AS LEVEL
MATHEMATICS

7356/2 Paper 2
Report on the Examination

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General

This year's paper proved to be more accessible than in 2022, and this was reflected in a rise in the average score obtained by students. There was good work seen in questions 3, 4, 7 and 8 in Section A as well as questions 15, 16 and 17 in Section B.

Students are reminded to look for links between parts of questions which are designed to provide structure to help them in their responses. This was the case in question 3, for example, where the integration in part (a) was required to find the equation of the curve in part (b).

Students must take care to ensure that their responses are clear and legible, there were cases where examiners were challenged when trying to read what had been written.

The level of algebraic skills was improved in this session, but still too many numerical and sign errors were made.

There is evidence that students are now more aware of the Large Data Set and have some knowledge of its contents and the characteristics of its data.

Question 1

This question, involving the simplification of an expression using the laws of logarithms, proved to be a good start for the majority of students, with around 75% of all students correctly identifying $3a$ as the correct answer.

Question 2

This question proved to be more challenging than expected, with around a third of students correctly identifying $-\frac{3}{5}$ as the correct answer. The ratio for $\cos\theta$ could be obtained from $\sin\theta$ using a right-angled triangle or even directly from a calculator using $\cos(\sin^{-1} 0.8)$. However, the calculator gave 0.6 using this method, which may explain why the majority of students incorrectly identified $\cos\theta$ as $\frac{3}{5}$, not recognising that as θ was an obtuse angle, $\cos\theta$ was negative.

Question 3

This question was generally well done. In part (a), the majority of students were able to integrate the expression successfully, although there were errors made in re-writing $\frac{8}{x^2}$ as $8x^{-2}$ before integrating, and a significant number of students omitted the constant of integration, which had consequences for part (b).

In part (b), the majority of students recognised that to find the equation of a curve from $\frac{\mathrm{d}y}{\mathrm{d}x}$ it was necessary to integrate. The integration needed was done in part (a), so the equation of the curve was $y=\frac{x^4}{2}-\frac{8}{x}+c$. The constant c was found by substituting x = 2 and y = 0 from the given point which gave the final equation $y=\frac{x^4}{2}-\frac{8}{x}-4$

A small minority of students incorrectly substituted x = 0 and y = 2 which received no credit.

Students who omitted the constant of integration in part (a) were unable to score any marks in part (b) unless they recovered the constant in their working.

Question 4

This question proved to be a significant test of applying the laws of logarithms. Most students were able to correctly simplify at least one of:

- $\ln(x+1)+\ln(x-1)=\ln((x+1)(x-1))$
- $2\ln 7=\ln 7^2=\ln 49$
- $\ln 15-\ln 49=\ln\left(\frac{15}{49}\right)$

Applying these rules to the given equation led to $\ln(x^2-1)=\ln\left(\frac{15}{49}\right)$ which when solved gave $x=\pm\frac{8}{7}$. Very few students recognised the need to check the validity of these solutions.

When $x=-\frac{8}{7}$, $x-1<0$ so $\ln(x-1)$ is undefined. Hence the only valid solution was $x=\frac{8}{7}$.

A few students didn't put brackets around the x^2-1 term which led to some confusion when dealing with their $\ln x^2-1$ term.

Question 5

This question incorporated some basic trigonometry knowledge along with the rationalisation of surds.

Most students correctly identified that $\tan 15°=\frac{\sin 15°}{\cos 15°}$ and used the given ratios to obtain

$$\tan 15°=\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}$$

To get the required form of the answer, it was necessary to multiply the top and bottom of the fraction by the conjugate of the denominator, i.e. $\sqrt{6}-\sqrt{2}$

This crucial step, along with the subsequent simplification, proved to be challenging for many students. The numerator and denominator could then be multiplied out to give

$$\frac{6-\sqrt{12}-\sqrt{12}+2}{\sqrt{6}+\sqrt{12}-\sqrt{12}-2} = \frac{8-2\sqrt{12}}{4} = 2-\sqrt{3} \text{ as } \sqrt{12}=2\sqrt{3}$$

Errors noted in this question included:

- simplifying $\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}$ on the calculator without any working, which received no credit
- multiplying top and bottom by $\sqrt{6}+\sqrt{2}$

Question 6

Most students made a good start to this question by correctly equating the right-hand sides of the two equations and rearranging to form a quadratic equation. As the line cut the curve at two distinct points the discriminant of this equation was greater than zero, so $(p-5)^2-24>0$.

