nearest pound**. Explain how this affects the calculations. [2]

7 **In this question you must show detailed reasoning.**

It is given that $f(z) = z^3 - 13z^2 + 65z - 125$.

The points representing the three roots of the equation $f(z) = 0$ are plotted on an Argand diagram.

Show that these points lie on the circle $|z| = k$, where k is a real number to be determined. [9]

8 Prove that $n! > 2^n$ for $n \geq 4$. [5]

- 9 (i) Find the value of k such that $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ k \end{pmatrix}$ are perpendicular. [2]

Two lines have equations $l_1 : \mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ and $l_2 : \mathbf{r} = \begin{pmatrix} 6 \\ 5 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

- (ii) Find the point of intersection of l_1 and l_2 . [4]

- (iii) The vector $\begin{pmatrix} 1 \\ a \\ b \end{pmatrix}$ is perpendicular to the lines l_1 and l_2 .

Find the values of a and b .

[5]

END OF QUESTION PAPER

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