The range of values of p, in exact form, could then be obtained from the simplified quadratic inequality, most simply by expanding and using a calculator to obtain the solution of the inequality directly.

This gave $p < 5-\sqrt{26}$ or $p > 5+\sqrt{26}$.

Some students seemed unaware of the discriminant, or thought that x was part of the discriminant, and so made little progress, however there were many fully correct solutions seen, which displayed confident algebraic skills.

The main errors noted were:

- errors rearranging the quadratic equation
- using the wrong inequality signs in the final answer

Question 7

This question on transformation of curves was well received, with each part being answered correctly by over half of all students.

In part (a), the translation was commonly identified, the only issue was using $a + 2$ for the x-coordinate rather than $a - 2$.

In part (b), the stretch in the y-direction moved the maximum point to (a, $8b$). A few students incorrectly moved the x-coordinate to $4a$.

In part (c), some students stated the coordinate after the transformation rather than the scale factor of the stretch as was requested.

Question 8

There was a noticeable improvement in the overall quality of responses to this proof question compared with previous series. There were some excellent solutions seen which displayed strong algebraic and logic skills.

Many students were careless in defining their consecutive odd numbers. If $k - 1$ and $k + 1$ were used, then it had to be stated that k was even to ensure that the two numbers were definitely odd. The most progress was made by students who used either $2k - 1$ and $2k + 1$ or $2k + 1$ and $2k + 3$. Each of these odd numbers had to be cubed and then added. To complete the proof a factor of 4 had to be clearly identified along with a short conclusion summarising what had been shown. Students who used $k - 1$ and $k + 1$ had to use the fact that k was even to complete their proof successfully.

Unfortunately, there was still a significant minority of students who stated a few numerical examples and thought the proof had been completed by exhaustion. This method received no credit.

Errors noted in this question included:

- defining non-consecutive odd numbers
- defining one odd and one even number
- using $k – 1$ and $k + 1$ without stating that k was even and then not using the fact that k was even to finish proving the final result
- expanding the cubes of the numbers incorrectly

Question 9

This question proved to be challenging for students, although good marks were still picked up in parts (a) and (b).
In part (a), as the result was given, there was no credit given for simply validating the formula by substituting $n = 10$ and $P = 24$. The given inverse proportion relationship between P and n had to be used to form an equation with a constant. Then $P = 24$ and $n = 10$ could be substituted to obtain the constant of proportionality and hence the given result.
In part (b), a similar approach to part (a) was initially required. This time the relationship could be expressed as $C = \frac{k}{n^2}$. Substituting $C = 60$ and $n = 10$ gave $k = 6000$.
The inequality $\frac{240}{n} > \frac{6000}{n^2}$ could then be solved leading to $n > 25$.
An alternative approach to solving the inequality was a numerical trial and error method using different values of n in the formulae for P and C, finding that when $n = 24$, $P < C$ and when $n = 25$, $P > C$. This method received full credit.

Errors in part (b) included:

- Incorrectly calculating 60×10^2 as 600
- Giving the final answer as $n = 25$ or $n \geq 25$

In part (c), it was necessary to interpret the solution to part (b) for the artist in real terms. If the artist sells more than 25 items, they will make a profit. Some students just stated **make** more than 25 items, but clearly no profit can be made unless the items are sold.

Question 10

This optimisation question proved to be extremely challenging for many students. Some marks were obtained in part (a), but over 25% of all students made no attempt at part (b), despite being directed to use calculus and having the formula to work with from part (a)(ii).
In part (a)(i) the area of the equilateral triangle could be found using $\frac{1}{2}ab\sin C$, with $a = b = x$ and $C = 60^\circ$. Some students did not show enough working to justify the given answer.
In part (a)(ii) the total area A was the sum of the areas of the rectangle and the equilateral triangle. The area of the rectangle was correctly stated as xy by most students, however many did not

realise that the perimeter of 66 could be used to express y in terms of x. Rearranging $3x + 2y = 66$ enabled y to be substituted into the area formula to give $A = x\left(33 - \frac{3}{2}x\right) + \frac{\sqrt{3}}{4}x^2$.

This could then be expanded and simplified to obtain the given result. Some sign errors were made during this process.

In part (b) there was a clear direction in the question to use calculus. As a maximum value for the area was needed, this required differentiation. There were some good attempts at the differentiation which gave $\frac{\mathrm{d}A}{\mathrm{d}x} = 33 - \frac{1}{2}\left(6 - \sqrt{3}\right)x$. 'Fully justify your answer' indicated it was necessary to make a statement such as 'at a stationary point $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$'. Solving and rearranging $33 - \frac{1}{2}\left(6 - \sqrt{3}\right)x = 0$, gave $x = \frac{66}{6 - \sqrt{3}}$, which could be evaluated on a calculator to obtain $x = 12 + 2\sqrt{3}$.

To finish, it was necessary to show that the area was a maximum at this value of x using any appropriate method: the simplest was to show $\frac{\mathrm{d}^2A}{\mathrm{d}x^2} = -\frac{1}{2}\left(6 - \sqrt{3}\right) < 0$, indicating a maximum.

Only a small number of students achieved full marks for this question.

A few students tried to use integration rather than differentiation, and some made sign errors when differentiating. This led to negative values for x which should have encouraged the checking of the work as x could clearly not be negative.

Question 11

There were some good responses in part (a) of this question. Many students recognised that the radius of the circle passing through P was perpendicular to the tangent at P. The equation of L_1 could then be rearranged to get its gradient, and from that the gradient of the radius could be obtained using the perpendicular gradients rule. Finally, the equation of the radius could be found using any appropriate method to obtain the required answer of $y = 7x - 37$.

There was no credit given for just validating the given equation by substituting $x = 6$ and $y = 5$ to show that the coordinates of the point P satisfied the equation of L_1.

Part (b) proved to be very challenging, with many unable to make any progress. To find the equation of the circle, a clear strategy to find the centre and radius was needed. To find the centre, the equation of the line through $Q(0, 3)$ perpendicular to the line $y = x + 3$ could be found. The two radii intersect at the centre of the circle, so solving their equations simultaneously (using a calculator) gave the centre of the circle as (5, -2). The radius of the circle could then be found using the distance formula from (5, -2) to either (0, 3) or (6, 5), to obtain $\sqrt{50}$. The final equation of the circle could then be stated as $\left(x - 5\right)^2 + \left(y + 2\right)^2 = 50$.

A correct alternative approach was to find the equation of the perpendicular bisector of PQ to obtain the equation of a different line passing through the centre.

Question 12

This question proved to be difficult. Only 35% identified the mass of an individual bag of nuts as the continuous variable.

Question 13

This question was very well done. Over 70% of students were able to identify the correct frequency density of 2.

Question 14

Students are strongly advised to describe a method for generating a simple random sample using random numbers from a calculator. Firstly, the employees needed to be numbered 1 to 93. Secondly, random integers in the range 1 to 93 needed to be chosen from a calculator (or random number generator.) Finally, some explanation of how to deal with repeats was required along with some reference to obtaining 20 different employees.

Despite comments in previous examination reports, there were still examples of students drawing bits of paper out of hats or boxes, or using wheels and spinners, etc. These methods received no credit unless the above points were clearly covered.

Question 15

This question was very well done by the vast majority of students. In part (a) for an even number to be obtained it was just necessary for the ball drawn out of bowl C to be even. Most students recognised that there were 3 balls in bowl C that would lead to an even number, however, some students did not realise that 0 is an even number.

In part (b) to obtain the number 703 you needed 7 from Bowl A, 0 from Bowl B and 3 from Bowl C. The three probabilities corresponding to these outcomes could then be multiplied together to obtain the answer $\frac{1}{48}$.

In part (c), there were two numbers that could be obtained that were divisible by 111, namely 222 and 333. The probabilities for each of these ($\frac{1}{288}$ and $\frac{1}{96}$) had to be evaluated separately before being added to obtain $\frac{1}{72}$. 75% of all students earned full marks on this part of the question.

Question 16

There were some very good attempts at this question. Most students recognised that it was necessary to find the value of p. For a discrete probability distribution, the sum of the probabilities must equal 1. This led to $5p + 0.65 = 1$, giving $p = 0.07$. The values of $P(X \geq 3)$ and $P(Y \leq 4)$ could then be found. The final step was to compare these two probabilities and conclude that the claim was correct. There were some careless errors seen in combining the probabilities which prevented some students achieving full marks for this question, but, nevertheless, nearly half of all students did achieve full marks.

Question 17

In part (a) most students correctly found the mean to be 23.7, but a significant number thought that the mean had to be an integer, and so rounded the figure to 23 or 24. On this occasion this was accepted, provided the value of 23.7 was seen before rounding.

Parts (b) and (c) were well answered.

In questions such as part (d) students are advised to consider exactly what range is needed in terms of X before using the calculator. As $P(X > 26)$ was requested, this means, in effect, that $P(X \geq 27)$ is required. This can then be adjusted to evaluate on the calculator.
$P(X > 26) = P(X \geq 27) = 1 – P(X \leq 26) = 0.0984$

Question 18

This was a different type of question on hypothesis testing which would have been unfamiliar to students who may have been expecting the typical full hypothesis test for p. It was only required to state the Null and Alternative hypotheses, and to then interpret the given result of the test in context. The test was to see if there had been an increase in the proportion, so this was a one-tailed test for p. For part (a), the Null and Alternative hypotheses were given by: H_0: $p = 0.25$ and H_1: $p > 0.25$

In part (b) some students did not seem familiar with the term 'critical region', and hence concluded incorrectly. It was necessary to state that as the test statistic was in the critical region, H_0 is rejected. This needed to be followed by a statement summarising what this meant in context, such as 'there is insufficient evidence to suggest that the proportion of customers buying a loaf of bread had increased'. This conclusion had to include 'proportion' (or percentage) of customers not number of customers as well as requiring 'insufficient evidence' or similar. It should be noted that it cannot be said that providing the free samples has or has not caused any change in the proportion. You cannot apply 'causality' to any hypothesis test. The inclusion of such statements was condoned on this occasion. This part proved to be challenging to obtain full marks, with many students getting caught up in descriptive terms about the bread itself, such as its taste.

Question 19

This question started off with a box plot before moving on to questions linked to the Large Data Set. Students are now more aware of the Large Data Set and the information it contains, as well as some of the more unusual characteristics of the data itself. In part (a) the median could be read from box plot A. Any value from 1650 to 1675 was accepted.

In part (b)(i), the claim that Box Plot B was incorrectly drawn was due to the implication from the box plot that at least one car has a mass of 0 kg. It had to be clear in the student's response that '0' was a mass or, alternatively, 0 kg had to be stated.

For part (b)(ii), the claim is not valid because the LDS does contain masses of 0 kg for convertible cars: to get the mark both a comment about the masses of 0 kg and the validity of the claim was required.

Part (c) required more detailed knowledge of the LDS to be able to say that Box Plot B must be from 2002, because that is the only one of the two years included in the LDS where there are any recorded masses of 0 kg. Other relevant comments, with clear reference to box plot B, such as there having been an increase in average mass from 2002 to 2016 were accepted.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.